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Problems of determining the degree of concentration for galaxy clusters

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In this article, an analysis of observational data on galaxy clusters is performed. Surface density distributions for 40 galaxy clusters are calculated and the values of the concentration degree are determined. Empirical formulas for the relationship between the concentration degree and the physical parameters of galaxy clusters are found.

Keywords: Galaxy clusters: regular, irregular, richness, galaxy, concentration

1 Introduction

Clusters of galaxies are the most massive gravitationally bound formations in the Universe, and they occupy a special place in modern astrophysics and cosmology. Like all objects in the Universe, the clusters were formed from tiny fluctuations that were embedded during the inflationary expansion of the Universe (Abell 1959, Abell et al. 1989, Kluge & Bender 2023, Kopylova & Kopylov 2022, Piraino-Cerda et al. 2024). These disturbances occurred when the Universe was 400 thousand years old. Their study is important for understanding the evolution of the Universe, the distribution of dark matter and baryonic matter. Rich clusters contain a large number (from several hundred to several thousand) of galaxies, hot intergalactic gas and dark matter, which determines their mass (Vikhlinin et al. 2014).

Clusters of galaxies are one of a continuous series of objects: galaxies ⇒ groups ⇒ clusters ⇒ supercluster ⇒ large-scale structure. Galaxies are usually combined into groups and clusters. Galaxy groups contain 3–30 bright galaxies, while galaxy clusters contain 30–300 or more bright galaxies. Clusters of galaxies are usually divided into two groups: regular and irregular. Regular clusters are characterized by a more clearly expressed concentration of galaxies toward the center, higher density and mass, and, as a rule, they are significantly more massive than irregular ones. Most members of

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regular clusters are galaxies with absent or very weak star formation (elliptical and lenticular galaxies), whereas irregular clusters of galaxies contain many spiral and irregular galaxies with active star formation. This difference indicates the influence of the denser environment of galaxies and the intergalactic gas, in which they move, on the evolution of galaxies.

At present, clusters of galaxies have been well studied and a number of results have been obtained on these objects. However, the term 'richness of galaxy clusters' has not yet been specifically explained. If a galaxy cluster contains from several hundred to several thousand galaxies, then it can be called rich. About 10 percent of all galaxies are part of rich clusters, consisting of thousands of galaxies surrounded by hot gas.

Rich clusters of galaxies include: galaxies (elliptical and spiral galaxies grouped around a massive center), intergalactic gas (hot X-ray gas that fills the space between galaxies), dark matter (invisible mass that makes up to 80 percent of the cluster's mass).

Clusters of galaxies are formed by the merging of smaller groups of galaxies under the influence of gravity, a process that produces heated gas, shock waves, and bursts of energy in the X-ray range.

It should be noted that many observational studies of galaxy clusters have been published so far. In this paper, we consider the main properties of clusters of galaxies, the results of the analysis of their observational data and the problems of determining the degree of concentration towards their center. We studied the distributions of galaxies in clusters with $R_L > 100$ (here $R_L = L_{200}/L$, L is the luminosity of a galaxy cluster, L_{200} is the luminosity at a radius of r_{200}, r_{200} is the radius of a cluster for which the average density of clusters is 200 times greater than the critical density of the Universe) based on observational data from the Sloan Digital Sky Survey (SDSS) database and catalogs (Wen et al. 2012, 2015, 2024). As a result, we compiled a consolidated catalogue containing 213 of the richest clusters of galaxies (Kutlimuratov et al. 2024). From the catalogue, 40 rich clusters of galaxies were selected, the surface density and degrees of concentration were calculated, and a search for empirical relationships between the main physical characteristics was performed.

2 Calculation of surface density and concentration degree for clusters of galaxies

To determine the concentration degree, we need observational data of surface density for clusters of galaxies. Note that we did not find surface density values for clusters of galaxies based on searches and observational data. Therefore, based on ready-made images given in the SDSS database, their values were calculated. Surface density values can be determined, as is well known, by counting the number of galaxies in the rings around the cluster center. Obviously, the greater the number of rings, the more confidently and accurately we find the behavior of the observed density by the distance from the center to the periphery of the cluster. When drawing rings, the difference in the surface of the rings should be the same. According to formula (1), we find the area of the first circle:

$$S_n = \pi R_n^2. \tag{1}$$

When drawing subsequent rings, it is necessary to take into account that the surface areas of these rings must be the same. For this, we took the standard area of the first ring, to which all the subsequent ones are equal:

$$S_1 = \Delta S_1 = \Delta S_2 = ... = \Delta S_n.$$

We selected 20 rings and calculated the surface density values. Then the number of galaxies in each ring is determined. The apparent surface density was determined by the formula $F = N\backslash S$ (F is measured in arcmin^{-2}).

As we know, clusters of galaxies are mostly spherical in shape and, assuming spherical symmetry of the cluster, the surface density distribution can be taken to be close to a power law. Based on this, we generalized King's model and represent the surface density distribution function $\sigma(r)$ of the cluster as

$$\sigma(r) = \sigma_0/[1 + (r/r^*)^2]^\gamma, \tag{2}$$

where γ, r^* and σ_0 are free parameters, and γ characterizes the degree of concentration of galaxies towards the center of the galaxy cluster, r^* is a value that is related to the radius of the cluster core r_c and σ_0 is the visible density at the center. Next, we performed mutual comparisons of the visible surface densities of the distribution of galaxies in clusters. Note that the accuracy of finding the values of the free parameters of model (2), especially the degree of concentration (γ) of galaxies toward the center, for specific clusters depends to a large extent on the accuracy of determining the observed density in them. In order to determine the values of the three free parameters in (2) for specific clusters of galaxies, it is necessary to minimize the function

$$\sigma(\gamma, r^*, \sigma_0) = \sum_k [\sigma(r_k, \gamma, r^*, \sigma_0) - \sigma_{obs}^{(k)}]^2, \tag{3}$$

which describes the behavior of the sum of the squares of the difference between the theoretical (σ) and observed values ($\sigma_{obs}^{(k)}$) of the density in the rings around the center of the cluster. Minimization of the function in (3) can be adequately performed by the simplex method, which has long been well tested in many problems of astrophysics (see, for example, Nuritdinov et al. (2021)). Using the function minimization method (3), based on observational data we determined the values of the parameter γ, the radius of the cluster core r_c and the surface density at the center of the cluster σ_0. The results are presented in Table 1 below.

Interestingly, the surface density distribution is well described by these parameters. As can be seen, the values of the degree of concentration γ lie in the range from 0.037 to 0.57.

Table 1 The degree of concentration for galaxy clusters.

Name	σ_0	r_c	γ
J001051.4+290940	155.6814	0.1931	0.2462
J002016.1+000446	78.4472	0.8172	0.3175
J002712.5-193045	85.6719	0.574	0.2415
J002800.9+244744	241.8503	0.231	0.3669
J004118.5+252609	72.5782	0.2149	0.1508
J004511.7+084111	40.9774	2.4684	0.3908
J023127.6+065856	82.7679	0.5312	0.2619
J023952.7-013419	386.835	0.1613	0.3569
J083057.3+655031	117.9669	0.4227	0.2607
J085007.9+360414	112.2458	1.1409	0.3528
J090912.7+105829	203.5664	0.1259	0.2804
J091609.0-002226	77.5699	1.4599	0.3187
J091753.4+514338	386.4694	0.1315	0.3351
J092048.3+302818	132.4687	0.1242	0.1712
J094951.8+170711	133.7621	0.514	0.2986
J100226.8+203102	109.7053	0.6505	0.2510
J105417.5+143904	160.7753	0.2526	0.2664
J111450.3-121351	171.6555	0.1563	0.244
J112358.8+212850	83.036	0.5638	0.2263
J113553.9+400132	14.2845	2.1784	0.254
J115235.4+371543	18.4877	0.1334	0.1006
J115914.9+494748	33.2696	0.4127	0.2469
J121218.5+273255	40.0439	0.6385	0.3299
J122902.5+473721	39.3388	0.2603	0.2649
J123048.9+103247	17.9791	2.1219	0.3066
J124932.5+495344	16.9474	0.054	0.0371
J130650.0+463333	20.964	2.5626	0.5735
J131129.5-012028	121.2828	1.0006	0.3661
J131505.2+514903	149.471	0.516	0.3095
J133238.4+503336	215.0104	0.1807	0.2719
J133520.1+410004	173.7122	0.1898	0.2133
J140102.1+025242	94.3957	1.1394	0.3554
J141424.1-002240	20.8051	0.2613	0.1289
J144431.8+311336	236.0683	0.0694	0.2307
J153940.5+342527	162.4453	0.248	0.2380
J155820.0+271400	221.4657	0.1008	0.2334
J160319.0+031645	104.5931	0.5564	0.2300
J164019.8+464242	371.6218	0.0326	0.2324
J164325.4+132236	132.0536	0.2691	0.2209
J212823.4+013536	99.4691	1.4662	0.3122

3 Empirical relationships

However, from the point of view of studying clusters of galaxies, the presence of a correlation between the degree of concentration γ and their individual physical characteristics is more important. We also searched for empirical relationships between

parameter γ and the main physical characteristics of clusters, namely, with the radius of clusters, the richness of clusters, and the redshift. Also, the relationship between γ and the radius of clusters of galaxies. Data analysis shows the presence of an empirical relationship

$$\gamma = (-0.197 \pm 0.101) + (0.345 \pm 0.076) r_{500}. \quad (4)$$

Therefore, the smaller the value of γ, the denser the clusters. In this case, the correlation coefficient $\rho = 0.61$. Also, the relationship between γ and cluster richness. We found a good correlation ($\rho = 0.59$) and the following statistical formula:

$$\gamma = (0.109 \pm 0.037) + (0.001 \pm 2.3 \cdot 10^{-4}) R_{L500}. \quad (5)$$

As can be seen from the empirical relationship (5), an increase in the degree of richness of clusters also implies an increase in the value of the degree of concentration, i.e. an increase in the brightness of a cluster leads to an increase in γ. The relationship between γ and redshift. The correlation coefficient between these quantities is $\rho = 0.62$. The corresponding empirical formula is

$$\gamma = (0.142 \pm 0.029) + (0.474 \pm 0.104) z. \quad (6)$$

As can be seen, with an increase in the value of the redshift, there is an increase in γ, i.e. a gradual decrease in the concentration of clusters. It should be noted that there is a very weak dependence of γ on the radius (r_{200}) and the apparent size of the clusters. The correlation coefficient is 0.19 and 0.22.

4 Conclusion

The analysis of observational data on clusters of galaxies based on the SDSS DR8 database has been performed. Based on the observational data, the number of galaxies in clusters has been calculated and the surface density distributions in clusters of galaxies have been found. The concentration degree, cluster core radius, and surface density at the center for 40 rich clusters of galaxies have been determined using the function minimization method based on the observational data. The correlation coefficients between the concentration degree and the main physical parameters have been calculated. Empirical formulas of the relationship between the concentration degree γ and the physical parameters of rich clusters of galaxies have been found.

References

Abell G.O. (1959). Astrophysical Journal Supplement, 3, 211

Abell G.O., Corwin H.G.Jr., Olowin R.P. (1989). Astrophysical Journal Supplement, 70, 1

Kluge M., Bender R. (2023). The Astrophysical Journal Supplement Series, 267, 2, 41, 32

Kopylova F.G., Kopylov A.I. (2022). Astrophysical Bulletin, 77, 4, 347

Kutlimuratov S., Tadjibaev I., Otojanova N, Djumaeva G. (2024). Uzbek Journal of Physics, 26, 2, 7
Nuritdinov S.N., Tadjibaev I.U., Rastorguev A.S. (2021). Astronomy Letters 47, 3, 163
Piraino-Cerda F., Jaffe Y.L., Lourenço A.C., Crossett J.P., Salinas V., Kim D., Sheen Y.K., Kelkar K., Pallero D., Bravo-Alfaro H. (2024). Monthly Notices of the Royal Astronomical Society, 528, 1, 919
Vikhlinin A.A., Kravtsov A.V., Markevich M.L., Sunyaev R.A., Churazov E.M. (2014). Physics-Uspekhi, 57, 4, 339
Wen Z.L., Han J.L., Liu F.S. (2012). Astrophys. J. Suppl. Ser., 199, 34
Wen Z.L., Han J.L. (2015). Astrophys. J., 807, 178
Wen Z.L., Han J.L. (2024). Astrophys. J. Suppl. Ser., 272, 39