Astronomical and Astrophysical Transactions (*AApTr*), 2017, Vol. 30, Issue 1, pp. 95–114,
ISSN 1055-6796, Photocopying permitted by license only, ©Cambridge Scientific Publishers

Testing dark energy models through large scale structure

O. Avsajanishvili[1,*], L. Samushia[1,2], N.A. Arkhipova[3], T. Kahniashvili[1,4,5]

[1]*Abastumani Astrophysical Observatory, Ilia State University, 3–5 Cholokashvili Ave., Tbilisi, 0194, Georgia*
[2]*Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS, 66506, USA*
[3]*Astro Space Center, P.N. Lebedev Physical Institute, 84/32 Profsoyuznaya Str., Moscow, 117997 Russia*
[4]*McWilliams Center for Cosmology and Department of Physics, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213 USA*
[5]*Department of Physics, Laurentian University, Ramsey Lake Road, Sudbury, ON P3E 2C, Canada*

Received 15 December, 2015

Material presented in this paper was reported at the 5th International Gamow Conference, Odessa, Ukraine, August 2015. The paper is based on Avsajanishvili et al. (2014) (Avsajanishvili, O., Arkhipova, N.A., Samushia, L., & Kahniashvili, T. 2014, Eur. Phys. J., C74, 3127).

We explore the scalar field model of dark energy with the Ratra–Peebles ϕCDM potential. Cosmic expansion and the large scale structure growth rate are studied. The equations for background expansion and for the growth rate of matter perturbations are solved jointly. The obtained theoretical results are compared with the observational data. Recent measurements of the growth rate and the baryon acoustic oscillation peak positions are used to constrain the matter density parameter $\Omega_{\rm m}$ and the model parameter α that describes the steepness of the scalar field potential. The larger value of the parameter α implies stronger time dependence of the scalar field. For the Ratra–Peebles model the expansion of the Universe occurs faster with the parameter α increasing. The scalar field begins to be dominant earlier with the value of the parameter α increasing. The Ratra–Peebles ϕCDM model predicts a slower growth rate than the ΛCDM model. The Linder γ-parametrization works well for the Ratra–Peebles ϕCDM model in the range of the redshifts $z \in (0; 5)$.

Keywords: Dark energy, scalar field, expansion rate, growth rate, growth index, large scale structure

*Email: olga.avsad@gmail.com

1 Introduction

According to the cosmological observations our Universe expands with an acceleration (Perlmutter et al., 1999; Riess et al., 1998, 2007). There are several models for explanation of this phenomenon. The most common approach is to assume that order of 70% of the energy density of the Universe is present in the form of dark energy (DE).

The simplest model of DE admits that DE is the vacuum energy that is given in the form of the time-independent cosmological constant Λ. This model is referred to as a *concordance* model since it is in a very good agreement with all available today cosmological observations. The ΛCDM model, however, suffers from the fine tuning and the coincidence problems (Carroll, 2001; Padmanabhan, 2003; Peebles & Ratra, 2003; Martin, 2012). To alleviate these problems, other models of DE have been proposed (Caldwell & Steinhardt, 1998; Armendariz-Picon et al., 1999; Amendola, 2000; Wetterich, 1995; Kamenshchik et al., 2001; Capozziello et al., 2003; Dvali et al., 2000; Shi & Baugh, 2015; Duniya, 2015; Chen & Xu, 2016; Pradhan et al., 2015). The main alternatives of the ΛCDM model are the models involving a dynamical scalar field, so called the ϕCDM models. This family of models avoids some theoretical foundation difficulties, namely: the fine tuning problem, having a more natural explanation for the observed low energy scale of DE (Zlatev et al., 1999). For the ΛCDM model the equation of state parameter w is a constant and is equal to minus one. For the ϕCDM models the equation of state parameter is time dependent (we will use below the scale factor function $\mathrm{w}(a)$) and approaches to minus one today, i.e. $\mathrm{w}(a_0) \to -1$, here a_0 is the today value of the scale factor normalized to be one, $a_0 \equiv a_{\mathrm{today}} = 1$ (Caldwell & Linder, 2005; Yoo & Watanabe, 2012).

Depending on the value of the equation of state parameter today, the ϕCDM models are divided into two classes: the phantom models (Scherrer & Sen, 2008a) ($-1 < \mathrm{w} < -1/3$) and the quintessence models (Schimd et al., 2007) ($\mathrm{w} < -1$). The quintessence models are subdivided into the thawing models, for which the evolution of the scalar field is fast (Scherrer & Sen, 2008b; Linder, 2015; Lima et al., 2015) and the tracking (freezing) models, for which the evolution of the scalar field is slow, compared to the Hubble expansion (Caldwell et al., 1998; Caldwell & Linder, 2005; La Vacca & Kristiansen, 2009; de Putter & Linder, 2008; Chiba et al., 2013). In the tracking models, the scalar field has a tracking solution, in which the scalar field energy density, remaining subdominant, tracks at first the radiation and then the matter energy densities (Brax & Martin, 2002; Steinhardt et al., 1999). At late times, the scalar field becomes the dominant component and starts to behave as a component with the effective negative pressure that leads at the late stages to the accelerated expansion of the Universe. The simplest representation of such a model is the scalar field model, when the scalar field potential is given through an inverse power law, Ratra–Peebles potential (Ratra & Peebles, 1988). We will refer to this model below as the ϕCDM Ratra–Peebles model.

The background expansion history (as well as the large scale structure growth rate) is different for the scalar field (ϕCDM) model and for the concordance (ΛCDM) model. Thus the ϕCDM model can be distinguished from the ΛCDM model through the high precision measurements of distances and growth rates over a wide redshift range (Samushia et al., 2012; di Porto et al., 2012; Pavlov et al., 2012; Bueno Belloso

et al., 2011; Fu et al., 2010; Lee & Ng, 2010; Pace et al., 2012; Ahn et al., 2009; de Putter & Linder, 2008; Silvestri & Trodden, 2009).

We present the solutions of the joint equations for the background expansion and for the growth rate of matter perturbations. We use a compilation of the recent measurements of the growth rate and the baryon acoustic oscillation (BAO) to put constraints on the α and the $\Omega_{\rm m}$ parameters.

This paper is organized as follows. In Sec. 2 we investigate the dynamics and the energy of the Ratra–Peebles ϕCDM models. In Sec. 3 we explore the parametrization of the equation of state parameter $\mathrm{w}(a)$ in the Ratra–Peebles ϕCDM model by the different models. In Sec. 4 we study the influence of the Ratra–Peebles ϕCDM models on the growth rate. In Sec. 5 we present the comparison of the obtained theoretical results with the observational data. We discuss our results and conclude in Sec. 6. We use the natural units with $c = \hbar = 1$ throughout this paper.

2 Background dynamics in the Ratra–Peebles ϕCDM model

2.1 Background equations

We study the ϕCDM model with the Ratra–Peebles potential given by (Ratra & Peebles, 1988):

$$V(\phi) = \frac{\kappa}{2} M_{\rm pl}^2 \phi^{-\alpha}, \tag{1}$$

where $M_{\rm pl}$ is the Planck mass, κ is a model parameter, and α is a positive constant, which determines the steepness of the scalar field potential. Current observational data suggest that α can not be larger than $\alpha \leq 0.7$ (Samushia, 2009; Pavlov et al., 2012; Farooq & Ratra, 2013; Schimd et al., 2007). A larger value of the α parameter implies a stronger time dependence of the scalar field potential $V(\phi)$. In the limit of $\alpha = 0$, the Ratra–Peebles ϕCDM model reduces to the ΛCDM model.

The equation of motion for the scalar field is (Ratra & Peebles, 1988; Sahni, 2002):

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{2}\kappa\alpha M_{\rm pl}^2 \phi^{-(\alpha+1)} = 0, \tag{2}$$

where an over-dot represents the derivative with respect to a physical time t, $H(a) = \dot{a}/a$ is the Hubble parameter, the scale factor is $a = 1/(1+\mathrm{z})$, and z is a redshift.

The energy density and the pressure of the scalar field are (Ratra & Peebles, 1988; Sahni, 2002):

$$\rho_\phi = \frac{M_{\rm pl}^2}{32\pi}\Big(\dot{\phi}^2 + \kappa M_{\rm pl}^2 \phi^{-\alpha}\Big), \tag{3}$$

$$P_\phi = \frac{M_{\rm pl}^2}{32\pi}\Big(\dot{\phi}^2 - \kappa M_{\rm pl}^2 \phi^{-\alpha}\Big), \tag{4}$$

and the corresponding equation of state w parameter is

$$\mathrm{w} = \frac{\dot{\phi}^2 - \kappa M_{\rm pl}^2 \phi^{-\alpha}}{\dot{\phi}^2 + \kappa M_{\rm pl}^2 \phi^{-\alpha}}. \tag{5}$$

The scalar field energy density parameter is defined by

$$\Omega_\phi(a) = \frac{1}{12H_0^2}(\dot{\phi}^2 + \kappa M_{\rm pl}^2 \phi^{-\alpha}), \tag{6}$$

and the first Friedmann equation for the Ratra–Peebles ϕCDM model in the spatially-flat Universe is:

$$E^2(a) = \Omega_{\rm r0} a^{-4} + \Omega_{\rm m0} a^{-3} + \frac{1}{12H_0^2}(\dot{\phi}^2 + \kappa M_{\rm pl}^2 \phi^{-\alpha}), \tag{7}$$

where $E(a) = H(a)/H_0$, with $H_0 = 100h$km/s/Mpc is a value of the Hubble parameter today; $\Omega_{\rm r0}$, $\Omega_{\rm m0}$, and $\Omega_{\phi 0}$ the dimensionless density parameters for radiation, matter and DE at present time. During the late stages of the expansion of the Universe (after radiation–matter equality) we can neglect the radiation term in Eq. (7). In what follows we will assume fiducial values $\Omega_{\rm m0} = 0.315$, $\Omega_{\phi 0} = 0.685$, $h = 0.673$ consistent with (Ade et al., 2014).

2.2 Initial conditions

We integrate the set of equations Eq. (2), Eq. (6), and Eq. (7) numerically, starting from $a_{\rm in} = 5 \cdot 10^{-5}$ to the present time $a_0 = 1$. We assume the following initial conditions for the scalar field amplitude and its time derivative:

$$\phi_{\rm in} = \left[\frac{1}{2}\alpha(\alpha+2)\right]^{1/2} a_{\rm in}^{\frac{4}{\alpha+2}}, \tag{8}$$

$$\phi'_{\rm in} = \left(\frac{2\alpha}{\alpha+2}\right)^{1/2} a_{\rm in}^{\frac{2-\alpha}{2+\alpha}}, \tag{9}$$

$$\kappa = \left(\frac{\alpha+6}{\alpha+2}\right)\left[\frac{1}{2}\alpha(\alpha+2)\right]^{\alpha/2}, \tag{10}$$

where a prime denotes the differentiation with respect to the scale factor a. To obtain these initial conditions we use the fact that the expansion of the Universe during matter and radiation domination epochs has a power-law form and use the ansatz

$$a(t) = a_\star \left(\frac{t}{t_\star}\right)^n, \qquad \phi(t) = \phi_\star \left(\frac{t}{t_\star}\right)^p, \tag{11}$$

where $a_\star \equiv a(t_\star)$ and $\phi_\star \equiv \phi(t_\star)$ are the scale factor and the scalar field values at $t = t_\star$. The index n depends on the dominant component driving the expansion of the Universe and is $n = 1/2$ in radiation dominated epoch and $n = 2/3$ in matter dominated epoch. We solve the set of equations Eq. (2), Eq. (6), and Eq. (7), during the radiation (and/or the matter dominated) epochs, to obtain the general expressions for κ, and the scalar field amplitude ϕ, and its time-derivative $\dot{\phi}$ (which depend only on the α parameter and value of the index n; the details are given in Appendix A of (Avsajanishvili et al., 2014)):

$$\phi = n\alpha(\alpha+2)^{1/2}\left(\frac{a}{a_\star}\right)^{2/n(\alpha+2)}, \tag{12}$$

$$\kappa = \frac{4n}{M_{\rm pl}^2 t_\star^2}\Big(\frac{6n+3n\alpha-\alpha}{\alpha+2}\Big)[n\alpha(\alpha+2)]^{\alpha/2}. \tag{13}$$

We set $t_\star = M_{\rm pl}^{-1}$ and obtain Eq. (8), Eq. (9), and Eq. (10) assuming the initial conditions are set in radiation dominated epoch ($n = 1/2$).

2.3 The dynamics and the energy of the Ratra–Peebles ϕCDM model

In this subsection we examine the evolution of the equation of state parameter $\mathrm{w}(a)$ and its scale factor derivative $\mathrm{w}'(a)$ for different values of the α parameter (see Fig. 1).

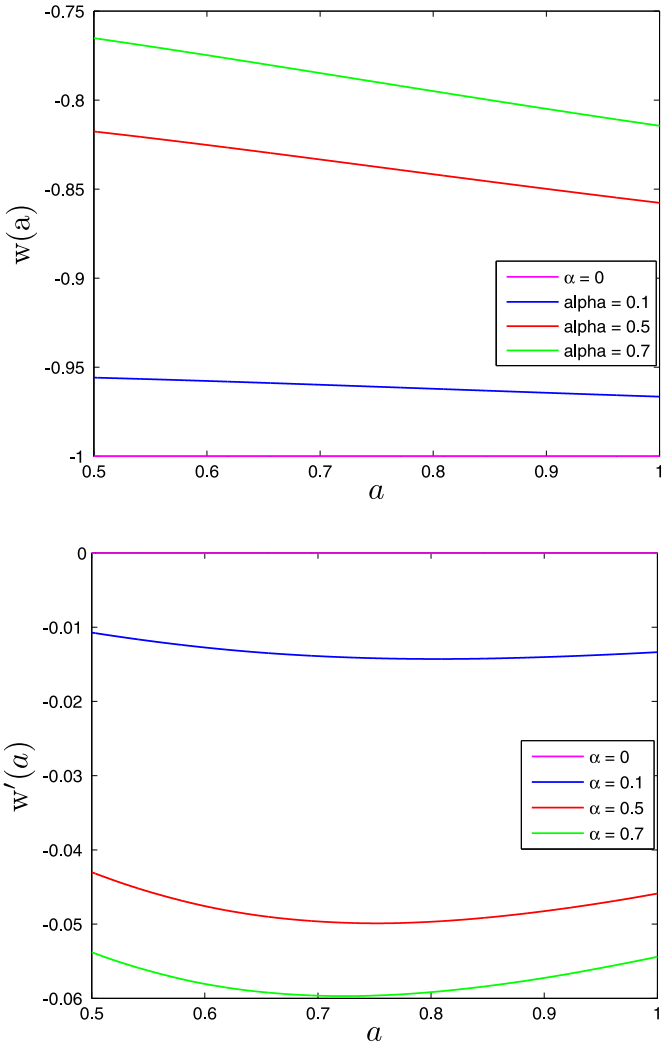


Figure 1 1a (upper panel) The DE equation of the equation of state parameter $\mathrm{w}(a)$ for different values of the α parameter. 1b (lower panel) The scale factor derivative $\mathrm{w}'(a)$ of the equation of state parameter for different values of the α parameter.

The equation of state parameter $w(a)$ is a decreasing function of time (with the increasing scale factor): that is a specific feature of the freezing models. In fact, a large value of the α parameter results in a stronger time dependence for the equation of state parameter $w(a)$ (see Fig. 1a) and its scale factor derivatives $w'(a)$ (see Fig. 1b).

Next, we investigate the influence of the scalar field ϕ on the expansion rate of the Universe. For the Ratra–Peebles ϕCDM model this expansion occurs faster with increasing value of the α parameter (see Fig. 2a). The ΛCDM limit corresponds to the slowest rate of the expansion of the Universe.

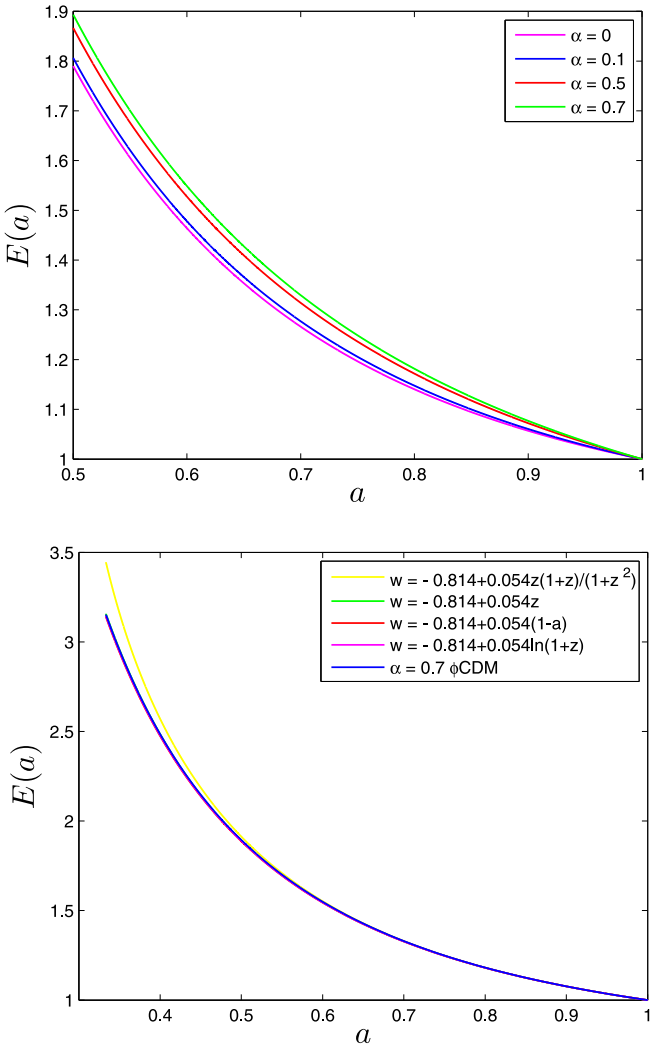


Figure 2 2a (upper panel) The normalized Hubble expansion rate $E(a)$ for the Ratra–Peebles ϕCDM model for different values of the α parameter. 2b (lower panel) The normalized Hubble expansion rate $E(a)$ for the BA, CH, GE, CPL parametrizations of $w(a)$ and for the Ratra–Peebles ϕCDM model, $\alpha = 0.7$.

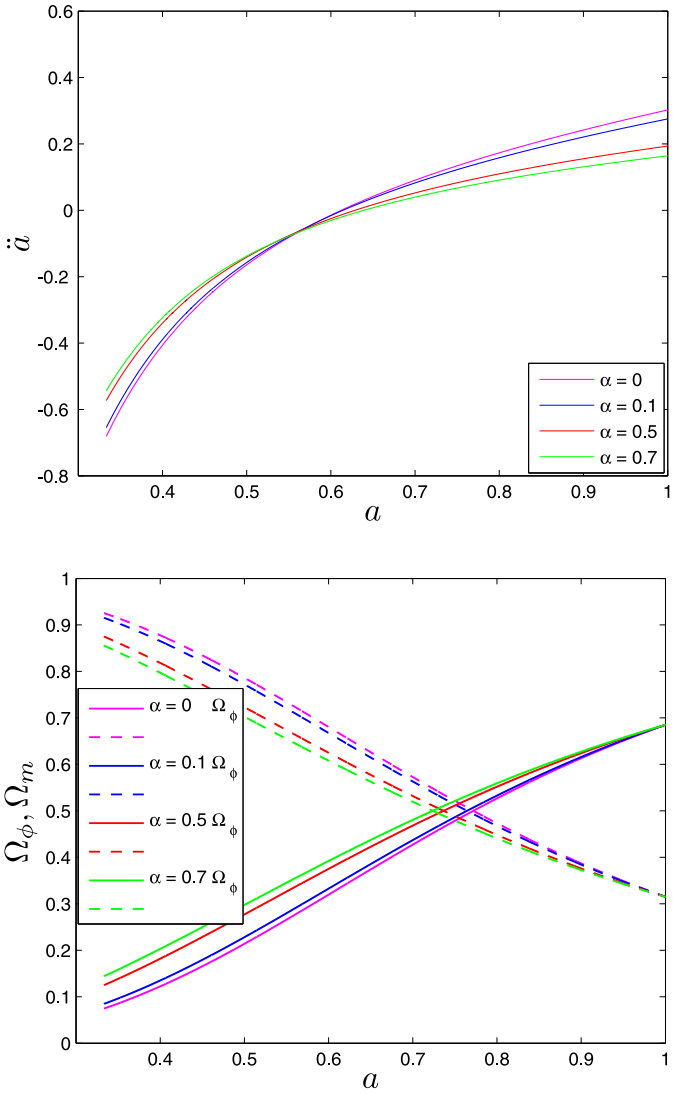


Figure 3 3a (upper panel) The second derivative of the scale factor. 3b (lower panel) The matter energy density $\Omega_{\rm m}(a)$ (dashed lines) and the scalar field energy density $\Omega_\phi(a)$ (solid lines) as functions of the scale factor for different values of the α parameter.

We also study the background dynamics (see Fig. 3a), and the evolution of the energy density of the matter and DE components, $\Omega_{\rm m}$(a) and Ω_ϕ(a) respectively (see Fig. 3b) for the Ratra–Peebles ϕCDM model. As we can see in the Ratra–Peebles ϕCDM model DE begins to be a dominant component earlier than in the ΛCDM model, and the effect is stronger for the larger value of the α parameter (see Fig. 3b), and thus the duration of the matter dominated epoch becomes shorter.

3 Parametrization of the equation of state parameter w(a) in the Ratra–Peebles ϕCDM model

There are several ways to parametrize the equation of state parameter w(a), including:

- Coorkay and Huterer (CH) (Cooray & Huterer, 1999)
  $w(a) = w_0 + w_a z$,

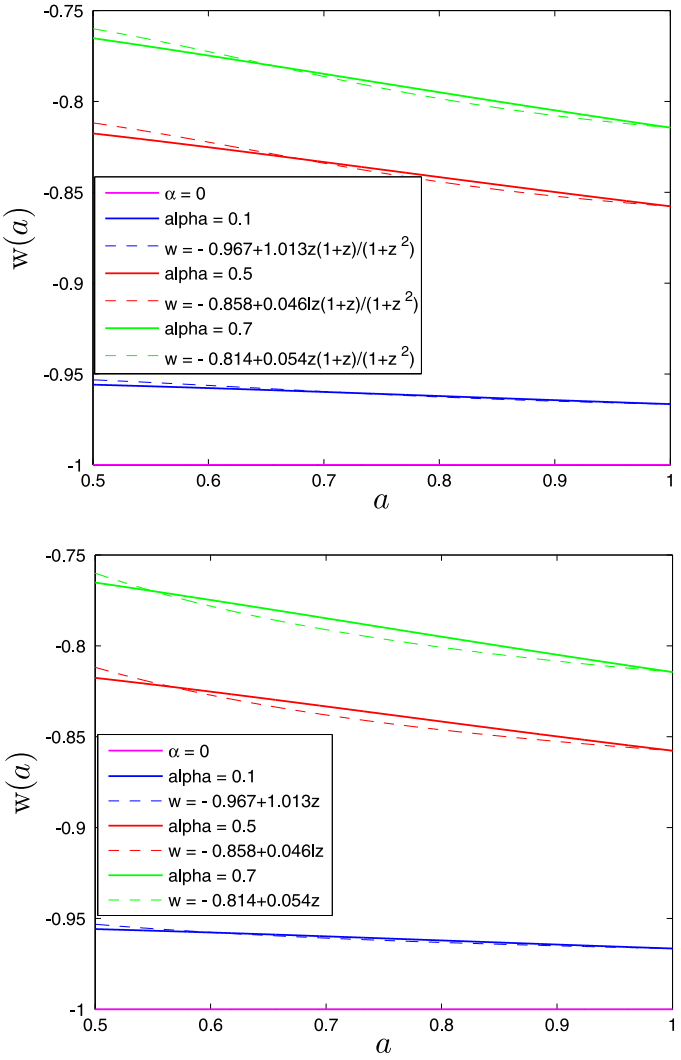


Figure 4 4a (upper panel) w(a) for different values of the α parameter along with predictions computed for the BA parametrization with the corresponding best-fit values for w_0 and for w_a. 4b (lower panel) w(a) for different values of the α parameter along with predictions computed for the CH parametrization with the corresponding best-fit values for w_0 and for w_a.

- Gerke and Estathiou (GE) parametrization (Efstathiou, 1999)
  $w(a) = w_0 + w_a \ln(1+z)$,
- Chevallier and Polarski, and Linder (CPL) parametrization (Chevallier & Polarski, 2001; Linder, 2003)
  $w(a) = w_0 + w_a(1-a)$,
- Barboza and Alcaniz (BA) parametrization (Barboza & Alcaniz, 2008)
  $w(a) = w_0 + w_a \frac{z(1+z)}{1+z^2}$,

where w_0 corresponds to the present day of the equation of state parameter $w(a)$, and $w_a = (dw/dz)|_{z=1}$.

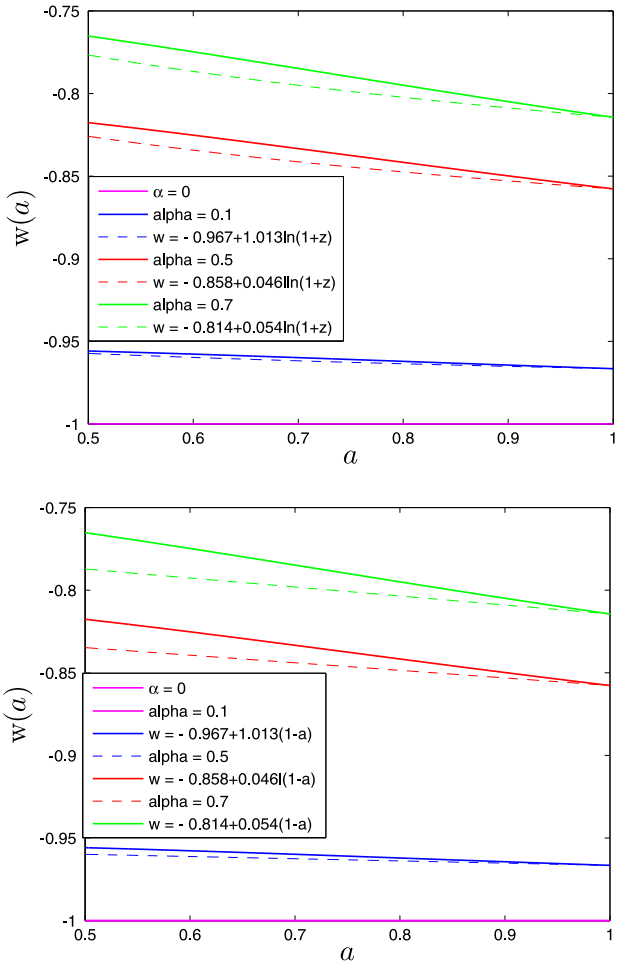


Figure 5 5a (upper panel) $w(a)$ for different values of the α parameter along with predictions computed for the GE parametrization with the corresponding best-fit values for w_0 and for $w(a)$. 5b (lower panel) $w(a)$ for different values of the α parameter along with predictions computed for the CPL parametrization with the corresponding best-fit values for w_0 and for w_a.

The parametrization of the equation of state parameter w(a) is used as a method to distinguish the different DE models among themselves (Scherrer, 2015). In particular, this approach can be used to distinguish the ΛCDM and the ϕCDM models at present moment. We present our results on Figs. 4–5. The BA and CH parametrizations fit the equation of state parameter w(a) in the Ratra–Peebles ϕCDM model in the range of the redshifts z $\in (0;1)$ better than the GE and CPL parametrizations. The CH, GE, CPL, BA w(a) parametrizations and the function w(a) in the Ratra–Peebles ϕCDM model, for $\alpha = 0.7$ in the range of the redshifts z $\in (0;1)$ are represented on Fig. 6a.

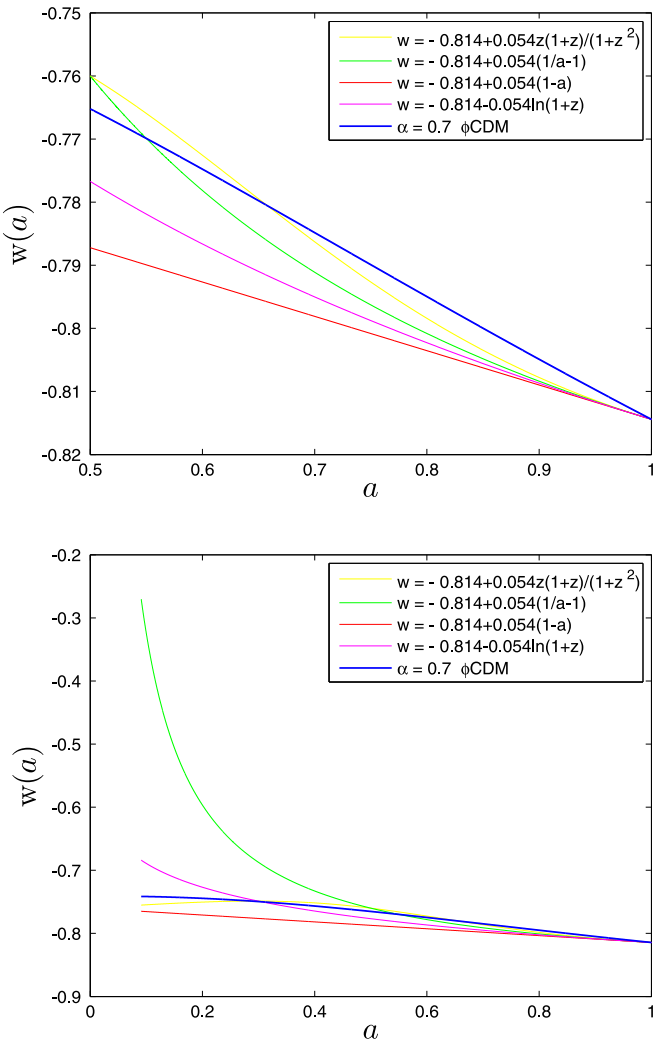


Figure 6 6a (upper panel) The CH, GE, CPL, and BA parametrizations of the w(a) and w(a) for the Ratra–Peebles ϕCDM model, $\alpha = 0.7$ in the range of the redshifts z $\in (0;1)$. 6b (lower panel) The BA, CH, GE, CPL parametrizations of the w(a) and w(a) for the Ratra–Peebles ϕCDM model, $\alpha = 0.7$ in the range of the redshifts z $\in (0;10)$.

But for the early epochs (large redshifts), the CPL and BA w(a) parametrizations approximate the equation of state parameter w(a) in the Ratra–Peebles ϕCDM model for the large redshifts better than the BA and CH parametrizations (see Fig. 6b).

We evaluate the Universe expansion rate for the BA, CH, GE and CPL parametrizations (Carroll, 2004):

$$E^2(a) = \Omega_{\rm m0} a^{-3} + \Omega_{\phi 0} e^{-3\int_1^a \frac{{\rm d}a}{a}(1+{\rm w}(a))}, \tag{14}$$

and we compare them with the value of the expansion rate for the Ratra–Peebles ϕCDM model. The expansion rates for all aforementioned parameterizations of w(a) fit well the expansion rate for the Ratra–Peebles ϕCDM model for $\alpha = 0.7$ in the range of the redshifts z $\in (0; 0.6)$ (see Fig. 2b).

4 Growth factor of matter density perturbations in the Ratra–Peebles ϕCDM model

In this section we investigate the influence of the scalar field (of the Ratra–Peebles ϕCDM model) on the growth of structure (Pavlov et al., 2014; Taddei & Amendola, 2015). We integrate numerically the linear perturbation equation Eq. (15) with respect to the relative density contrast $\delta \equiv \delta\rho_m/\overline{\rho}$, where $\overline{\rho}$ and $\delta\rho_m$ are the mean density and overdensity of the matter component respectively. We apply the initial conditions: $\delta(a_{\rm in}) = \delta^{'}(a_{\rm in}) = 5 \cdot 10^{-5}$ (Pace et al., 2010; Campanelli et al., 2012).

$$\delta^{''} + \Big(\frac{3}{a} + \frac{E^{'}}{E}\Big)\delta^{'} - \frac{3\Omega_{\rm m0}}{2a^5E^2}\delta = 0, \tag{15}$$

Equation (15) describes completely the dynamical evolution of matter perturbations, assuming that the perturbed fluid is a perfect one. We study the evolution of the perturbations through the linear growth factor $D(a) = \dfrac{\delta(a)}{\delta(a_0)}$, where $\delta(a_0)$ is a value of the density contrast today. We normalize the linear growth factor $D(a)$ to be unity today, i.e. $D(a_0) = 1$.

For the Ratra–Peebles ϕCDM model a larger value of the α parameter implies a stronger time dependence of the linear growth factor $D(a)$, see Fig. 7a. This is because the growth of matter perturbations occurs only during matter dominated epoch (Frieman et al., 2008). The Hubble expansion takes place faster for larger values of the α parameter (see Fig. 2a), while the scalar field energy domination begins earlier, see Fig. 3b. As a result, the matter perturbations have less time to grow. To reach the same amplitude of matter perturbation $\delta(a_0)$ today, the models where the scalar field has a larger value of α parameter will require larger initial amplitudes.

The growth rate $f_2(a) = {\rm d}\ln D(a)/{\rm d}\ln a$ strongly depends on the fractional matter density $f_1(a) = \Omega_{\rm m0}a^{-3}/E^2$ and this dependence can be parametrized by a power-law relationship (Wang & Steinhardt, 1998)

$$f_2(a) \approx [f_1(a)]^{\gamma}, \tag{16}$$

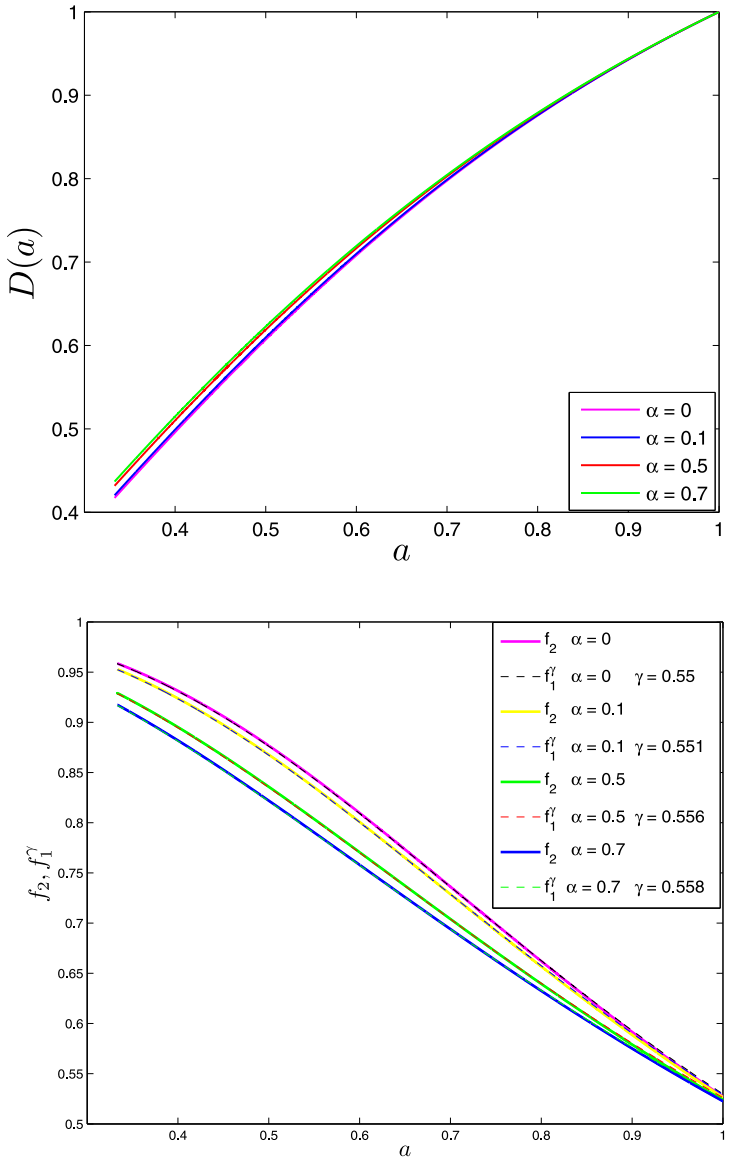


Figure 7 7a (upper panel) The linear growth D(a) as a function of the scale factor for different values of the α parameter. 7b (lower panel) The logarithmic growth rate as a function of the scale factor f_2 for different values of the α parameter (solid lines) along with the predictions f_1^{γ} (dashed lines), computed for the corresponding best-fit values of the γ parameter.

where γ is a growth index, the value of which depends on the model parameters. Assuming GR is correct, the value of γ-parameter depends on dark energy parameters as (Linder, 2005):

$$\gamma = 0.55 + 0.05(1 + \mathrm{w}_0 + 0.5\mathrm{w}_a), \quad \text{if} \quad \mathrm{w}_0 \geq -1. \tag{17}$$

For the ΛCDM model (with $\mathrm{w}_a = 0$, $\mathrm{w}_0 = -1$), the growth index $\gamma \approx 0.55$.

We study the applicability of the power-law approximation Eqs. (16) for the Ratra–Peebles ϕCDM model. This approximation works well for the Ratra–Peebles ϕCDM model (see Fig. 7b). The value of the growth index γ for the ϕCDM model depends on the α parameter. The value of the growth index γ for the ϕCDM model increases with increasing value of the α parameter, and it is slightly higher than one for the ΛCDM

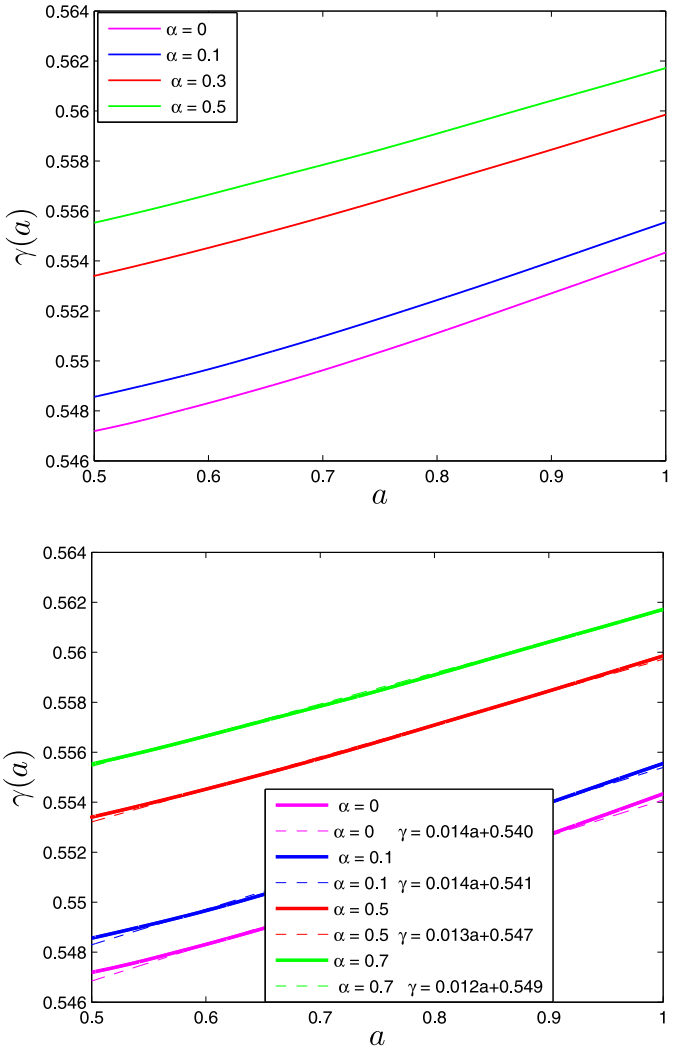


Figure 8 8a (upper panel) The $\gamma(a)$ function for different values of the α parameter in the range of the redshifts $\mathrm{z} \in (0; 1)$. 8b (lower panel) The $\gamma(a)$ function for different values of the α parameter along with predictions, computed from the linear parametrizations.

model ($\gamma \approx 0.55$). The growth rate of matter perturbations occurs slower with the increasing value of the α parameter (see Fig. 7b). These results show that the Hubble expansion and the growth of matter perturbations are interrelated and oppositely directed processes, and the faster Hubble expansion for the larger α parameter (see Fig. 2a) leads to the suppression of the growth of matter perturbations.

We examine the applicability of the Linder γ-parametrization on the large redshifts. This parametrization occurs in the range of the redshifts z $\in (0; 5)$ (see Fig. 9a) and for larger values of the redshifts the Linder γ-parametrization is not applicable.

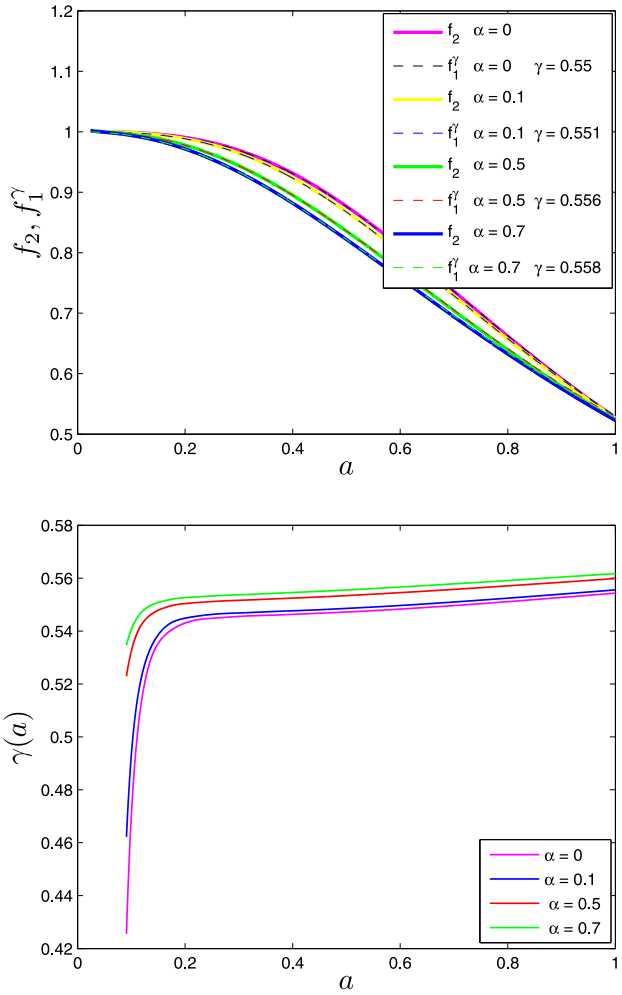


Figure 9 9a (upper panel) The logarithmic growth rate as a function of the scale factor f_2 for different values of the α parameter (solid lines) along with the predictions f_1^γ (dashed lines), computed for the corresponding best-fit values of the γ parameter in the range of redshifts z $\in (0; 10)$. 9b (lower panel) The $\gamma(a)$ function for different values of the α parameter in the range of the redshifts z $\in (0; 10)$.

We consider also the evolution of the $\gamma(a)$ function with its dependence on the scale factor a (Wu et al., 2009) for the Ratra–Peebles ϕCDM model. From Eqs. (16) we have (Linder & Cahn, 2007; Basilakos, 2015; Mehrabi et al., 2015):

$$\gamma(a) = \frac{\ln f_2(a)}{\ln f_1(a)}.$$

The $\gamma(a)$ functions in the range of the redshifts $z \in (0; 1)$ for different values of the α parameters are represented on Fig. 8a. We approximate the $\gamma(a)$ function for different values of the α parameters by the linear functions (see Fig. 8b).

Next, we investigate the behavior of the $\gamma(a)$ function on the large redshifts, see Fig. 9b. The linear dependence of the $\gamma(a)$ function breaks off in the range of the redshifts from $z \approx 3$ for $\alpha = 0$ till $z \approx 5$ for $\alpha = 0.7$. Thus the linear dependence of the $\gamma(a)$ function breaks off earlier for the ΛCDM model than for the ϕCDM models. Comparing Fig. 9a and Fig. 9b we see that the termination of the applicability of the Linder γ-parametrization for different values of the α parameter coincides with the moments of the termination of the linear dependence of the $\gamma(a)$ function.

5 Comparison with observations

In this section we carry out the observational constraints on the α and the $\Omega_{\rm m}$ parameters using χ^2 analysis, where calculated values of the growth rates are compared with the observational ones, obtained from the redshift space distortion surveys. For this purpose we use a compilation of the growth rate measurements from (Gupta et al., 2012).

The 1 and 2σ confidence contours resulting from this likelihood are presented on Fig. 10a. The likelihood contours in the $\alpha - \Omega_{\rm m}$ plane obtained from the growth rate data alone are highly degenerate. If we fix $\alpha = 0$ we get $\Omega_{\rm m} = 0.278 \pm 0.03$ which is within 1σ of the best-fit value obtained by Planck collaboration (Ade et al., 2014). To break the degeneracy between the $\Omega_{\rm m}$ and the α parameters we carry out the χ^2 BAO/CMB analysis (Giostri et al., 2012). Where we construct the ratio of the angular distance $d_{\rm A}$ and the distance scale $D_{\rm V}$:

$$\eta(z) \equiv d_{\rm A}(z_{\rm bao})/D_{\rm V}(z_{\rm bao}).$$

Assuming Gaussianity of the errorbars we compute

$$\chi^2_{\rm bao} = \boldsymbol{X}^{\rm T}\boldsymbol{C}^{-1}\boldsymbol{X}, \tag{18}$$

and a likelihood function

$$\mathcal{L}^{\rm bao}(\alpha, \Omega_{\rm m}, H_0) \propto \exp(-\chi^2_{\rm bao}/2), \tag{19}$$

where $\boldsymbol{X} = \eta_{\rm th} - \eta_{\rm m}$ and $\boldsymbol{C}$ is the covariance matrix of the measurements. To marginalize over parameter H_0 in $\mathcal{L}^{\rm bao}$ we take a Gaussian prior of $H_0 = 74.3 \pm 2.1$ from (Freedman et al., 2012). We assume that $\mathcal{L}^{\rm f}$ and $\mathcal{L}^{\rm bao}$ are independent and the combined likelihood is simply a product of the two. The results are presented on Fig. 10b.

As a result of the BAO/CMB analysis we have obtained the restrictions on the parameters $\Omega_{\rm m}$ and α. $\Omega_{\rm m}$ is now constrained to be within $0.26 < \Omega_{\rm m} < 0.34$ at 1σ confidence level. For α parameter we get $0 \leq \alpha \leq 1.3$ at 1σ confidence level.

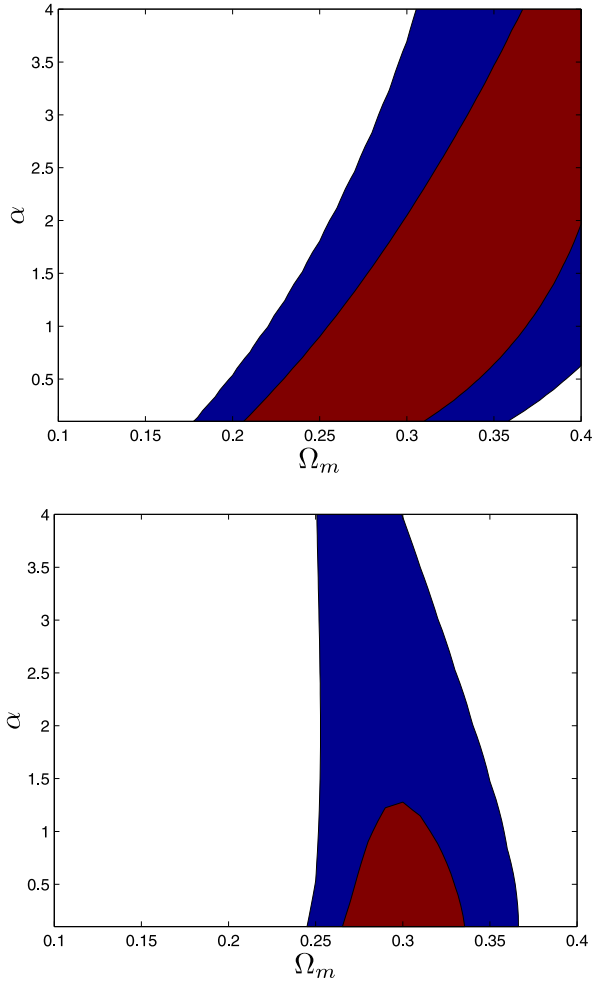


Figure 10 1σ and 2σ confidence level contours on the parameters $\Omega_{\rm m}$ and α for the Ratra–Peebles ϕCDM model. 10a (upper panel) The constraints obtained from the growth rate data (Gupta et al., 2012). 10b (lower panel) The constraints obtained after adding the BAO measurements and the cosmic microwave background (CMB) distance prior as in (Giostri et al., 2012) for the BAO/CMB distance prior.

6 Discussion and Conclusions

Analyzing the obtained results we can conclude that the Ratra–Peebles ϕCDM model differs from the ΛCDM model in a number of ways. These distinctive features are generic and do not depend on the specific values of the model parameters. In the Ratra–Peebles ϕCDM model the expansion rate of the Universe is always faster than for the ΛCDM model. The DE dominated epoch sets in earlier than for the ΛCDM model. The scalar field model predicts a slower growth rate than the ΛCDM model.

Below we summarize our results:

We have investigated the parametrization of the equation of state parameter $w(a)$ for the Ratra–Peebles ϕCDM model by the Coorkay–Huterer, Gerke–Estathiou, Chavalier–Polarsry–Linder, and Barboza–Alcaniz models. We have found that the Barboza–Alcaniz and Coorkay–Huterer parametrizations fit well the equation of state parameter $w(a)$ in the Ratra–Peebles ϕCDM model in the range of the redshifts $z \in (0; 1)$. While the Gerke–Estathiou and Chevallier–Polarski–Linder parametrizations fit well the equation of state parameter $w(a)$ for the Ratra–Peebles ϕCDM model on the large redshifts.

The expansion rates calculated for all aforementioned parametrizations of $w(a)$ fit well the expansion rates for the Ratra–Peebles ϕCDM model for all values of the α parameter in the range of the redshifts $z \in (0; 0.6)$.

We have explored the Linder γ-parametrization for the Ratra–Peebles ϕCDM and the ΛCDM models. The Linder γ-parametrization works well for both models and it is applicable in the range of the redshifts $z \in (0; 5)$.

We have found that the effective $\gamma(a)$ function can be approximated as a linear function in the range of the redshifts $z \in (0; 5)$, which coincides with the range of the redshifts applicability of the Linder γ-parametrization.

We have explored the observable predictions of the scalar field model. We have used a compilation of the BAO, the growth rate and the distance prior from the CMB to constrain the model parameters of the scalar field model. When only the growth rate data is applied, there is a strong degeneracy between the Ω_m and the α parameters.

Adding the BAO data and the distance prior from the CMB break the degeneracy resulting in $\Omega_m = 0.3$ and $\alpha \leq 1.3$ with the best fit of $\alpha = 0$.

Acknowledgements

We appreciate useful discussions with Leonardo Campanelli, Bidzina Chargeishvili, Gennady Chitov, Vasil Kukhianidze, Tengiz Mdzinarishvili, Anatoly Pavlov, Bharat Ratra, Robert Scherrer, and Alexander Tevzadze. We acknowledge partial support from Shota Rustaveli Georgian National Science Foundation grant FR/264/6-350/14, the Swiss NSF grant SCOPES IZ7370-152581, the CMU Berkman foundation, the NSF grants AST-1109180, and the program "The origin, structure and evolution of objects of the Universe" of the Presidium of the Russian Academy of Sciences (P-41).

References

Ade, P. A. R. et al. 2014, Astron. Astrophys., 571, A16
Ahn, C., Kim, C., & Linder, E. V. 2009, Phys. Rev., D80, 123016
Amendola, L. 2000, Phys. Rev., D62, 043511
Armendariz-Picon, C., Damour, T., & Mukhanov, V. F. 1999, Phys. Lett., B458, 209
Avsajanishvili, O., Arkhipova, N. A., Samushia, L., & Kahniashvili, T. 2014, Eur. Phys. J., C74, 3127
Barboza, Jr., E. M. & Alcaniz, J. S. 2008, Phys. Lett., B666, 415

Basilakos, S. 2015, Mon. Not. Roy. Astron. Soc., 449, 2151
Brax, P. & Martin, J. 2002, in Semaine de l'Astrophysique Francaise (sf2a) Paris, France, June 24–29, 2002
Bueno Belloso, A., García-Bellido, J., & Sapone, D. 2011, J. Cosmology Astropart. Phys., 10, 10
Caldwell, R. R., Dave, R., & Steinhardt, P. J. 1998, Phys. Rev. Lett., 80, 1582
Caldwell, R. R. & Linder, E. V. 2005, Phys. Rev. Lett., 95, 141301
Caldwell, R. R. & Steinhardt, P. J. 1998, Phys. Rev., D57, 6057
Campanelli, L., Fogli, G. L., Kahniashvili, T., Marrone, A., & Ratra, B. 2012, European Physical Journal C, 72, 2218
Capozziello, S., Carloni, S., & Troisi, A. 2003, Recent Res. Dev. Astron. Astrophys., 1, 625
Carroll. 2004, San Francisco: Addison-Wesley, 0906, 328
Carroll, S. M. 2001, Living Rev. Rel., 4, 1
Chen, Y. & Xu, L. 2016, Phys. Lett., B752, 66
Chevallier, M. & Polarski, D. 2001, Int. J. Mod. Phys., D10, 213
Chiba, T., De Felice, A., & Tsujikawa, S. 2013, Phys. Rev., D87, 083505
Cooray, A. R. & Huterer, D. 1999, Astrophys. J., 513, L95
de Putter, R. & Linder, E. V. 2008, JCAP, 0810, 042
di Porto, C., Amendola, L., & Branchini, E. 2012, MNRAS, 419, 985
Duniya, D. 2015, arXiv:1505.03436
Dvali, G. R., Gabadadze, G., & Porrati, M. 2000, Phys. Lett., B485, 208
Efstathiou, G. 1999, Mon. Not. Roy. Astron. Soc., 310, 842
Farooq, O. & Ratra, B. 2013, Physics Letters B, 723, 1
Freedman, W. L., Madore, B. F., Scowcroft, V., et al. 2012, ApJ, 758, 24
Frieman, J., Turner, M., & Huterer, D. 2008, Ann. Rev. Astron. Astrophys., 46, 385
Fu, X., Wu, P., & Yu, H. W. 2010, Eur. Phys. J., C68, 271
Giostri, R., Vargas dos Santos, M., Waga, I., et al. 2012, J. Cosmology Astropart. Phys., 3, 27
Gupta, G., Sen, S., & Sen, A. A. 2012, JCAP, 1204, 028
Kamenshchik, A. Yu., Moschella, U., & Pasquier, V. 2001, Phys. Lett., B511, 265
La Vacca, G. & Kristiansen, J. R. 2009, J. Cosmology Astropart. Phys., 7, 36
Lee, S. & Ng, K.-W. 2010, Phys. Rev., D82, 043004
Lima, N. A., Liddle, A. R., Sahln, M., & Parkinson, D. 2015, arXiv:1501.02678
Linder, E. V. 2003, Phys. Rev. Lett., 90, 091301
Linder, E. V. 2005, Phys. Rev., D72, 043529
Linder, E. V. 2015, Phys. Rev., D91, 063006
Linder, E. V. & Cahn, R. N. 2007, Astropart. Phys., 28, 481
Martin, J. 2012, Comptes Rendus Physique, 13, 566
Mehrabi, A., Basilakos, S., Malekjani, M., & Davari, Z. 2015, arXiv:1510.03996
Pace, F., Fedeli, C., Moscardini, L., & Bartelmann, M. 2012, MNRAS 422, 1186
Pace, F., Waizmann, J.-C., & Bartelmann, M. 2010, MNRAS, 406, 1865
Padmanabhan, T. 2003, Phys. Rept., 380, 235
Pavlov, A., Farooq, O., & Ratra, B. 2014, Phys. Rev., D90, 023006
Pavlov, A., Samushia, L., & Ratra, B. 2012, ApJ, 760, 19
Peebles, P. J. E. & Ratra, B. 2003, Rev. Mod. Phys., 75, 559
Perlmutter, S. et al. 1999, Astrophys. J., 517, 565
Pradhan, A., Saha, B., & Rikhvitsky, V. 2015, Indian J. Phys., 89, 503

Ratra, B. & Peebles, P. J. E. 1988, Phys. Rev., D37, 3406
Riess, A. G. et al. 1998, Astron. J., 116, 1009
Riess, A. G. et al. 2007, Astrophys. J., 659, 98
Sahni, V. 2002, Class. Quant. Grav., 19, 3435
Samushia, L. 2009, PhD thesis, Proquest Dissertations And Theses 2009. Section 0100, Part 0606 107 pages; [Ph.D. dissertation].United States – Kansas: Kansas State University; 2009. Publication Number: AAT 3380411. Source: DAI-B 70/12, Jun 2010
Samushia, L., Percival, W. J., & Raccanelli, A. 2012, MNRAS, 420, 2102
Scherrer, R. J. 2015, Phys. Rev., D92, 043001
Scherrer, R. J. & Sen, A. A. 2008a, Phys. Rev., D78, 067303
Scherrer, R. J. & Sen, A. A. 2008b, Phys. Rev., D77, 083515
Schimd, C., Tereno, I., Uzan, J.-P., et al. 2007, Astron. Astrophys., 463, 405
Shi, D. & Baugh, C. M. 2015, arXiv:1511.00692
Silvestri, A. & Trodden, M. 2009, Rept. Prog. Phys., 72, 096901
Steinhardt, P. J., Wang, L.-M., & Zlatev, I. 1999, Phys. Rev., D59, 123504
Taddei, L. & Amendola, L. 2015, JCAP, 1502, 001
Wang, L.-M. & Steinhardt, P. J. 1998, Astrophys. J., 508, 483
Wetterich, C. 1995, Astron. Astrophys., 301, 321
Wu, P., Yu, H. W., & Fu, X. 2009, JCAP, 0906, 019
Yoo, J. & Watanabe, Y. 2012, International Journal of Modern Physics D, 21, 30002
Zlatev, I., Wang, L.-M., & Steinhardt, P. J. 1999, Phys. Rev. Lett., 82, 896