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Estimation of model parameters in the selected double imaged quasars

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This article presents the results of modeling based on observations of the gravitationally lensed quasars SDSS J0806+2006 and SDSS J2124+1632 observed at the Maidanak Observatory in 2017-2022 and 2018-2022. The value of the Hubble constant H_0 = 61.4±7 for the SDSS J0806+2006 GLK is estimated, and the redshift of the lensing galaxy for the SDSS J2124+1632 GLK is found Zg = 0.245±0.071.

Keywords: Gravitational lensed quasar, modeling, Hubble constant

1 Introduction

Gravitational lensing reveals very interesting and unusual properties of matter in the Universe. For example, by studying gravitationally lensed quasars (GLQs), we can investigate the problem of dark matter and dark energy, as well as the mass and structure of cosmological objects. Among other things, one can determine the Hubble constant or the redshift of quasar sources (Eigenbrod, 2011; Chan et al., 2002; Wong et al., 2019, 2024). In this work, we estimate the Hubble constant based on the delay times calculated for selected gravitationally lensed quasars.

The Hubble constant H_0 is a fundamental parameter characterizing the rate of expansion of the Universe. Determining its exact value is one of the main tasks of modern cosmology. H_0 remains a mystery, especially in terms of discrepancies between different measurement methods. One independent method of estimation is the analysis of gravitationally lensed quasars. In such systems, light rays from a distant quasar are bent by the influence of a massive galaxy, forming multiple images. By measuring the delay in the brightness of these images due to the different path lengths of the light, cosmological distances and, from them, the Hubble constant can

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be determined. This method is becoming increasingly important in the context of the ongoing "Hubble constant crisis".

The Hubble constant problem is the discrepancy between the values of the Hubble constant H_0 obtained using different measurement methods. This problem arises because two main independent techniques produce inconsistent results:

1. The early Universe, as measured by the CMB method, Planck and WMAP satellites. It is based on data from the early period and relies on measurements of the anisotropy of the cosmic microwave background radiation and the predictions of the ΛCDM model. This method yields $H_0 \approx 67$–68 km/s·Mpc.
2. Astrophysical observations of the beginning of the Universe. This includes observations of Cepheids, Type Ia supernovae, and gravitational lensing of quasars. These observations are based on direct measurements of the distances to the objects and yield values of $H_0 \approx 73$–74 km/s·Mpc.

These two methods differ from each other by 5σ, which makes it highly unlikely that the error can be explained by statistical fluctuations or systematic effects. If this discrepancy is real, it could indicate: the need to revise the Standard Model of ΛCDM; the existence of new physics (such as the unknown interaction of dark matter and dark energy); or errors in distance calibration or other measurement methods. The method of delay times in gravitationally lensed quasars provides an independent estimate of H_0. So far, it has yielded results that are closer to those obtained by local methods, which strengthens the argument for the need to revise the Standard Cosmological Model.

As we have seen, the delay times between the GLQ abstract images can be used to determine the Hubble constant, which is the most important cosmological parameter (Eigenbrod (2011); Chan et al. (2002); Dalal&Kochanek (2002); Nierenberg et al. (2017)). If we calculate the Hubble constant for objects in the young and old Universe, given the significant difference between the measurements, independent measurements of quasars with different redshifts become particularly relevant (Gilman et al. (2019)).

We will try to determine the Hubble constant using the example of the SDSS J0806+2006 GLQ. We have been observing this system for several years, calculating the delay times. Now we will apply our previous results to find the Hubble constant.

2 Results and discussion

To calculate the Hubble constant, we must resort to numerical modeling methods and programs, since this quantity is not directly observable. One of the most widely used methods today is the GravLens program (Keeton (2004)) developed by Charles R. Keeton.

GravLens is a software package designed to model and analyze gravitational lensing. It is widely used in astrophysics to study the mass distribution in lensing galaxies, determine cosmological parameters, and analyze delay times in lensed systems.

The input parameters are the relative coordinates of the lensed abstract images and the lens, their redshifts, and relative brightnesses, and the model based on which they are calculated is indicated. As a result, the coordinates, relative brightness,

and Hubble constant of the source are obtained. We will apply this approach to the research object.

2.1 SDSS J0806+2006

This GLQ was discovered by Inada et al. (2006). Based on the analysis of the SDSS database, two new binary GLQs were found: $\alpha = 121^\circ.59875$, $\delta = +20^\circ.108861$. The distance between the lensed components is $\theta = 1.4''$. At the time of discovery, their apparent brightness in the R filter was found to be $m_A = 18^m.93$ and $m_B = 19^m.36$. Among the abstract components, there is a lensing galaxy, the brightness of which is found to be $\mu_G = 21^m.20$. Later, the angular distance between the components was determined – it turned out to be $1.49''$.

Elementary calculations based on the singular isothermal ellipsoid (SIE): Radius of the Einstein ring $b = 0.69''$, position angle $PA = 28^\circ$, axis ratio $q = 0.08$, brightness gain $\mu = 4.62$. Most importantly, a model delay time value of $\Delta t_{AB} \sim 32$ h^{-1} day was found in Sluse et al. (2008).

We calculated the delay time for this object based on 6 years of observational data. As a result, the delay time was determined to be $\Delta t_{AB} = 53 \pm 6.0$ days (Bekov et al. (2024)). In addition, the redshifts of the source galaxy and the lensing galaxy in this system were also known from observations: $z_q = 1.540$ and $z_g = 0.573$. The coordinates of the components are already known to us from catalogs. Now, based on the delay time we found and previous calculations, we can calculate the Hubble constant.

Known quantities are entered into the `garvlens` program via a text file. The Hubble constant $h = H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$ is used in normalized units of view. A model of the system is constructed for different possible values of h, the coordinates of the abstract images are found, compared with the observation results, and the degree of correspondence is calculated for each time

$$\chi^2_{\mathrm{test},j} = \min_i \left(\frac{|y_j - x_i|^2}{\sigma_i^2}, \frac{d_{i-1,i}(y_j)^2}{\sigma_i \sigma_{i-1}} \right).$$

Then the coordinates and relative luminances of the components do not change, and as a result the dependence $h = f(\chi^2)$ is obtained.

In our calculations, we used the alpha model – a set of seven parameters that corresponds to the well-known SIE model. From this graph, we can see that the most likely value of the Hubble constant is $H_0 = 61.4 \pm 7$. The other parameters derived are: the Einstein ring radius $\theta_E = 0.656''$, eccentricity $e = 0.26$, position angle $PA_e = 31^\circ$, gamma $\gamma = 0.01$, and position angle of gamma $PA_\gamma = 64^\circ$.

2.2 SDSS J2124+1632

This system is a new object discovered in 2018. It was discovered, spectroscopically confirmed, and the initial properties of 24 lensed quasars were calculated by the GAIA space observatory (Lemon et al. (2019)). The system consists of components A and B, the angular separation between which is $\theta = 3.02''$ arcsec. The redshift of the lensing galaxy is $z_q = 1.28$. At the time of discovery, their apparent brightnesses were

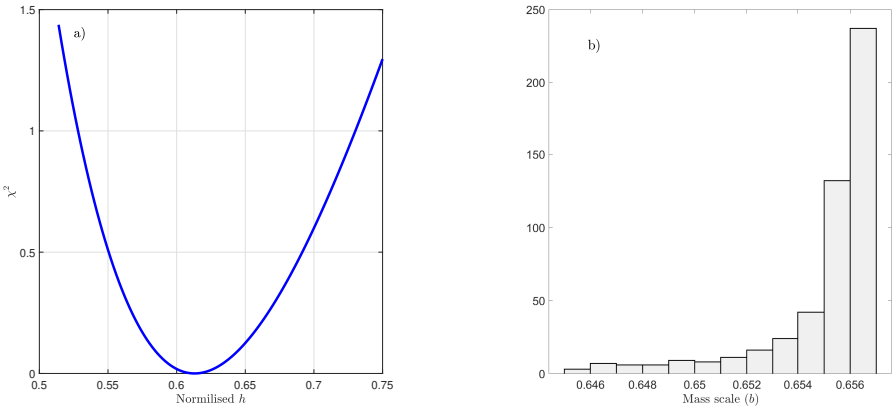


Figure 1 (a) Estimation of h versus χ^2; (b) Histogram for the Einstein ring radius via modeling.

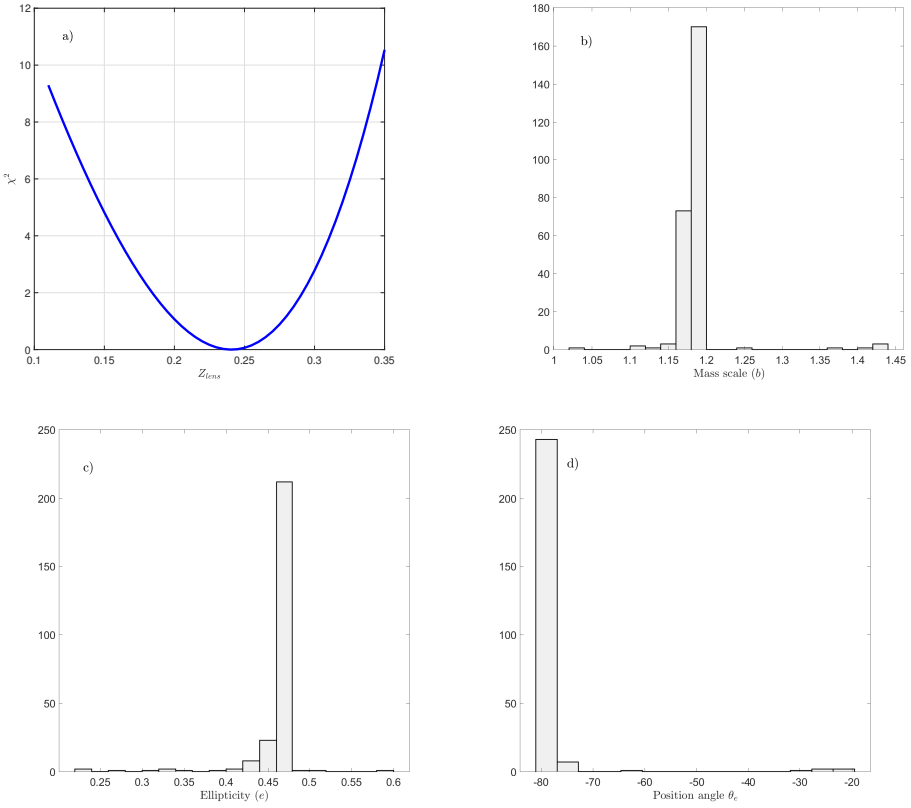


Figure 2 The histograms for the lensing galaxy redshift (a), the mass scale (b), ellipticity (c) and the position angle (d).

$m_A = 18.11^m$ and $m_G = 18.40^m$. Since the image of the galaxy is located very close to component B, its brightness and redshift could not be measured.

Calculations were made based on the appearance of the system using an isothermal model. The radius of the Einstein ring is $b = 1.41''$, the position angle is $PA = 176°$, the axis ratio is $q = 0.39$, and the luminosity enhancement is $\mu = 2.7$. We have observed this GLQ at the Maidanak Observatory for 5 years and found that the delay time between the components is $\Delta t_{AB} = 102 \pm 20.0$ days (Bekov et al. (2024)).

Using these data, we can build a model of this system. However, since the redshift of the lensing object is unknown, the Hubble constant cannot be calculated directly. Instead, we decided to solve the inverse problem: assuming the Hubble constant is known and taking it as a criterion, we can estimate the redshift of the lensing galaxy, while at the same time redefining other parameters of the model.

The modeling results are shown in the graphs of Figure 1 and Figure 2. The top-left graph shows the lensing redshift versus the degree of fit, while the remaining figures display histograms of various parameters.

According to the calculation results, the following parameters were determined: the lensing galaxy redshift: $Z_g = 0.245 \pm 0.071$, the Einstein ring radius: $\theta_E = (1.18 \pm 0.03)''$, the ellipticity: $\epsilon = 0.46 \pm 0.03$, the position angle: $PA = (-79.5 \pm 7.73)°$, the external shift: $\gamma = 0.13$, the angle of the external shift: $\theta_\gamma = 70.7°$.

Conclusion

The article is devoted to the study of the properties of matter in the Universe through the phenomenon of gravitational lensing. The article studies the problem of determining the Hubble constant and other unknown parameters based on the measured delay times of the gravitationally lensed quasars SDSS J0806+2006 and SDSS J2124+1632.

The results are compared with observations from different quasars, and the value of the Hubble constant for SDSS J0806+2006, $H_0 = 61.4 \pm 7$, is found using the alpha model. This value is consistent with the value obtained by the CMB method, and the following additional unknown parameters are also estimated: $\theta_E = 0.656$, $e = 0.26$, $PA_e = 31°$, $\gamma = 0.01$, $PA_\gamma = 64°$.

In the SDSS J2124+1632 system, the lensing galaxy redshift $z_g = 0.245 \pm 0.071$, Einstein ring radius $\theta_E = (1.18 \pm 0.03)''$, ellipticity $\varepsilon = 0.46 \pm 0.03$, position angle $PA = (-79.5 \pm 7.73)°$, external shift $\gamma = 0.13$, and its angle $\theta_\gamma = 70.7°$ were found.

Based on the results, we can conclude that the data obtained through gravitational lensing and quasars contribute to the improvement of cosmological models, and research in this area is ongoing.

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References

Bekov D., Akhunov T. et al., 2024. Astrophysical Bulletin, V79, no. 1, pp. 15–24
Bekov D., Akhunov T. at al., 2023 ACTA NUUz, Issue 3/2/1, p.503-507.
Chantry V., Sluse D., & P. Magain, 2010, Astronomy & Astrophysics, 522, A95
Chan J.H., Lemon C. et al., 2022. Astronomy & Astrophysics 659, A140.
Dalal N., Kochanek C.S., 2022 ApJ 572, 25.
Eulaers E. et al., 2010, Astronomy & Astrophysics, 553, A121
Eigenbrod A., 2011. "Gravitational lensing of quasars". London: Taylor&Francis Group, LLC, 152 p.
Gilman D., Birrer S. et al., 2019. MNRAS 487, 5721-5738.
Inada N. et al., 2006. AJ, 131:1934–1941
Keeton Ch., 2004. Software for Gravitational Lensing, Version 9 January
Lemon C.A., Auger M.W., McMahon R.G., 2019. MNRAS, 483, Issue 3, p.4242-4258.
Nierenberg A.M., Treu T. et al., 2017. MNRAS 471, 2224-2236.
Sluse D., Courbin F., et al., 2008. Astronomy & Astrophysics, 492, L39.
Wong K.C., Suyu S.H., Chen G.C.-F., et al., 2019 MNRAS, 498(1), 1420–1439.
Wong K.C., Dux F., Shajib A.J., et al., 2024. Astronomy & Astrophysics 689, A168.