Astronomical and Astrophysical Transactions (*AApTr*), 2025, Vol. 35, Issue 2, pp. 215–224,
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On the theory of the formation of lopsided structure in disk-shaped self-gravitating systems

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Received 27 April, 2025; accepted 27 May, 2025

Many disk galaxies exhibit a lopsided nature, with their nuclei clearly displaced from their geometric centers. This phenomenon is associated with gravitational instability during the early stages of the evolution of these objects. Based on this, the present work investigates the gravitational instability of lopsided perturbation modes in the context of a non-stationary disk model with an anisotropic velocity diagram. Corresponding non-stationary analogues of the dispersion equations for these perturbation modes have been derived. The results of the study are presented as critical dependencies of the initial virial ratio on the disk rotation parameter for different values of the superposition parameter. Based on the analysis of the obtained results, it is established that, within the framework of a non-stationary disk model, lopsided perturbation modes exhibit various types of instability depending on the values of the parameters of the non-stationary model of disk-shaped self-gravitating systems.

Keywords: Self-gravitating disk, non-stationary model, gravitational instability, instability increment, lopsided structure

1 Introduction

Lopsidedness is a widespread and intriguing feature observed in disk galaxies, characterized by asymmetry in the distribution of stars or gas relative to the galaxy's center (Weinberg (1995); Swaters et al. (2002, 1999); Heller et al. (2000); Jog & Maybhate (2006); Reichard et al. (2008); Espada et al. (2011); van Eymeren et al. (2011a,b)). Although, for example, spiral galaxies are often idealized as symmetric systems in theoretical models, lopsidedness is observed in about 30% of spiral galaxies (Bournaud et al. (2005b); Jog & Combes (2009)). It has a significant impact on galaxy evolution. Therefore, investigating the mechanisms behind the emergence of lopsidedness is one of the pressing issues in astrophysics and modern cosmology. Several

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mechanisms have been proposed to explain the origin of lopsidedness. For instance, tidal interactions between galaxies (Jog (1997, 2002); Andersen & Bershady (2002); Reichard et al. (2010); Wu et al. (2010); Saha et al. (2007)), asymmetric accretion of material from a satellite galaxy (Jog & Combes (2009); Bournaud et al. (2005a)), galaxy mergers (Jog & Maybhate (2006); Zaritsky & Rix (1997)), and individual modes of instability (Dury et al. (2008); Gulati et al. (2012); Junqueira & Combes (1996); de Rijcke et al. (2011) have been suggested as possible causes of disk lopsidedness. Consequently, the formation process of lopsided disk galaxies can also be explained by the instability of certain perturbation modes during the early stages of their evolution. Thus, to study the formation of lopsided structure in disk galaxies, it is essential to investigate the instabilities of lopsided modes within the framework of non-stationary disk models of galaxies (Mirtadjieva et al. (2011); Mirtadjieva & Nuritdinov (2012a); Mirtadjieva (2012b)), since large-scale structures in galaxies form during nonlinear, nonequilibrium stages of their evolution. To this end, in the present work we investigate the problem of lopsided structure formation in disk-shaped self-gravitating systems within the framework of a nonlinear non-stationary model with a composite nature. Non-stationary analogues of dispersion equations (NADE) for lopsided perturbation modes are derived, and the results of their numerical analysis are presented as critical dependencies of the initial virial ratio on the disk rotation parameter for various values of the superposition parameter.

2 Nonlinear non-stationary anisotropic disk model

As the initial state, we construct a radially oscillating disk model with an anisotropic velocity diagram based on the nonlinear isotropic model of Nuritdinov (Nuritdinov (1993)), which in turn is a non-stationary generalization of the equilibrium disk of Bisnovatyi-Kogan and Zeldovich (Bisnovatyi-Kogan & Zeldovich (1970)). This isotropic non-stationary model, in an inertial coordinate system, has the following phasespace density (Nuritdinov (1993); Nuritdinov et al. (2008)):

$$\Psi_{is}\left(\mathrm{r},\mathrm{v_r},\mathrm{v_\perp},\lambda,\Omega,t\right)= \\ =\frac{\sigma_0}{2\pi\Pi\sqrt{1-\Omega^2}}\left[\frac{1-\Omega^2}{\Pi^2}\left(1-\frac{\mathrm{r}^2}{\Pi^2}\right)-\left(\mathrm{v_r}-\mathrm{v_a}\right)^2-\left(\mathrm{v_\perp}-\mathrm{v_b}\right)^2\right]^{-1/2}\chi(\mathrm{R}-\mathrm{r}), \tag{1}$$

where $R(t)=R_0\Pi(t)$, $\Pi(t)=\dfrac{1+\lambda\cos\psi}{1-\lambda^2}$, is the pulsation coefficient, time $t=\dfrac{\psi+\lambda\sin\psi}{(1-\lambda^2)^{3/2}}$. Here ψ is the auxiliary variable instead of time t, and the amplitude of radial oscillations $\lambda=1-(2\mathrm{T}/|\mathrm{U}|)_0$ is expressed, evidently, through the value of the virial ratio at time $t=0$, i.e., $0\le\lambda\le1$. When $\lambda=0$, model (1) coincides with the equilibrium configuration of Bisnovatyi-Kogan and Zeldovich (Bisnovatyi-Kogan & Zeldovich (1970)). Also, Ω is the angular velocity of the disk rotation ($0\le\Omega\le1$), and χ is the Heaviside function. In (1), the following notations are introduced:

$$\mathrm{v_a}=-\lambda\frac{\mathrm{r}\sin\psi}{\sqrt{1-\lambda^2\Pi^2}},\quad \mathrm{v_b}=\frac{\Omega\mathrm{r}}{\Pi^2}, \tag{2}$$

which represent the components of the centroid velocity in the radial and transverse directions, respectively. Note also that the known normalization $\pi^2 G\sigma_0 = 2R_0\,(R_0 = 1)$ is adopted here, and all other designations are described in detail in Nuritdinov (1993); Nuritdinov et al. (2008). To create the anisotropic model, we use the method of averaging equation (1) over the parameter Ω (Mirtadjieva & Nuritdinov (2012a)). Introducing a weighting function $\rho(\Omega)$, we obtain:

$$\Psi_a = \int_{-1}^{+1} \rho(\Omega)\Psi_i d\Omega \Big/ \int_{-1}^{+1} \rho(\Omega) d\Omega. \tag{3}$$

From the standpoint of an analytically solvable model, a particularly interesting case is the weighting function:

$$\rho(\Omega) = \frac{8}{\pi}\Omega^2\sqrt{1-\Omega^2},$$

which satisfies the normalization condition $\int_{-1}^{1} \rho(\Omega)d\Omega = 1$. Then the distribution function (3) becomes:

$$\Psi_a\left(\mathrm{r}, \mathrm{v_r}, \mathrm{v_\perp}, t\right) = \frac{4\sigma_0}{\pi^2}\int_{-\sqrt{D}}^{\sqrt{D}} \frac{\left(X + \mathrm{r v_\perp}\right)^2}{\sqrt{D - X^2}} dX. \tag{4}$$

Here, $D = \left(1 - \dfrac{\mathrm{r}^2}{\Pi^2}\right)\left(1 - \Pi^2 \mathrm{v}_\perp^2\right) - \Pi^2\left(\mathrm{v_r} - \mathrm{v_a}\right)^2 \geq 0$ and $\Omega - \mathrm{r v_\perp} = \mathrm{X}$. Finally, we obtain the following result:

$$\Psi_a\left(\mathrm{r}, \mathrm{v_r}, \mathrm{v_\perp}, t\right) = \frac{2\sigma_0}{\pi}\left[2\mathrm{r}^2\mathrm{v}_\perp^2 + D\right]\chi(D). \tag{5}$$

To account for certain possible intermediate states between the isotropic model (1) and the anisotropic model (5), we construct the following composite model using the method of linear superposition:

$$\Psi_{\mathrm{comp}}\left(\mathrm{r}, \mathrm{v_r}, \mathrm{v_\perp}, \nu, t\right) = (1-\nu)\cdot\frac{2\sigma_0}{\pi}\left[2\mathrm{r}^2\mathrm{v}_\perp^2 + D\right]\chi(D) +$$
$$+\nu\cdot\frac{\sigma_0}{2\pi\sqrt{1-\Omega^2}}\left[\frac{1-\Omega^2}{\Pi^2}\left(1 - \frac{\mathrm{r}^2}{\Pi^2}\right) - \left(\mathrm{v_r} - \mathrm{v_a}\right)^2 - \left(\mathrm{v_\perp} - \mathrm{v_b}\right)^2\right]^{-1/2}\chi(R - r), \tag{6}$$

where the superposition parameter ν like Ω and λ, takes values in the interval [0; 1]. Below, against the background of this composite non-stationary disk model (6), we investigate the problem of instability of lopsided perturbation modes with the principal harmonic index N = 3 and N = 5, at a fixed value of the azimuthal wave number m = 1.

3 Investigation of gravitational instability of the lopsided mode $\mathbf{N = 3; m = 1}$

The instability of this perturbation mode leads to the displacement of galaxy nuclei from their geometric centers (Binney & Tremaine (1987); Fridman & Polyachenko

(1984)). The instability problem of this mode against the background of the isotropic model (1) was studied in our previous work (Mirtadjieva et al. (2011)). Using the method described in Mirtadjieva et al. (2011); Mirtadjieva (2012b) for the lopsided mode (3;1), we obtained the following Non-Stationary Analogue of the Dispersion Equation (NADE) based on the composite model (6):

$$(1+\lambda\cos\psi)\frac{\mathrm{d}^2\ell_\tau(\psi)}{\mathrm{d}\psi^2}+\lambda\sin\psi\frac{\mathrm{d}\ell_\tau(\psi)}{\mathrm{d}\psi}+\ell_\tau(\psi)=\frac{3B_{31}(\psi)}{8(1+\lambda\cos\psi)^4}(\lambda+\cos\psi)^{2-\tau}\sin^\tau\psi$$
$$(\tau=\overline{0-2}). \tag{7}$$

Here

$$B_{31}(\psi)=\nu\left\{\left[11q_1^2+\left(5\Omega^2-4\right)q_3-10i\Omega q_1\sqrt{q_2}\sin\psi\right]\ell_0(\psi)+\right.$$
$$+\left[10\left(3-\Omega^2\right)q_1q_2\sin\psi+10i\Omega\sqrt{q_2}\left(q_1^2-q_3\right)\right]\ell_1(\psi)+$$
$$\left.+\left[11q_3q_2+\left(5\Omega^2-4\right)q_1^2q_2+10i\Omega q_1q_2^{3/2}\sin\psi\right]\ell_2(\psi)\right\}+$$
$$(1-\nu)\left\{\left[11q_1^2-\frac{3}{2}q_3\right]\ell_0(\psi)+\left[25q_1q_2\sin\psi\right]\ell_1(\psi)+\left[11q_3q_2-\frac{3}{2}q_1^2q_2\right]\ell_2(\psi)\right\}$$
$$q_1=\lambda+\cos\psi,\quad q_2=1-\lambda^2,\quad q_3=q_2\sin^2\psi.$$

The NADE (7) is a system of three second-order differential equations, which cannot be analyzed analytically. Therefore, we examined it numerically using the method of stability analysis of periodic solutions (Malkin (1967)). Based on the results of the numerical analysis of NADE (7), we constructed critical diagrams of the initial virial ratio $(2\mathrm{T}/|\mathrm{U}|)_0$ as a function of the disk rotation parameter Ω for various values of the superposition parameter ν (Figures 1 and 2).

Analysis of the numerical results for NADE (7) at $\nu=0.25$ shows that in the range $0.402\le\Omega\le0.601$, a narrow zone of stability appears within the region of instability (Figure 1). From the resulting critical diagram, it is also evident that in the intervals $0\le\Omega\le0.02$ and $0.735\le\Omega\le1$, the lopsided mode $(3;1)$ is completely unstable on the background of model (6), regardless of the system's initial virial ratio. Regardless of the values of the disk's rotation parameter, the instability of this mode is always of an oscillatory nature.

From the critical diagram at the superposition parameter value $\nu=0.75$, it is evident that in the interval of rotation parameter values $0\le\Omega\le0.101$, a peninsula of stability appears within the region of instability (Figure 2). At $\Omega=0$, in the range of initial virial ratio values $0\le(2\mathrm{T}/|\mathrm{U}|)_0\le0.412$, the instability of the mode $(3;1)$ is aperiodic, while in the range $0.515\le(2\mathrm{T}/|\mathrm{U}|)_0\le0.612$, it is oscillatory. When the composite model of the non-stationary disk (6) includes rotation, only oscillatory instability is observed. In the range of rotation parameter values $0.15079\le\Omega\le0.43096$, there is an elongated island of instability in the form of a "petal." Additionally, it can be observed that at rotation parameter values $\Omega=0.115, 0.320, 0.651, 0.693$, and 0.775 , the superposition of the two models (1) and (5) resulted in the emergence of a resonance effect, causing the instability region to form peaks at points a, b, c, d, and p.

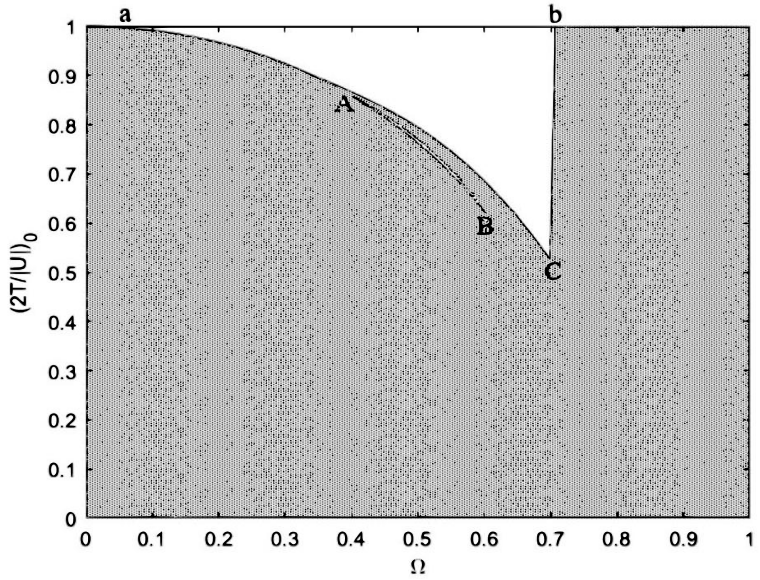


Figure 1 Critical dependence of the initial virial ratio on the rotation parameter for the lopsided mode (3; 1) at the superposition parameter value $\nu = 0.25$. The instability region is shaded. Points A (0.402; 0.858), B (0.601; 0.622), C (0.708; 0.509), with $a = 0.02, b = 0.735$.

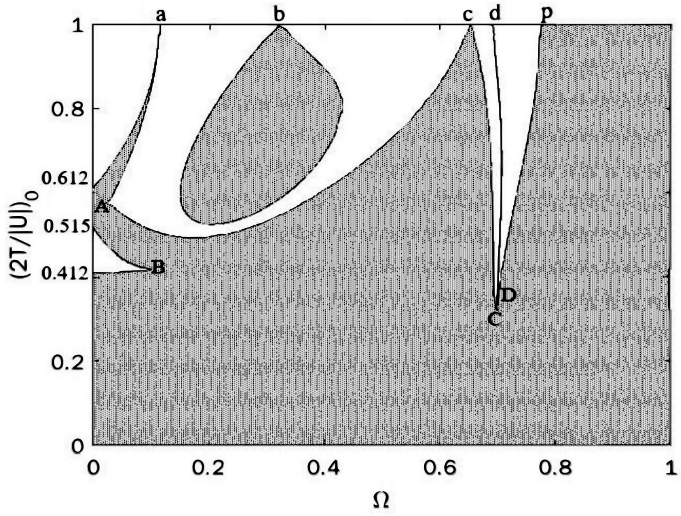


Figure 2 Critical dependence of the initial virial ratio on the rotation parameter for the lopsided mode (3;1) at the superposition parameter value $\nu = 0.75$. Points A (0.028; 0.576), B (0.101; 0.421), C (0.699; 0.331), D (0.701; 0.348), with $a = 0.115, b = 0.320, c = 0.651, d = 0.693$, $p = 0.775$.

4 Gravitational instability of the perturbation mode N = 5; m = 1

This case clearly falls under the category of large-scale perturbation modes. Against the background of model (6), the obtained NADE for the perturbation mode (5; 1) has the following form:

$$(1+\lambda\cos\psi)\frac{\mathrm{d}^2\gamma_\tau(\psi)}{\mathrm{d}\psi^2}+\lambda\sin\psi\frac{\mathrm{d}\gamma_\tau(\psi)}{\mathrm{d}\psi}+\gamma_\tau(\psi)=\frac{15}{64}B_{51}(\psi)(\lambda+\cos\psi)^{4-\tau}\sin^\tau\psi \quad (\tau=\overline{0-4}). \tag{8}$$

Here:

$$\begin{aligned}
B_{51}(\psi)=\nu\{&\left[29b_1^4\cos^4\psi-b_2\cos^3\psi\sin\psi+b_3\cos^2\psi\sin^2\psi-b_4\cos\psi\sin^3\psi+\right.\\
&\left.+b_5\sin^4\psi\right]\gamma_0(\psi)+\left[116b_1^4\cos^3\psi\sin\psi+b_2\left(q_1\cos^3\psi-3\cos^2\psi\sin^2\psi\right)+\right.\\
&\left.+2b_3\left(\cos\psi\sin^3\psi-q_1\cos^2\psi\sin\psi\right)+b_4\left(3q_1\cos\psi\sin^2\psi-\sin^4\psi\right)-\right.\\
&\left.-4b_5q_1\sin^3\psi\right]\gamma_1(\psi)+\left[174b_1^4\cos^2\psi\sin^2\psi-3b_2\left(\cos\psi\sin^3\psi-q_1\cos^2\psi\sin\psi\right)+\right.\\
&\left.+b_3\left(\sin^4\psi-4q_1\cos\psi\sin^2\psi+q_1^2\cos^2\psi\right)+3b_4\left(q_1\sin^3\psi-q_1^2\cos\psi\sin\psi\right)+\right.\\
&\left.+6b_5q_1^2\sin^2\psi\right]\gamma_2(\psi)+\left[116b_1^4\cos\psi\sin^3\psi+b_2\left(3q_1\cos\psi\sin^2\psi-\sin^4\psi\right)-\right.\\
&\left.-2b_3\left(q_1\sin^3\psi-q_1^2\cos\psi\sin\psi\right)+b_4\left(q_1^3\cos\psi-3q_1^2\sin^2\psi\right)-4b_5q_1^3\sin\psi\right]\gamma_3(\psi)+\\
&+\left[29b_1^4\sin^4\psi+b_2q_1\sin^3\psi+b_3q_1^2\sin^2\psi+b_4q_1^3\sin\psi+b_5q_1^4\right]\}\gamma_4(\psi)+\\
&+(1-\nu)\{\left[29p_1^4\cos^4\psi-p_2\cos^3\psi\sin\psi+p_3\cos^2\psi\sin^2\psi-p_4\cos\psi\sin^3\psi+\right.\\
&\left.+p_5\sin^4\psi\right]\gamma_0(\psi)+\left[116p_1^4\cos^3\psi\sin\psi+p_2\left(q_1\cos^3\psi-3\cos^2\psi\sin^2\psi\right)+\right.\\
&\left.+2p_3\left(\cos\psi\sin^3\psi-q_1\cos^2\psi\sin\psi\right)+p_4\left(3q_1\cos\psi\sin^2\psi-\sin^4\psi\right)-\right.\\
&\left.-4p_5q_1\sin^3\psi\right]\gamma_1(\psi)+\left[174p_1^4\cos^2\psi\sin^2\psi-3p_2\left(\cos\psi\sin^3\psi-q_1\cos^2\psi\sin\psi\right)+\right.\\
&\left.+p_3\left(\sin^4\psi-4q_1\cos\psi\sin^2\psi+q_1^2\cos^2\psi\right)+3p_4\left(q_1\sin^3\psi-q_1^2\cos\psi\sin\psi\right)+\right.\\
&\left.+6p_5q_1^2\sin^2\psi\right]\gamma_2(\psi)+116p_1^4\cos\psi\sin^3\psi+p_2\left(3q_1\cos\psi\sin^2\psi-\sin^4\psi\right)-\right.\\
&\left.-2p_3\left(q_1\sin^3\psi-q_1^2\cos\psi\sin\psi\right)+p_4\left(q_1^3\cos\psi-3q_1^2\sin^2\psi\right)-4p_5q_1^3\sin\psi\right]\gamma_3(\psi)+\\
&+\left[29p_1^4\sin^4\psi+p_2q_1\sin^3\psi+p_3q_1^2\sin^2\psi+p_4q_1^3\sin\psi+p_5q_1^4\right]\}\gamma_4(\psi).
\end{aligned}$$

Moreover,

$$\begin{aligned}
&b_1=\frac{1}{(1+\lambda\cos\psi)},b_2=(28i\Omega-116c)\sqrt{q_2}b_1^5,b_3=\left(174c^2+98\Omega^2-68-84ic\Omega\right)q_2b_1^6\\
&b_4=\left(136c-196c\Omega^2-116c^3+84i\Omega^3-56i\Omega+84ic^2\Omega\right)b_1^7q_2^{3/2},\\
&b_5=\left(8-28\Omega^2+21\Omega^4-68c^2+98c^2\Omega^2+29c^4-84ic\Omega^3+56ic\Omega-28ic^3\Omega\right)b_1^8q_2^2\\
&p_1=\frac{1}{(1+\lambda\cos\psi)},\quad p_2=-116c\sqrt{q_2}p_1^5,\quad p_3=\left(174c^2-19\right)q_2p_1^6,\\
&p_4=\left(38c-116c^3\right)p_1^7q_2^{3/2},\quad p_5=\left(\frac{9}{16}-19c^2+29c^4\right)p_1^8q_2^2.
\end{aligned}$$

Using the numerical results of NADE (8), critical diagrams of the initial virial ratio as a function of the disk rotation parameter were constructed for the superposition

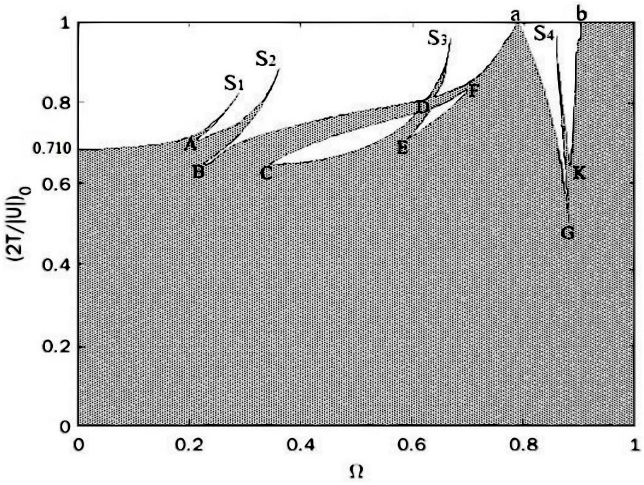


Figure 3 Critical dependence of the initial virial ratio on the rotation parameter for the lopsided mode (5;1) at the superposition parameter value $\nu = 0.25$. Points A (0.212; 0.712), B (0.224; 0.647), C (0.344; 0.648), D (0.609; 0.775), E (0.591; 0.711), F (0.719; 0.859), G (0.883; 0.498), K (0.884; 0.641), with $a = 0.787, b = 0.85$.

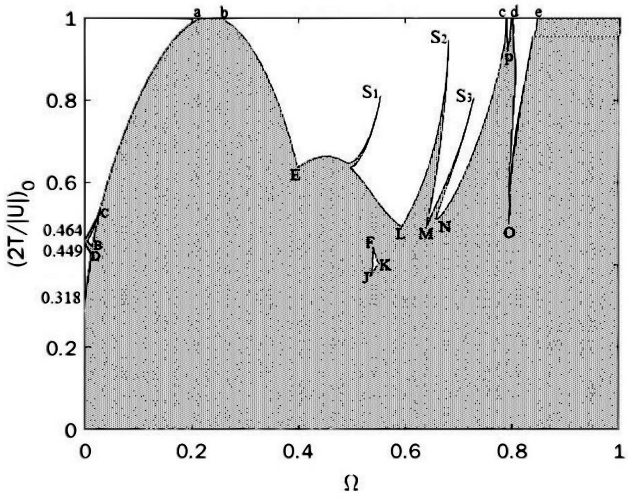


Figure 4 Critical dependence of the initial virial ratio on the rotation parameter for the lopsided mode $(5; 1)$ at the superposition parameter value $\nu = 0.75$. Points A (0.0042; 0.458), B (0.0134; 0.444), C (0.0280; 0.536), D (0.0113; 0.428), E (0.398; 0.636), L (0.593; 0.493), M (0.639; 0.489), N (0.657; 0.506), F (0.540; 0.435), J (0.532; 0.371), K (0.548; 0.404), O (0.794; 0.496), P (0.793; 0.918), with $a = 0.216, b = 0.258, c = 0.790, d = 0.799, e = 0.844$.

parameter values $\nu = 0.25$ and $\nu = 0.75$. Figure 3 shows the case for $\nu = 0.25$. Here, at $\Omega = 0$, the instability of mode $(5; 1)$ is aperiodic. However, when $\Omega \neq 0$, we only observe oscillatory instability. In the intervals $0.344 \leq \Omega \leq 0.609$ and $0.591 \leq \Omega \leq 0.719$, two stability islands are observed within the region of instability.

As the value of the rotation parameter increases, the instability region gradually expands with the appearance of additional branches located at points S1, S2, S3, and S4. At rotation parameter values $\Omega = 0.787$ and $\Omega = 0.850$, instability peaks are formed. In the range $0.850 \leq \Omega \leq 1$, the nonstationary composite model (6) becomes completely unstable with respect to mode (5;1). Analysis of the numerical results obtained for the superposition parameter value $\nu = 0.75$ shows that when the model has no rotation, in the range $0 \leq (2T/|U|)_0 \leq 0.318$, there is an aperiodic instability, while in the range $0.449 \leq (2T/|U|)_0 \leq 0.464$, oscillatory instability arises. It is also notable that, in the rotation parameter range $0 < \Omega < 0.0113$, a peninsula of stability appears inside the instability region, similar to the mode (3;1). In the intervals $0.0113 \leq \Omega \leq 0.042$ and $0.532 \leq \Omega \leq 0.547$, we observe "islands" of stability in the shape of a spherical triangle. Furthermore, as the rotation parameter increases, the instability region gradually expands due to additional branches, with the "peaks" located at points S1, S2, and S3. In the intervals of rotation parameter values $0.216 \leq \Omega \leq 0.258$ and $0.844 \leq \Omega \leq 1$, mode $(5;1)$ is completely unstable for all values of the initial virial ratio of the system (shown in Figure 4).

5 Conclusion

1. Non-stationary analogues of the dispersion equations (NADE) for lopsided perturbation modes, applied to a nonlinear non-stationary model of a self gravitating disk (6) with a composite nature, have been obtained.
2. Based on numerical calculations of these NADE, the critical values of the model parameters have been determined, and marginal diagrams of the initial virial ratio $(2T/|U|)_0$ as a function of the disk rotation parameter Ω have been constructed for different values of the superposition parameter ν of the model.
3. The analysis of the obtained results shows that lopsided perturbation modes exhibit different types of instability depending on the values of the parameters of the nonstationary composite model of disk-shaped self-gravitating systems.
4. It is shown that the superposition of two models (1) and (5) leads to the emergence of a resonant effect, causing the instability region to stretch up to a value of $(2T/|U|)_0 \approx 1$ at certain values of the disk rotation parameter.
5. It has been found that during the development of lopsided perturbation modes against the background of the non-linear non-stationary model (6), in the rotation parameter range $0 \leq \Omega \leq 0.6$, the instability rate of mode $(3;1)$ is higher than that of mode $(5;1)$, while in the interval $0.6 \leq \Omega \leq 1$, mode $(5;1)$ dominates over mode $(3;1)$.
6. It has been found that as the disk rotation parameter increases, the range of the initial virial ratio also expands, within which a lopsided structure can form in the system.

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