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Magnetic field growth in accretion disc: growth and nonlinear saturation

E.A. Mikhailov[1,2,3], E.N. Zhikhareva[2], M.V. Frolova[2]

[1]*Department of Theoretical Physics, P.N. Lebedev Physical Institute of RAS, Leninskii pr. 53, Moscow, 119991 Russia*
[2]*Faculty of Physics, M.V. Lomonosov Moscow State University, Leninskie gory 1 2, Moscow, 119991 Russia*
[3]*Skolkovo Institute of Science and Technology, Bolshoy Boulevard 30 1, Moscow, 121205 Russia*

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Astrophysical accretion discs associated with massive objects should have some magnetic fields. There are strong arguments that such fields should be, at least partly, connected with dynamo action that is based on properties of small- and large-scale motions in the object. Now there are some works connected with studies of the accretion disc dynamos, and it has been shown that the magnetic field can grow there. However, there is a question connected with field generation and its timescale. The properties of the accretion disc are quite different for various parts of the object, so the growth rate can differ dramatically, too. Another problem is connected with nonlinear saturation connected with the field conservation law. Here we describe different approaches to estimate the field growth and describe its structure.

Keywords: Magnetic fields, growth rate, thin disc model, *RZ* approximation

1 Introduction

Accretion discs play an important role in relativistic astrophysics [1–4]. They surround compact massive objects (black holes, neutron stars, white dwarfs) and contain rapidly rotating medium which can be strongly magnetized. The arguments for the existence of the magnetic field are based on the magnetohydrodynamic processes: they can explain transition of the angular momentum and the medium [1]. Also there are some proofs of the magnetic field at least for such a specific object as an accretion disc surrounding the black hole in the active nuclei of M 87 galaxy [5]. Radioastronomical observations show that there is a remarkable Faraday rotation for electromagnetic waves passing from it, so there should be a magnetic field that produces the rotation

*Email: e.mikhajlov@lebedev.ru

of the polarization plane. We do not know how suitable this example is for other objects, but it gives us strong evidence that the field can take place in accretion discs, too.

The origin of the magnetic field can be explained using different mechanisms. One of them is connected with transition with falling medium [6]. The field is frozen into the fluid, so we can apologize that it can be a source for the magnetic field. However, strong random turbulent motions can prevent transition of the regular magnetic field. Other suggestions are connected with interaction between the central object and the disc. There are some works that have shown that this effect is not very strong [7]. Basically it can be connected with the direction of the radial motions in the disc. Parts of the ionized medium move from the disc plane to the central object. So we cannot say that the field passes from the disc to the neutron star or black hole. Opposite effects can be associated with decretion discs [8], but it is a subject of another research study. One of the most interesting approaches is connected with so-called magnetorotational instability. First of all, it was proposed by Velikhov [9] and developed by Chandrasekhar [10]. As for astrophysical objects significant results are presented in [11] by Balbus and Hawley. It has been shown [12, 13] that magnetorotational instability can be important for accretion discs, too. Finally, the effect can be based on the large-scale dynamo action in the accretion disc. It is based on the joint action of the large-scale rotation of the disc and helicity of the turbulent motions.

As for the dynamo, it is usually described by a mathematically complicated process which is based on a system of three-dimensional equations [14]. Large-scale fields are connected with so-called mean field dynamo equation, also called Steenbeck–Krause–Rädler equation [15]. It is quite difficult to solve both theoretically and numerically. So, for most astrophysical objects where the magnetic fields are described by this law, the simplified models are used. Usually the researchers try to take some two-dimensional approximations. One of the oldest examples is connected with classical Parker dynamo which has been developed for a spherical layer modelling convective zone of the Sun [16]. As for the accretion discs it is useful to take the approaches that have been developed for galactic discs. It can be assumed that the disc is quite thin, so its thickness can be taken as a small parameter. This leads us to a pair of differential equations which give us an opportunity to estimate the field evolution [17, 18]. The theoretical approximations are not very difficult, also computations are not connected with extremely large resources and can be carried out using the quite modest possibilities of a personal computer. Also we can use methods of spectral analysis of the differential operator to describe the field evolution and give the estimates for the field growth rate. The thin disc model has been used for accretion discs magnetism and gave proper results [19, 20]. The thin disc approximation describes the field depending on two coordinates (distance from the central object and azimuthal angle). However, as for the galaxies it has been shown that the azimuthal structures are destroyed by strong rotation [22, 23]. Here we study the possibility of the existence of such structures in accretion discs.

Although the thin disc model gives convenient estimates, it has some serious flaws. It is based on the fact that the disc thickness is much smaller than its radius. Also it is recommended that these parameters do not change across the objects. (Sometimes we can take expanding discs using the thin disc model, but we note that it changes quite slowly.) As for the galaxies it is quite applicable: at least for the main part

the difference between thicknesses is smaller than their typical values. However, for the accretion discs the thickness can change dramatically and reach the numbers that are at least comparable with the radius. So for this case, we should use the more complicated model. For such problems an RZ model has been developed [24, 25]. It is quasi-two-dimensional, too: the field is described by its azimuthal component and other ones are principally smaller. Sometimes we should take the azimuthal component of the vector potential, but the main effects are realized even in this reduced case. The problem for this model is connected with much more difficult methods of the eigenvalues calculation. So, sometimes the spectra of the differential operator can be found only numerically or using some perturbation methods [25].

It is important that the field growth rate can be different for various regions of the disc. Concrete values depend on the model, but principally similar results can be given by both approaches.

Another problem which takes place both for the thin disc and RZ model is connected with nonlinear saturation. The energy conservation law forbids infinite magnetic field growth [14]. It is restricted by the transition of the turbulent motions energy to the magnetic field energy. When these values become comparable, the field growth stops. It is quite important to study the typical times of this field growth. They can be estimated using both theoretical estimates and numerical modelling.

The paper is organized as follows. First of all, we formulate thin disc approximation and its equations. After that we give a short description of analytical results describing the magnetic field evolution based on spectral analysis of the corresponding differential operators. Then we discuss numerical results showing the field evolution. Special attention is paid to evolution of the axisymmetric and non-axisymmetric structures and comparison of their stability. We show that the azimuthal details are destroyed as it has been shown for galaxies previously. After that we describe the RZ model. We describe the basic features of the magnetic field from the numerical point of view and compare the results with previous model. One of the most important features of this model is the possibility to study field evolution in the expanding disc. It is connected with other eigenvalues for corresponding differential operators and different rate for the magnetic field growth.

2 Thin disc approximation

Magnetic field evolution is connected with magnetohydrodynamics equations. If we are speaking about a regular magnetic field which can be obtained averaging the structures on lengthscales of turbulence, we should use some mean values. This leads to a mean field dynamo equation [15] (also called Steenbeck–Krause–Rädler equation). In the thin disc approximation [18], it is assumed that $\mathbf{B} \sim \cos\left(\frac{\pi z}{2h}\right)$, and $\alpha \sim z$. So the equations for the plane components of the magnetic field will be:

$$\frac{\partial B_r}{\partial t} = -\frac{\Omega l^2}{h^2} B_\varphi - \nu \frac{\pi^2 B_r}{4h^2} +$$

$$+\nu \left(\frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_r}{\partial \varphi^2} - \frac{1}{r^2}\frac{\partial B_\varphi}{\partial \varphi} \right), \quad (1)$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{d\Omega}{dr}B_r - \nu\frac{\pi^2 B_\varphi}{4h^2} +$$
$$+\nu\left(\frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_\varphi}{\partial \varphi^2} + \frac{1}{r^2}\frac{\partial B_r}{\partial \varphi}\right), \quad (2)$$

where Ω is the angular velocity, l is the turbulence lengthscale, ν is the diffusion coefficient, h is the half-thickness. Here radial B_r and azimuthal B_φ components of the magnetic field are studied.

Here we should take into account that $h = h_0\,(r/R)^{9/8}$, $\Omega = \Omega_0\,(r/R)^{-3/2}$ and $\rho = \rho_0\,(r/R)^{-21/16}$ where h_0, Ω_0 and ρ_0 are the values of the parameters for outer boundary of the accretion disc [20].

The radial and angular component of the Laplace operator (in brackets in (1) and (2)) are smaller than other parts of the equations. Also we can measure times in units of $\frac{h^2}{\nu}$, so the approximate equations will be:

$$\frac{dB_r}{dt} = -R_\alpha B_\varphi - \frac{\pi^2 B_r}{4}, \quad (3)$$

$$\frac{dB_\varphi}{dt} = -R_\omega B_r - \frac{\pi^2 B_\varphi}{4}, \quad (4)$$

where $R_\alpha = \frac{\Omega l^2}{\nu}$ characterizes alpha-effect and $R_\omega = -r\frac{d\Omega}{dr}\frac{h^2}{\nu}$ is the differential rotation. The field grows according to an exponential law $\sim \exp\gamma t$. As for the growth

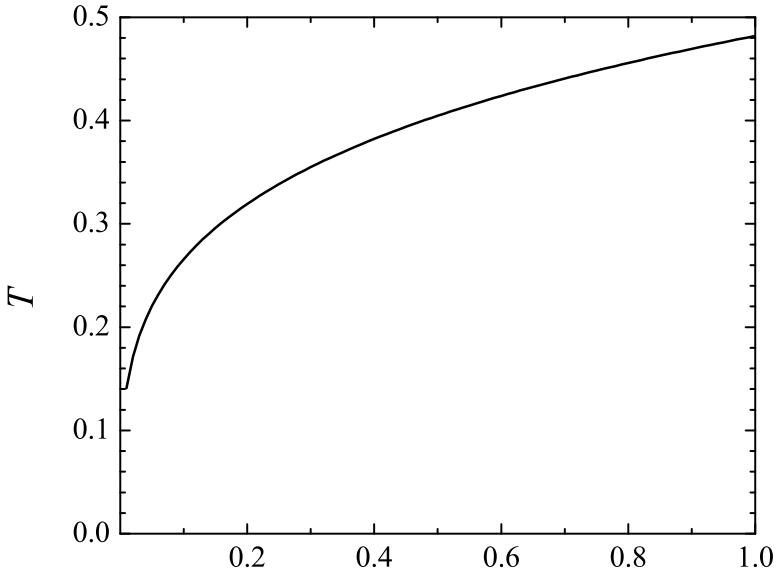


Figure 1 Time of the field growth for different distances from the center in dimensionless units.

rate we can obtain $\gamma = D^{1/2} - \frac{\pi^2}{4}$. Here $D = R_\alpha R_\omega$ is the so-called dynamo number which characterizes magnetic field evolution. Taking into account that $\nu = \frac{lu}{3}$ (u is the typical velocity of turbulent motions), the dynamo number can be found as $D = \frac{9h^2\Omega^2}{u^2}$.

The magnetic field cannot grow infinitely. The growth is saturated [26] if $\frac{B^2}{8\pi} = \frac{\rho u^2}{2}$. So-called equipartition field will be $B^* = 2u\pi^{1/2}\rho^{1/2}$. It will depend on the distance from center of the disc, decaying in the outer parts.

The typical time of the magnetic field growth can be estimated as:

$$T(r) = \frac{1}{\gamma(r)} \ln\left(\frac{B^*(r)}{B_0}\right). \tag{5}$$

Radial dependence of this time is shown in Fig. 1. We can see that in the outer parts of the disc the magnetic field grows slower.

More accurate results can be obtained using numerical modelling. We should solve the equations:

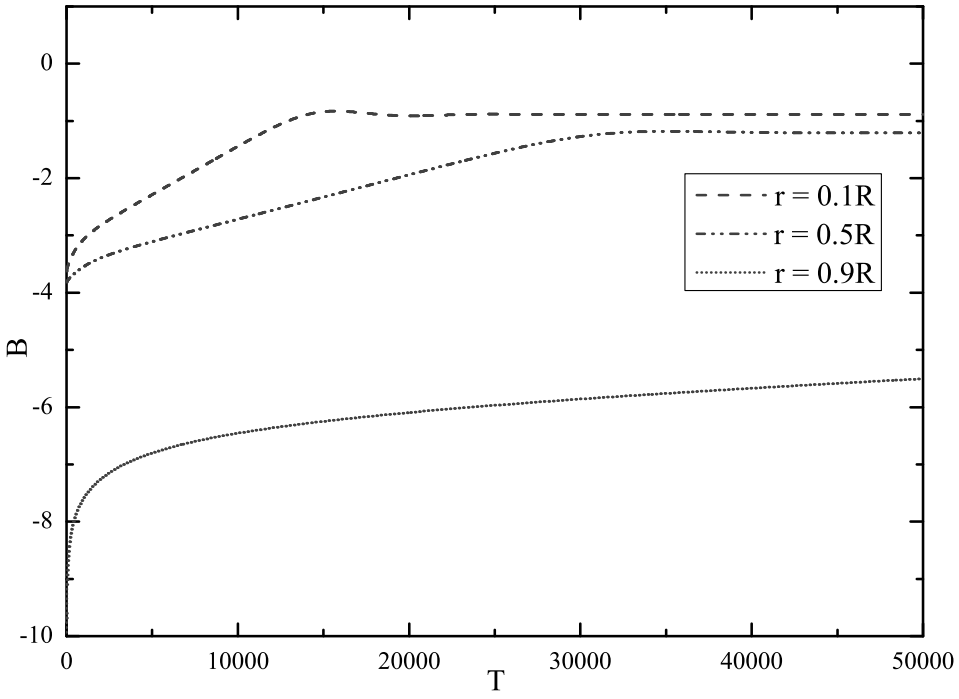


Figure 2 Time dependence of the magnetic field for different distances from the center for thin disc approximation.

$$\frac{\partial B_r}{\partial t} = -\frac{\Omega l^2}{h^2} B_\varphi \left(1 - \frac{B_r^2 + B_\varphi^2}{B^{*2}}\right) - \nu \frac{\pi^2 B_r}{4h^2} +$$

$$+\nu \left(\frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_r}{\partial \varphi^2} - \frac{1}{r^2}\frac{\partial B_\varphi}{\partial \varphi}\right), \tag{6}$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{d\Omega}{dr} B_r - \nu \frac{\pi^2 B_\varphi}{4h^2} +$$

$$+\nu \left(\frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_\varphi}{\partial \varphi^2} + \frac{1}{r^2}\frac{\partial B_r}{\partial \varphi}\right). \tag{7}$$

Figure 2 presents the magnetic field growth for different distance from the center.

Another problem is connected with evolution of radial structures. We have studied the magnetic field evolution for the case of the following initial conditions:

$$B_r|_{t=0} = 0, \tag{8}$$

$$B_\varphi|_{t=0} = B_0 \frac{r}{R}\left(1 - \frac{r}{R}\right)(1 + \kappa \cos(\varphi)). \tag{9}$$

Figure 3 shows the structure for the magnetic field which becomes nearly axisymmetric [21].

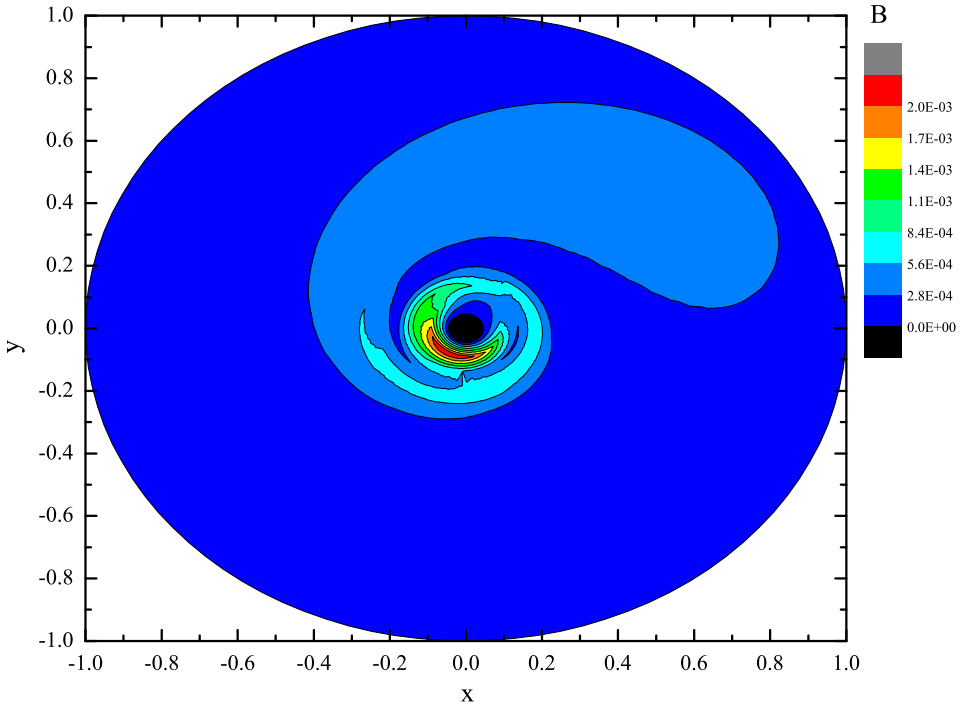


Figure 3 Azimuthal magnetic field $t = 300$ s for thin disc approximation.

3 *RZ* approach

To study the magnetic field in finite disc case, we use the so-called *RZ* model. Unlike the usual thin disc approximation, this model takes into account the vertical structure of the accretion disc. The *RZ* model is based on an axisymmetric description of the magnetic field, which allows us to consider the dependence of the magnetic structure on both the radial coordinate r and the vertical coordinate z simultaneously. The evolution of the magnetic field can be described by the following equation:

$$\frac{\partial B}{\partial t} = \sqrt{D}B\left(1 - \frac{B^2}{B^{*2}}\right) + h\sqrt{D}z\left(1 - \frac{B^2}{B_{\max}^2}\right)\frac{\partial B}{\partial z} +$$

$$+\lambda^2\left(1 - \frac{B^2}{B_{\max}^2}\right)\left(\frac{\partial^2 B}{\partial z^2} + \frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r} - \frac{B}{r^2}\right) + V_r\frac{\partial B}{\partial r},$$

$$-h(r) < z < h(r),$$

$$r_{\min} < r < R.$$

In this case, boundary conditions of zero field value at the edges are imposed:

$$B(r_{\min}, z, t) = B(1, z, t) = B(r, \pm h(r), t) = 0.$$

D is dynamo number, $h(r)$ is the ratio between the height and radius of the disc scale, the inital field is sinusoidal along the vertical direction.

The numerical solution of this equation takes an explicit finite difference format with first-order time coordinates and second-order spatial coordinates. To stabilize the solution, the time step is constrained based on the Courant condition. The grid is uniformly distributed in the r and z directions, and a truncation condition is added where the height varies with radius.

The simulation results show that the magnetic field is initially concentrated in the central part of the disc. Over time, the magnetic field begins to dissipate and smear outward, as shown in Figure 4. This process is due to competing diffusion effects caused by the accretion flow and radial transport.

For quantitative analysis of magnetic field changes over time, the field evolution for different r is studied. The plot of the $B(t)$ dependence shows that at the beginning of time the magnetic field grows exponentially and then reaches saturation, which is associated with the nonlinear constraint introduced through $1 - B^2/B^{*2}$. Figure 4 shows the relationship between the evolution of the magnetic field at fixed points of the disc for different radii at $z = 0$. The magnetic field decays at small radii and grows at large radii, which corresponds to a positive growth rate of γ in these regions.

Of particular interest is the estimate of the time required for the magnetic field to reach a stable state. For this purpose, a theoretical estimate of the magnetic field growth rate is carried out based on the spectral analysis of the expanding disc equation [27]:

$$\gamma_{n,m} = -\frac{\pi^2 n^2}{4} + \frac{\sqrt{D}}{2} - \frac{h^2 D}{48} - \lambda^2\pi^2\left(m + \frac{3}{4}\right)^2,$$

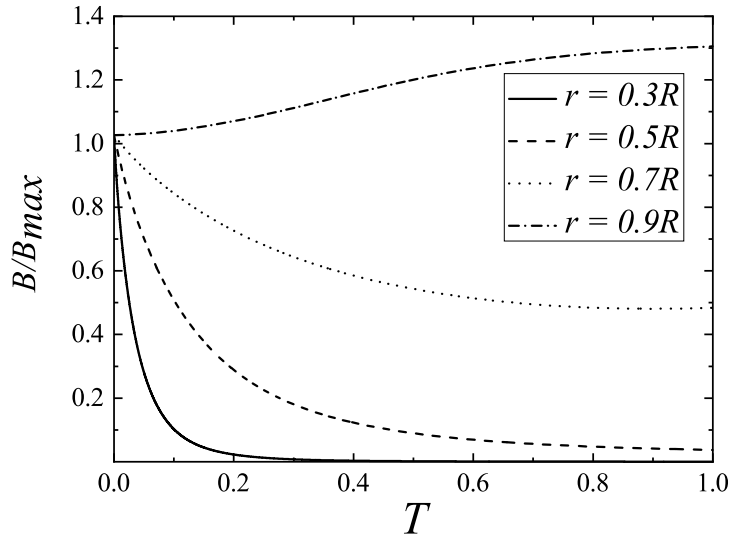


Figure 4 Dependence of the time to reach a stable state on coordinate r for RZ model.

where m and n are numbers for r and z, respectively. For our calculations we take the highest eigenvalue for which $n = 1, m = 1$.

In Figure 5 the dependence of the time to reach a stable state is shown for $z = 0$, i.e. in the equatorial plane of the accretion disc. The data are given for four radii: $r = 0.3R$, $0.5R$, $0.7R$ and $0.9R$, where R is the outer radius of the accretion disc. At the beginning of time ($t = 0$), the magnetic field has the same value at all points, which corresponds to the initial perturbation. However, the subsequent evolution

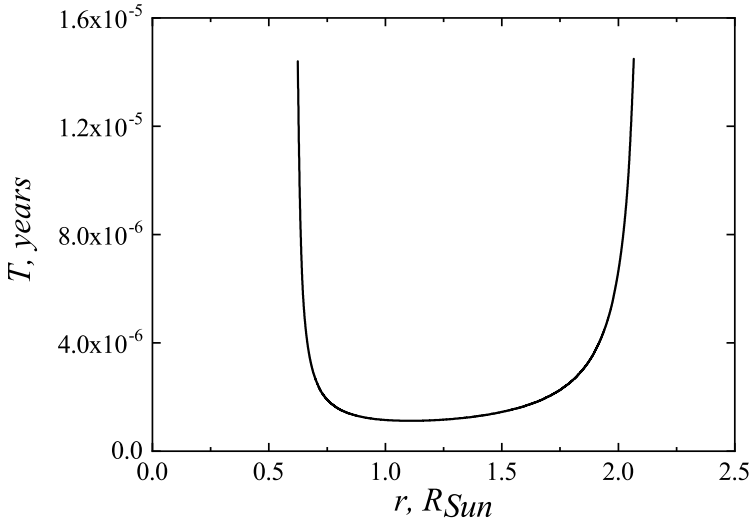


Figure 5 Dependence of the time to reach a stable state for RZ model.

changes significantly with radius. In regions close to the center of the accretion disc (for example, $r = 0.3R$ and $r = 0.5R$), an exponential decrease in the magnetic field can be observed. This is due to the fact that in these radial regions the growth rate of $\gamma(r)$ is negative, which is explained by both the enhanced diffusion of the magnetic field and the geometric factors of the RZ model.

For radii located outside the disc ($r = 0.7R$ and especially $r = 0.9R$), the magnetic field behaves fundamentally differently. At these distances, the magnetic field grows exponentially, which corresponds to a positive growth rate of $\gamma(r)$. This phenomenon can be explained by the fact that in these regions the dynamo number $D(r)$ and the disc height $h(r)$ create favorable conditions for efficient generation of the magnetic field.

The dependence of $T(r)$ (calculated similarly to (6)) is plotted along the radius of the accretion disc. As can be seen from Figure 5, $\gamma(r)$ reaches its maximum value in the middle of the accretion disc and decreases toward the inner and outer edges of the accretion disc. Contradictory to the thin disc model, it does not grow monotonically with radii, which can be connected with effects of different thickness.

Thus, this figure shows the spatial inhomogeneity of the dynamo process in the accretion disc and demonstrates the importance of taking into account the radial dependence of the model parameters when analyzing the evolution of the magnetic field over time.

Figure 6 shows the spatial dependence of the field. Maximum value of the field moves to the outer parts of disc.

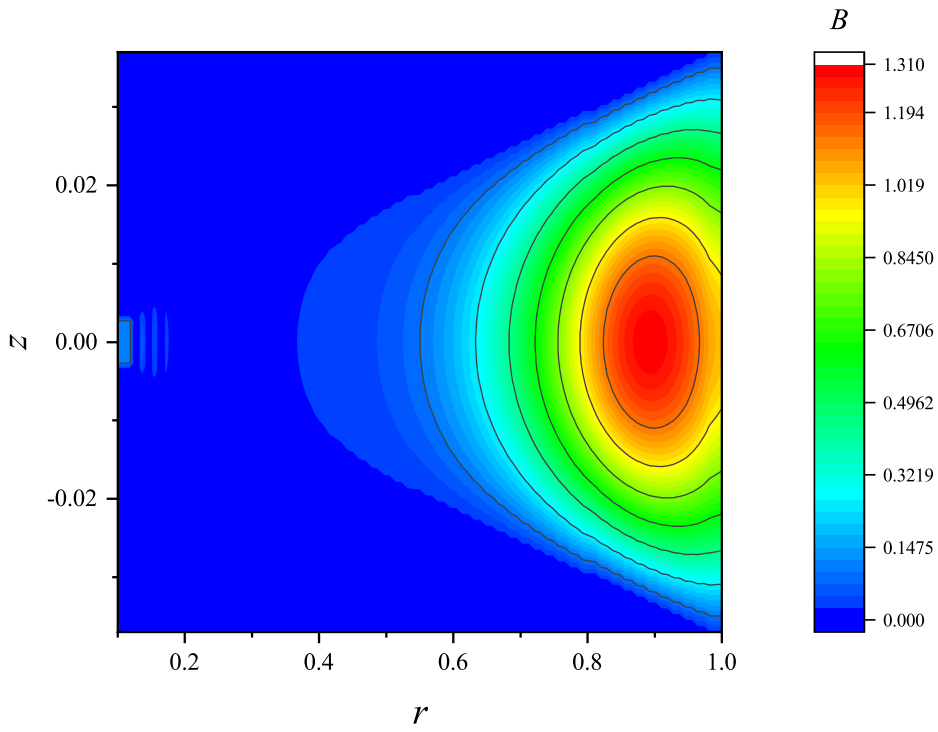


Figure 6 Magnetic field structure for RZ approach.

4 Conclusions

We have studied the magnetic field growth in accretion discs using two sufficiently different approaches. First of all, we have studied the field evolution using the thin disc approximation based on the algebraic model for vertical dependence. This model allows us to study the field in the nonaxisymmetric case. However, it has been shown that the field non-uniformities are destroyed by the rotation. After that we used more effective RZ approximation which takes into account that the field depends on the z coordinate according to a differential law. We have studied typical timescales for the magnetic field growth for both cases, and typical field structures.

It can be said that the magnetic field in the nonaxisymmetric case can be modelled using the thin disc approximation. However, vertical structures are described better using the RZ approach.

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