Astronomical and Astrophysical Transactions (*AApTr*), 2024, Vol. 34, Issue 3, pp. 267–276,
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Application of machine analysis methods for classification of resonant motion of small bodies of the Solar System

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Received 4 March, 2024; accepted 20 April, 2024

This paper examines the possibility of using machine analysis methods to study the features of the resonance dynamics of asteroids and artificial Earth satellites. The paper considers two options for automatic classification of resonant arguments: using convolutional neural networks and by analyzing the argument values. In the course of experimental studies, it was shown that both programs are capable of identifying and classifying the resonant arguments of small bodies, which makes it possible to speed up the process of analyzing the dynamics of these objects. However, the second option is more visual, while the neural network is essentially a black box. The paper also presents the examples of application of the developed software system to the analysis of the dynamics of asteroids and artificial Earth satellites.

Keywords: Asteroids, artificial Earth satellites, orbital resonances, classification of resonances, convolutional neural networks

1 Introduction

Currently, the use of machine analysis methods is becoming part of scientific research. Using these methods, a large amount of data can be processed more rapidly. This paper examines the possibility of using machine analysis methods to study the motion of small bodies of the Solar System.

Small bodies of the Solar System can bring great danger to the Earth. Therefore, the study of asteroid dynamics is one of the important scientific problems. One aspect of this research is the study of orbital resonances with planets. The nature of the asteroid-planet geometric configuration influences the motion of asteroids. Thus, a stable resonance can be a protective mechanism against the close encounters of objects with planets, while an unstable one, on the contrary, can lead to regular approaches, which will result in a change in the asteroid's orbit and cause chaotic motion. The study of orbital resonances in the motion of artificial Earth satellites and space debris

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is also of undoubted value, since the complex gravitational influence of the Earth on an artificial Earth satellite introduces perturbations into its motion that can lead to the chaotic motion and deviation of the orbit from the nominal one.

In the process of studying the classification of the orbits of small bodies according to the type of resonance relationships, the need arises to process a large amount of data. When performing this work manually, it is quite labor-intensive and time-consuming and raises the need to automate the process. Within the framework of this work, two approaches to solving the problem of automated separation of resonant and non-resonant motion will be considered.

2 Study of orbital resonances in the motion of asteroids

Orbital resonances arise when there is commensurability between the mean motions of the asteroid and the major planet. In this case, the main characteristics being studied are the resonant (critical) argument (1) and its first time derivative (resonance band) (2) (Murray, Dermott, 1999; Ellis, Murray, 2000, Nesvorný et al., 2003).

$$\beta = k_1\lambda' - k_2\lambda + k_3\varpi' + k_4\varpi + k_5\Omega' + k_6\Omega \tag{1}$$

$$\alpha = k_1\dot{\lambda}' - k_2\dot{\lambda} \tag{2}$$

where λ is the mean longitude, ϖ is the longitude of the periapsis, and Ω is the longitude of the ascending node. In these formulas, quantities with a prime refer to the elements of the asteroid, while those without a prime refer to the elements of the major planet. The quantities k_1, k_2, k_3, k_4, k_5, and k_6 are integers.

Orbital frequencies (mean motions) can be expressed explicitly through the semimajor axes of the asteroid and planet. This means that the resonance condition (1) with given k_1 and k_2, but different k_3, k_4, k_5, and k_6, will correspond to the same values of the semimajor axes. This structure is called a resonant multiplet. When constructing a multiplet, the following two conditions must be met (Murray, Dermott, 1999; Ellis, Murray, 2000, Nesvorný et al., 2003):

$$k_1 - k_2 + k_3 + k_4 + k_5 + k_6 = 0 \tag{3}$$

$$k_5 + k_6 = 0, 2, 4, ... \tag{4}$$

The behavior of the argument β shows the stability of the resonance or its absence. If β librates, that is, oscillates around a certain center with amplitude of less than 360°, then, we can conclude that the resonance is stable. A change in the argument from 0 to 360° (circulation) without a specific center indicates the absence of resonance. The change from libration to circulation and vice versa indicates a mixed type of behavior, in which resonance is defined as unstable.

2.1 Automation using convolutional neural networks

The first approach to developing software that allows us to solve the problem of automating the classification of small body orbits according to the resonant argument

type is based on the use of convolutional neural networks (Nikolenko et al., 2018) and includes three stages. At the first stage, the time series were marked by assigning a label to each of the classes: 1 (libration), 2 (mixed type), or 3 (circulation). To solve this problem, a script was written that allows you to interactively mark up data and write the results into a csv file for further conversion and construction of a dataset.

At the second stage, the data were converted into an image, which made it possible to use ultra-precise neural networks. For this purpose, a script has also been written, implemented as a Python class, which converts time series into three-channel images and forms a directory structure in which images are saved for further generation of a set of data. At the third stage, a resonance classification system was developed to determine the presence or absence of resonance.

To train the artificial intelligence, initial data were used, which were text files containing time series of critical (resonant) arguments of asteroids obtained during the study of the object evolution. The training set contained 3000 samples (the original set had to be reduced due to imbalance of classes, since the proportion of libration was more than 65%). After rebalancing the classes, the final training set contained 34% resonances of mixed type, 33% librations, and 33% circulations. The learning results were assessed using the Confusion Matrix (error matrix) and Accuracy (proportion of correct answers) tools. During the experiment, various architectures were considered: *resnet18*, *resnet50*, *efficientnet_b7*, and *convnext_base*. Each architecture was trained several times with different numbers of epochs. The results of this experiment showed that in our case, *convnext* trained on five epochs is the best model in terms of predictive ability. Its prediction accuracy is 92.8 percent. The disadvantage of this automation method is the need for preliminary training of neural networks and the use of architecture as a black box. To eliminate these shortcomings, we considered another approach to automation.

2.2 Automation by analyzing the values of the resonant argument

The idea of the second approach (Sekhar et al., 2016) is that at the first stage, a one-dimensional array of zero values is created (the size of the array is determined by the partition step, for example, at a partition of 1°, the array size will be [1..360]). When the values of the resonant argument fall within a certain interval, the corresponding value of the array element is assigned one. Thus, using such an array, we can track the values that the argument takes in the range from 0 to 360°. A study of the classification of various types of resonances using this method showed difficulties in classifying the mixed type and the case of limiting libration. To solve this problem, we modified this algorithm by considering partitioning of the time interval into several equal subintervals (N). Thus, to classify resonant arguments, N arrays are analyzed and the following condition is applied: if the arrays consist only of ones, then, such a case is classified as circulation; if all arrays contain zeros, then, it is classified as libration. In all other cases, resonance is defined as mixed. To evaluate the accuracy, manual classification was done in the same way as is done in the program, followed by comparison of the results. As the comparison showed, the accuracy of this classification method was 75%, which is certainly not enough when solving problems of asteroid dynamics. The question arose of how to improve the classification accuracy.

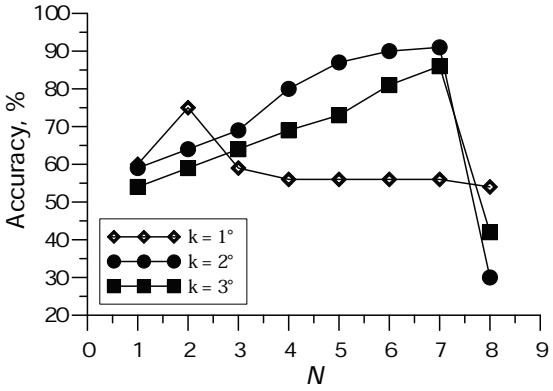


Figure 1 Results of estimation of program classification accuracy.

As part of solving this problem, we considered different numbers of partitions of time intervals and different numbers of partitions by degrees. Our goal was to find such a combination to compensate for strong differences in data, during which some elements of the arrays remained empty, and thereby to improve the accuracy of the classification. The result of this study can be seen in Figure 1. Here, the x-axis shows the number of partitions of the time interval (N), and the y-axis shows the assessment of accuracy in comparison with manual classification. Three steps of the partition by degrees were considered: 1° (shown with diamonds), 2° (shown with circles), and 3° (shown with squares). As can be seen from the figure, the maximum accuracy corresponds to the partition of the interval into 7 subintervals for 2° partition (in this case, the accuracy is 91%). Note that the decrease in the resulting accuracy in the case of partition into 8 subintervals is most likely due to the short length of the time series. Next, we consider the examples of using the developed program.

2.3 Examples of classification of resonant arguments of asteroids

When studying the evolution of an object, at the first stage of classification using the IDA software package (Galushina, Letner, 2021), text files were obtained containing time series of critical (resonant) arguments of asteroids. These series are formed for all possible values of parameters k_3, k_4, k_5, and k_6 according to formulas (3) and (4). Note that for different resonance relationships, the number of such files is different. For example, for a 7/1 resonance with a planet, there will be 44 files, which is quite labor-intensive for research without using automation. Subsequently, using an automation program, the entire multiplet was studied and graphs were plotted for all possible combinations of parameters k_3, k_4, k_5, and k_6.

Asteroids with small perihelion distances were chosen as objects of study. For these objects, a preliminary search for orbital resonances was performed using the modified IDA program. As a result of the study, it was determined that 33 objects move in the vicinity of resonances with one or several planets simultaneously (Galushina et al., 2023). Figure 2 shows an example of the classification of resonant arguments for the asteroid 2013 YC, which is at 3/1 resonance with Jupiter. The multiplet includes 6 arguments.

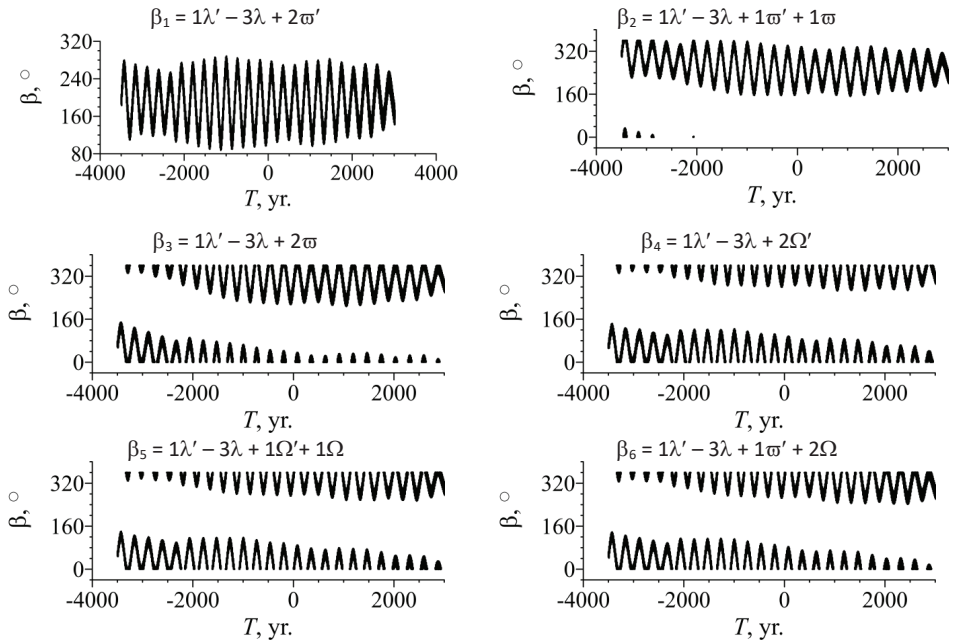


Figure 2 Behavior of the resonant multiplet arguments corresponding to the 3/1 resonance of the asteroid 2013 YC with Jupiter.

The developed software system classified this type of the resonance relationship as libration for the entire multiplet. Indeed, it can be seen from Figure 2 that in this case, the critical arguments of the resonant multiplet librate and this type of the resonance is characteristic of the entire multiplet. In addition, the developed program allows us to automatically build graphs and display percentage ratios of different classes.

3 Study of orbital resonances in the motion of artificial Earth satellites

As mentioned above, not only asteroids, but also objects located in the near-Earth space are affected by orbital resonances. The uneven distribution of masses inside the Earth creates a complex gravitational effect on an artificial satellite, introducing perturbations into its motion that can lead to chaotic motion and deviation of the orbit from the nominal one. This may create a risk of collision with other objects, including functioning ones.

To test the program, we consider as an example a 1/2 resonance with the Earth's rotation speed. In this case, the critical (resonant) arguments and frequencies of orbital resonance with the Earth's rotation speed are calculated in accordance with the Allan's method (Allan, 1967a; 1967b) supplemented in (Kuznetsov et al., 2012; Tomilova et al. 2020) using formulas (5) and (6).

$$\begin{aligned}
\Phi_1 &= M + \Omega + \omega - 2\theta \\
\Phi_2 &= M + \omega + 2(\Omega - \theta) \\
\Phi_3 &= M + 2(\Omega + \omega - \theta) \\
\Phi_4 &= \Phi_1 - 2\Omega \\
\Phi_5 &= \Phi_3 + 2(\Omega - 2\omega)
\end{aligned} \tag{5}$$

$$\begin{aligned}
\dot{\Phi}_1 &= \dot{M} + \dot{\Omega} + \dot{\omega} - 2\dot{\theta} \\
\dot{\Phi}_2 &= \dot{M} + \dot{\omega} + 2(\dot{\Omega} - \dot{\theta}) \\
\dot{\Phi}_3 &= \dot{M} + 2(\dot{\Omega} + \dot{\omega} - \dot{\theta}) \\
\dot{\Phi}_4 &= \dot{\Phi}_1 - 2\dot{\Omega} \\
\dot{\Phi}_5 &= \dot{\Phi}_3 + 2(\dot{\Omega} - 2\dot{\omega})
\end{aligned} \tag{6}$$

where Ω is the longitude of the ascending node, ω is the perigee argument, M is the mean anomaly of the satellite's orbit, and θ is the Greenwich sidereal time.

3.1 Methodology for automatic classification of resonances of artificial Earth satellites

The classification of resonances was based on the approach (Sekhar et al., 2016) described above. Each set of data on the evolution of the resonant argument was divided into cells, forming a grid with a step of 30° along the argument value axis and with a step of 5 years along the time axis. To identify the libration of the resonant argument, an additional array was formed with a 15° partition. An illustration of the resulting partitions is shown in Figures 3 and 4.

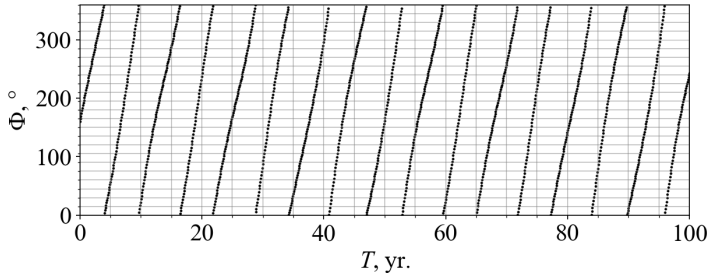


Figure 3 Partition of the resonant argument values for automatic classification.

If the resonant argument circulates (changes monotonically) and almost all grid cells are filled, as shown in Figure 3, then, in this situation no resonance is observed. During classification, the maximum number of empty grid cells allowed for circulation varied from 3 to 30.

The case shown in Figure 4, in which there are zero elements (rows) in the array, highlighted in dark gray, is classified as libration.

However, there is often a case in which it is impossible to select completely empty rows in a partition, but there are groups of cells in it that do not contain at least one

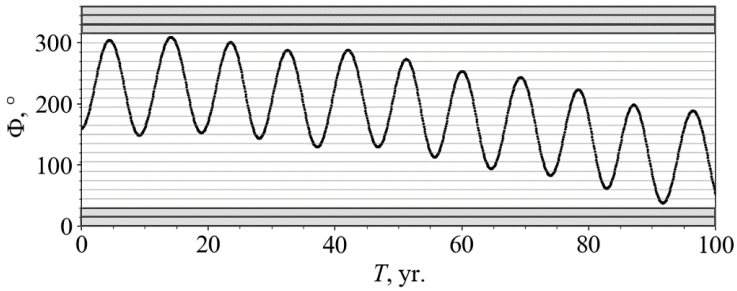


Figure 4 Additional partition of the resonant argument values for automatic classification of libration.

point. Such cases are classified as unstable resonances because the resonant argument experiences both librational and circulation changes with time. An example is shown in Figure 5.

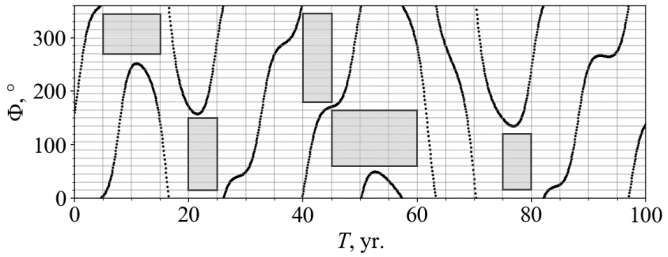


Figure 5 Partition grid for a mixed change in the resonant argument in time.

3.2 Results of automatic classification

The operation of the classifier was tested based on a study of the orbits of model objects uniformly distributed along the semimajor axis in the range from 26500 to 26600 km with a step of 0.5 km. The inclination of the orbits varied from 0 to 180° with a step of 2°. The initial eccentricity of the orbits was 0.001, the longitude of the ascending node Ω_0 and the perigee argument ω_0 were equal to 0°. Numerical modeling of the dynamics of objects was carried out over a time interval of 100 years. In total, the motion of 18291 objects was simulated.

As can be seen from Figure 6, the automatic classification technique makes it possible to identify the regions of resonant motion with fairly high accuracy. However, this approach has a drawback. The fact is that in the region of stable 1/2 resonance with the Earth's rotation speed, the resonant arguments are characterized by libration with a shifting center (Figure 7), but this behavior is classified as mixed, corresponding to an unstable resonance. Here, the best result was obtained for the Φ_1 component.

The classification accuracy for various arguments using the proposed algorithm is presented in Table 1.

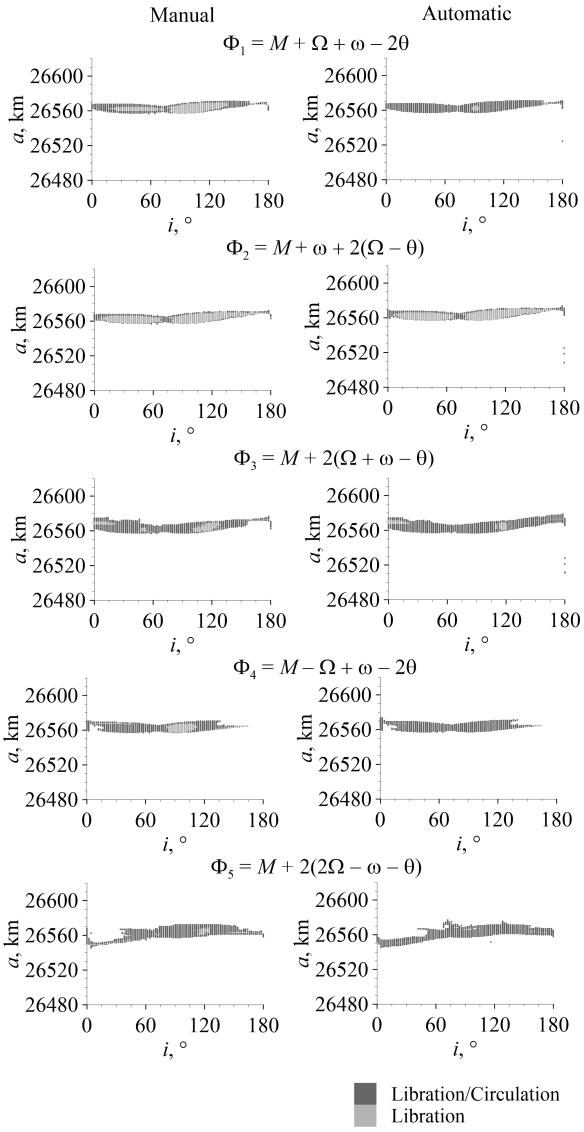


Figure 6 Regions of action of orbital 1/2 resonances with the Earth's rotation speed during manual and automatic processing

As can be seen from Table 1, for components Φ_1–Φ_4, the algorithm gives fairly high accuracy already at 3 empty cells. Further increase in this parameter only slightly increases the accuracy. At the same time, to classify the behavior of the resonant arguments Φ_5, it is necessary to increase the number of empty cells. This can be explained by the fact that the order of the resonance relationship Φ_5 (see formula 5) is higher than that of the other components of the resonant multiplet, and its behavior has a more complex structure.

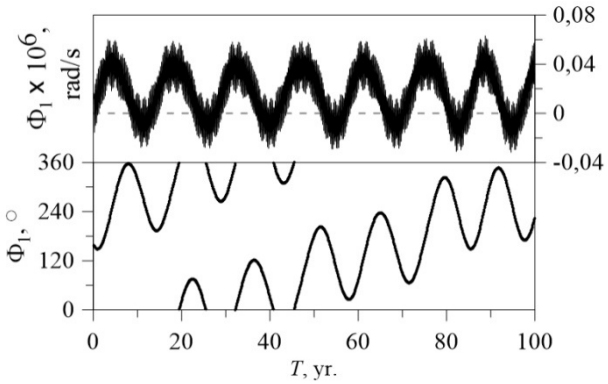


Figure 7 An example of libration with a shifting center.

Table 1 Accuracy of automatic classification of orbital resonance.

Allowed number of empty cells	Resonant argument				
	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5
3	95.99%	99.64%	96.87%	98.26%	87.22%
10	96.00%	99.71%	97.38%	98.22%	93.32%
20	96.00%	99.75%	97.41%	98.20%	95.26%
30	96.00%	99.74%	97.32%	98.10%	95.77%

4 Conclusions

The paper considers two options for automatic classification of resonant arguments: using convolutional neural networks and by analyzing the argument values. In the course of experimental studies, it was shown that both programs are capable of identifying and classifying the resonant arguments of small bodies, which makes it possible to speed up the process of analyzing the dynamics of these objects. However, the second option is more visual, while the neural network is essentially a black box. The paper also presents the examples of application of the developed software system to the analysis of the dynamics of asteroids and artificial Earth satellites.

ACKNOWLEDGEMENTS

This work was supported by the Russian Science Foundation (Scientific Project No. 19-72-10022), https://rscf.ru/en/project/19-72-10022/.

This research was supported in part through computational resources of the supercomputer "SKIF Cyberia" of Tomsk State University.

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