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"Big Bang" as a result of the curvature-driven first-order phase transition in the early cold Universe

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We suggest that the "Big Bang" may be a result of the first-order phase transition driven by changing scalar curvature of the 4D space-time in the expanding cold Universe, filled with nonlinear scalar field ϕ and neutral matter with equation of state $p = \nu\varepsilon$ (where p and ε are pressure and energy density of matter). We consider a Lagrangian for scalar field in curved space-time with nonlinearity ϕ^4, which along with the quadratic term $-\xi R\,|\phi|^2$ (where ξ is interaction constant and R is scalar curvature) contains a term $\sim \xi R\left(\phi + \phi^+\right)$ linear in ϕ. Due to this term the condition for the extrema of the potential energy of the scalar field is given by a cubic equation. Provided $\nu > 1/3$ the scalar curvature $R = [\kappa(3\nu - 1)\varepsilon - 4\Lambda]$ (where κ and Λ are Einstein's gravitational and cosmological constants) decreases along with decreasing ε in the process of the Universe's expansion, and at some critical value $R_c < 0$ a first-order phase transition occurs, driven by an "external field" parameter proportional to R. Given certain conditions the critical radius of the early Universe at the point of the first-order phase transition may reach arbitrary large values, so this scenario of unrestricted "inflation" of the Universe may be called "hyperinflation". Beyond the point of phase transition the system is rolling down into the potential minimum releasing the potential energy of scalar field with subsequent powerful heating of the Universe playing the role of "Big Bang".

Keywords: Early Universe, equation of state, inflation

1 Introduction

The question about the cause of the "Big Bang" initiating the hot phase in the development of our Universe is still a central question of cosmology. Starting with the works of Kirzhnits and Linde (1972) and Guth (1981), various inflation scenarios of the initially hot Universe were investigated. In these scenarios the first- or second-order phase transitions induced by temperature occurred in the process of expansion

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and cooling of the Universe with subsequent spontaneous braking of the symmetries of various interactions and production of fields and particles from vacuum. The common shortcoming of the hot Universe scenarios is the existence of critical fluctuations during the second-order phase transitions or the formation of domains ("bubbles") of new phase in the case of the first-order phase transitions, leading to a strong large-scale spatial inhomogeneity of matter and anisotropy of the relic radiation in the modern Universe, which contradicts astronomical observations.

Thereby Linde (1982) proposed a scenario of "chaotic inflation" for the early cold Universe, when the energy density of vacuum was determined by the potential energy density of a nonlinear scalar field $U(\phi) \leq M_P^4$ (where $M_P \sim 1\Big/\sqrt{G}$ is the Planck mass, presuming $\hbar = c = 1$, and G is the Newton gravitational constant). For the potentials $U(\phi)$ described by powers of ϕ with sufficiently small interaction constants and large initial values of the field amplitude $\phi \gg M_P$, the initial quantum fluctuations on the Planck scale $l_P \sim 1/M_P$ expand ("inflate") gigantically, by many orders of magnitude exceeding the observable size of the present-day Universe, which provides explanation for its flat geometry, isotropy and high degree of large-scale homogeneity, as well as for the absence of domain walls and 't Hooft (1974) – Polyakov (1974) monopoles in our world.

In the inflation scenarios the heating of the cold Universe to high temperatures (so-called "reheating", actually playing the role of "Big Bang") occurs as the result of dissipation of the energy of oscillations of the scalar field's amplitude, with energies of about 10^{12}–10^{14} GeV, in the vicinity of the potential minimum. The attenuation of these oscillations is due to both the expansion of the Universe and the production of various particles and antiparticles (Kofman et al. (1997)).

It is good to bear in mind though that, given the small scale and high scalar curvature of the early Universe, the interaction of the fundamental scalar field with gravitation should have played a significant role in the processes of its evolution, which was not considered by Linde (1982) and Kofman et al. (1997).

As was shown by Zee (1979) and Smolin (1979), the term of a general form $-\xi R\left|\phi\right|^2$ in the Lagrangian of the Higgs field with $\mu \neq 0$, where the dimensionless constant ξ may be treated as the constant of interaction of scalar and gravitational fields, leads in the framework of standard model to generation of mass of the order of M_P only for anomalously large values of the constant $\xi \geq 10^{34}$.

In this paper, contrary to the Guth (1981) scenario with the first-order temperature-driven phase transition in the expanding Universe, we propose an alternative scenario of the evolution of the early cold Universe with the first-order phase transition, induced by the parameter of an "external field" proportional to scalar curvature. It is shown that this transition is possible given the following conditions:

(i) the Universe born in a rather large quantum fluctuation is filled with some fundamental scalar field ϕ, described by the Lagrangian which in the curved 4-space with $R \neq 0$ includes a term $\sim \xi R(\phi + \phi^+)$ linear in ϕ, playing the role of an "external field", along with the quadratic in ϕ term $-\xi R\left|\phi\right|^2$.

(ii) the early Universe with finite cosmological constant Λ contains neutral cold matter, described by the equation of state $p = \nu\varepsilon$ with $\nu > 1/3$, which ensures the decreasing of the scalar curvature $R = [\kappa(3\nu - 1)\varepsilon - 4\Lambda]$ along with the decreasing of the matter's energy density in the process of the Universe expansion.

Given certain conditions the critical radius of the early Universe at the point of the first-order phase transition may reach arbitrary large values, so this scenario of unrestricted "inflation" of the Universe may be called "hyperinflation".

2 Modified Lagrangian of a nonlinear scalar field in curved space-time and first-order phase transition in the early Universe

In the model considered henceforth the interaction of the complex scalar nonlinear field with gravitation is given as $-\xi R\left|(\phi-\phi_0)\right|^2$, where ϕ_0 is the vacuum average of the scalar field.

As the result, the Lagrangian of the scalar field with nonlinearity ϕ^4 contains linear and quadratic in ϕ terms, which are proportional to scalar curvature R:

$$\tilde{L}=\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi^+\right)+\frac{1}{2}\left(\mu^2-2\xi R\right)\left|\phi\right|^2-\frac{1}{4}g^2\left|\phi\right|^4+\xi R\phi_0(\phi+\phi^+)-\xi R\phi_0^2. \quad (1)$$

Assuming $\phi=(\Phi+\phi')$, where $\Phi(R)$ is the real (classical) part of the field amplitude ϕ for $R\neq 0$ and ϕ' is its complex (quantum) part, and varying (1) with respect to ϕ' we obtain in the linear approximation $|\phi'|\ll\Phi$ the equation for the bosonic field ϕ' with the curvature-dependent mass of the scalar boson:

$$m_B(R)=\sqrt{3g^2\Phi^2\left(R\right)-(\mu^2-2\xi R)}. \quad (2)$$

In the zero-order approximation in ϕ', (1) gives the expression for the potential energy density of the real part of the scalar field Φ:

$$U(\Phi,R)=\frac{1}{4}g^2\Phi^4-\frac{1}{2}(\mu^2-2\xi R)\Phi^2-2\xi R\phi_0(\Phi-\phi_0/2)+U_0, \quad (3)$$

where U_0 is some arbitrary constant, which should ensure the zero minimal value of the potential (3).

The condition of the existence of the extrema of potential (3) is given by the cubic equation with respect to amplitude Φ:

$$\frac{\partial U}{\partial\Phi}=g^2\Phi^3-\left(\mu^2-2\xi R\right)\Phi-2\xi R\phi_0=0. \quad (4)$$

As will be shown below, in a certain parametric domain equation (4) has three real roots, so the change in the scalar curvature may lead to the first-order phase transition.

For the further analysis of equations (3) and (4) it is convenient to introduce dimensionless variables $x=\Phi/\phi_0$ and $V=U/\mu^2\phi_0^2$:

$$V\left(x,h\right)=\frac{x^4}{4}-\left(1-h\right)\frac{x^2}{2}-hx+\frac{h}{2}+V_0; \quad (5)$$

$$\frac{\partial V}{\partial x}=x^3-\left(1-h\right)x-h=0, \quad (6)$$

where $h=2\xi R/\mu^2$ is the dimensionless "external field" and $V_0=U_0/\mu^2\phi_0^2$.

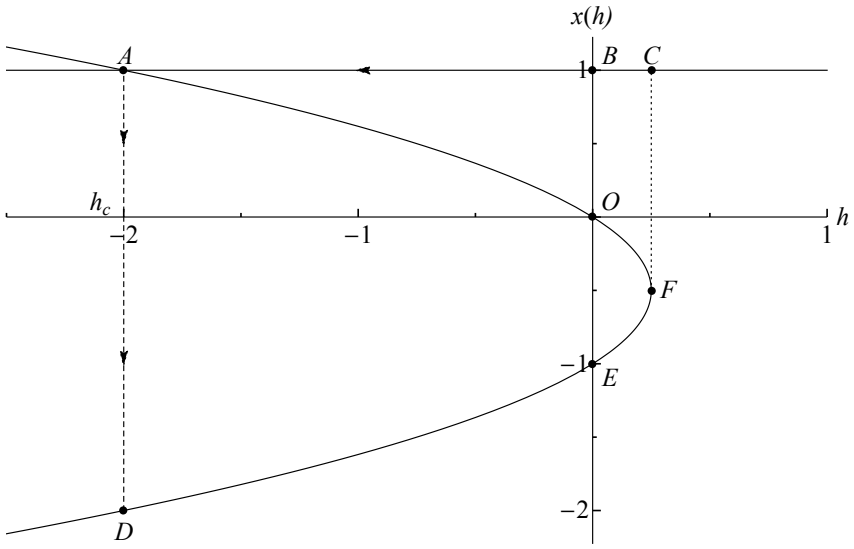


Figure 1 The h dependencies of three real roots $x_i\,(h)$ of the cubic equation (6) in the range $-2 \leq h \leq 0.25$.

The three real roots $x_i\,(h)$ of the cubic equation (6) are shown in Fig. 1 for $-2 \leq h \leq 0.25$. For all $h > -2$ there is a positive h-independent root $x = 1$, which corresponds to the value $\Phi = \phi_0 \equiv \mu/g$ of the amplitude of the scalar field (the ABC branch). This positive root determines the constant position of the right minimum of the potential (5), the negative root (the DEF branch) gives the position of the left minimum, while the sign-changing root (the AOF branch) describes the position of the potential maximum, i.e. it corresponds to the absolutely unstable state. The segments AB and EF on the positive and negative branches correspond to metastable states.

3 Evolution of the early cold Universe before the first-order phase transition

Suppose that in homogeneous space filled with nonlinear scalar field in ground state at zero temperature with equilibrium amplitude ϕ_0 and minimal potential energy density $U_{\min}(\phi_0) = 6.75\mu^2\phi_0^2$ at some (initial) moment appears a rather large quantum fluctuation, spontaneously producing some matter, which is neutral with respect to all charges and characterized by the equation of state $p = \nu\varepsilon$, where p and ε are pressure and energy density of the matter, with dimensionless coefficient ν satisfying condition $\nu > 1/3$.

Suppose also that due to space isotropy the form of the quantum fluctuation is close to spherical, while its initial radius considerably exceeds the Planck length l_P, so the description of the evolution of the spherically symmetric incipient Universe may neglect quantum effects. Consequently, further evolution of this big fluctuation

may be studied using the classical general relativity equations for the homogeneous isotropic closed Universe:

$$\dot{a}^2 + 1 = \frac{\tilde{\kappa}}{3}(\varepsilon + \lambda)\, a^2; \quad \ddot{a} = -\frac{\tilde{\kappa}}{6}(\varepsilon + 3p - 2\lambda)\, a, \tag{7}$$

where a is the scale (radius) of the Universe, $\dot{a}$ and $\ddot{a}$ – its first and second proper time derivatives, $\tilde{\kappa} = \kappa/\left(1 + 2\xi\kappa\phi_0^2\right)$ is the Einstein gravitational constant, renormalized due to interaction of scalar and gravitation fields, and parameter λ is the vacuum energy density, which is related to the Einstein cosmological constant $\lambda = \Lambda/\tilde{\kappa}$.

With account for a possible time dependence of λ, equations (7) give the energy conservation law:

$$3\frac{\dot{a}}{a} + \frac{\dot{\varepsilon} + \dot{\lambda}}{\varepsilon + p} = 0, \tag{8}$$

as well as the expression for the scalar curvature:

$$R = -\frac{6}{a^2}\left(a\ddot{a} + \dot{a}^2 + 1\right) = \tilde{\kappa}\left[(3\nu - 1)\varepsilon - 4\lambda\right]. \tag{9}$$

Contrary to the scenario of "chaotic inflation" (Linde (1982)) where the vacuum energy density is defined as $\lambda = U(\phi) + \dot{\phi}^2/2$, due to the constancy of the scalar field amplitude $\phi = \phi_0$ and minimal density of the potential energy $U(\phi) = U^{(1)}_{\min}(\phi_0)$ on the phase trajectory ABC we shall assume

$$\lambda = U^{(1)}_{\min}(\phi_0) = 6.75\mu^2\phi_0^2 = const. \tag{10}$$

In the capacity of matter filling the early cold Universe we shall consider non-relativistic degenerate Fermi-gas with the equation of state $p = 2\varepsilon/3$. Assuming $\lambda = const$, in accordance with (8) and (9) we have:

$$\varepsilon(t) = \varepsilon_0 \cdot \left[a_0/a(t)\right]^5; \quad R(t) = \tilde{\kappa}[\varepsilon(t) - 4\lambda]. \tag{11}$$

Here ε_0 and a_0 are the initial values of the energy density of the matter and radius of the nucleus of the Universe, which satisfy conditions $a_0 > l_P$ and $\varepsilon_0 \le \varepsilon_P$, where $\varepsilon_P = M_P^4$ is the Planck energy density.

In the process of the Universe expansion the scalar curvature decreases in time with reduction of $\varepsilon(t)$ in accordance with the power-low dependence. In the framework of the proposed model of the nonlinear scalar field this corresponds to the decreasing of the dimensionless parameter of the "external field" $h(t) = 2\xi R(t)/\mu^2$.

On the assumption of the quantum origin of the Universe it is convenient to introduce dimensionless variables $\tilde{a}(\tau) = a(t)/l_P$ and $\tau = t/t_P$ (where t_P is the Planck time). In this case the scalar curvature and dimensionless "external field" parameter h, with account for the energy conservation (see (11)), are written as:

$$R(\tau) = -\Lambda \cdot \left[4 - \frac{\varepsilon_0}{\lambda} \cdot \left(\frac{\tilde{a}_0}{\tilde{a}(\tau)}\right)^5\right]; \quad h(\tau) = -\frac{\tilde{\xi}\cdot\beta}{\left(1+\tilde{\xi}\right)} \cdot \left[4 - \frac{\tilde{\varepsilon}_0}{\beta}\left(\frac{\tilde{a}_0}{\tilde{a}(\tau)}\right)^5\right], \tag{12}$$

where $\tilde{a}_0 = a_0/l_P$, $\tilde{\xi} = 2\xi\kappa\phi_0^2$, $\tilde{\varepsilon}_0 = \varepsilon_0/\mu^2\phi_0^2$ and $\beta = \lambda/\mu^2\phi_0^2$ are the dimensionless parameters of the present model.

In order to describe the dynamics of the Universe we use the first of the equations (7) for the velocity of the expansion (along with the energy conservation law):

$$\frac{d\tilde{a}}{d\tau} = \left\{ b \left[1 + \frac{\tilde{\varepsilon}_0}{\beta} \cdot \left(\frac{\tilde{a}_0}{\tilde{a}\,(\tau)} \right)^5 \right] \cdot \tilde{a}^2\,(\tau) - 1 \right\}^{1/2}; \quad b\left(\tilde{\xi}\right) = \frac{\beta}{3} \frac{\Omega_P}{\tilde{\varepsilon}_P} \frac{1}{1+\tilde{\xi}}, \tag{13}$$

where $\Omega_P = \kappa \varepsilon_P l_P^2 = 25.1327...$ is a universal constant expressed in terms of the world constants, while $\tilde{\varepsilon}_P = \varepsilon_P / \mu^2 \phi_0^2$ is an additional dimensionless model parameter dependent on the parameters of the universal scalar field μ and $\phi_0 = \mu/g$.

The described scenario assumes that the Universe's expansion may continue only till the time τ_c when parameter $h\,(\tau)$ reaches its critical value $h_c \equiv h\,(\tau_c) = -2$ in the point A on the phase trajectory ABC (see Fig. 1) and the dimensionless radius becomes equal to the limit value $\tilde{a}_c \equiv \tilde{a}\,(\tau_c)$.

From (11) for $h = -2$ we obtain the ratio $\tilde{a}_c/\tilde{a}_0$, dependent on the model parameters β, $\tilde{\varepsilon}_0$ and $\tilde{\xi}$:

$$\frac{\tilde{a}_c}{\tilde{a}_0} = \left[\frac{\tilde{\xi}}{2\left[(2\beta - 1)\,\tilde{\xi} - 1 \right]} \tilde{\varepsilon}_0 \right]^{1/5}. \tag{14}$$

The inequality $\tilde{a}_c > \tilde{a}_0$ necessary in the case of the Universe's expansion leads to the following restriction on the parameters $\tilde{\varepsilon}_0$ and $\tilde{\xi}$:

$$\tilde{\varepsilon}_0 > 2 \frac{(2\beta - 1)\,\tilde{\xi} - 1}{\tilde{\xi}}; \quad \tilde{\xi} \geq 1/\,(2\beta - 1). \tag{15}$$

These conditions, together with $\tilde{a}_0 > 1$, $\dot{\tilde{a}}^2 > 0$ and (15) are the complete set of restrictions, applied to the parameters of the proposed model (see Fig. 2).

According to (14), when $\tilde{\xi}$ tends to its minimal value $\tilde{\xi}_{\min} = 1/(2\beta - 1)$ the radius of the Universe in the point of phase transition goes to infinity, $a_c \to \infty$.

Thus, in the point of phase transition the maximal radius of the early cold Universe diverge $a_c \to \infty$, as well as the total released energy $E_c \to \infty$, for all possible initial values of a_0 and E_0, if $\tilde{\xi} \to \tilde{\xi}_{\min}$.

The value of the dimensionless radius of the Universe in the point of phase transition $\tilde{a}_c$ is shown in Fig. 2 in function of the parameters $\tilde{\xi}$ and $\tilde{\varepsilon}_0$ for $\tilde{0}_0 = 5$, $\beta = 6.75$ and $\tilde{\varepsilon}_P = 4660$. We can see that $\tilde{a}_c \to \infty$ for $\tilde{\xi} \to \tilde{\xi}_{\min} = 0.08$, and significant expansion of the Universe, with $\tilde{a}_c \gg \tilde{a}_0$, is possible only in a narrow range of the values of $\tilde{\xi}$ when $(\tilde{\xi} - \tilde{\xi}_{\min}) \ll 1$.

Let us make estimates of the scalar field parameters in the framework of the proposed model of the early cold Universe. As was shown in Section 3, a physically reasonable assessment of the dimensionless constant of interaction of scalar and gravitational fields $\xi \approx 0.04$ may be achieved with assumption $\kappa\phi_0^2 \approx 1$, which corresponds to a rather large ratio $\phi_0/\phi_H \approx 10^{16}$ of the vacuum averages of the fundamental scalar field $\phi_0 = \mu/g$ and the Higgs field $\phi_H = \mu_H/g_H$.

On the other hand, the choice of the big value for the dimensionless parameter $\tilde{\varepsilon}_P \equiv \varepsilon_P/\mu^2\phi_0^2 = 4660$, with $\varepsilon_P = M_P^4$, corresponds to the ratio $\mu\phi_0/\mu_H\phi_H \approx 10^{32}$. Both these choices may by consistent only if $\mu/\mu_H \approx 10^{16}$ and $g/g_H \approx 1$, which also gives $\mu \approx 0.1 M_P$ and $\phi_0 = \mu/g \approx 0.274 M_P$ for $g \approx g_H \approx 0.364$. The ratio of the

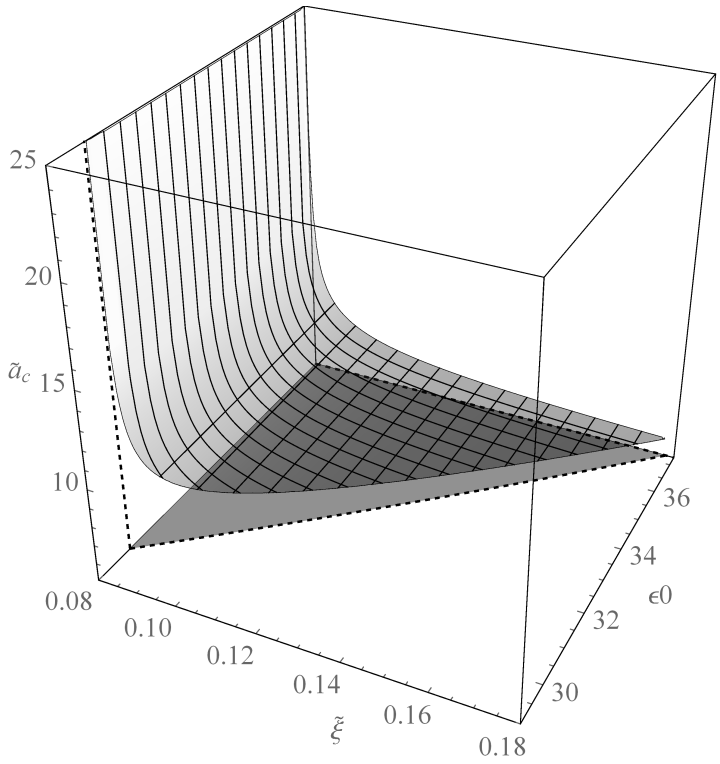


Figure 2 The dimensionless radius of the Universe $\tilde{a}_c$ at the moment of phase transition (see (14)) in function of parameters $\tilde{\xi}$ and $\tilde{\varepsilon}_0$ for $\tilde{a}_0 = 5$, $\beta = 6.75$ and $\tilde{\varepsilon}_P = 4660$. The dark region in the plane $\tilde{\xi} - \tilde{\varepsilon}_0$ is the domain of allowable values of these parameters.

potential energy density of the scalar field to the Planck energy density then equals $\Delta U/\varepsilon_P = 6.75\mu^2\phi_0^2/M_P^4 \approx 0.00145$.

Accordingly, the fundamental scalar field in the early cold Universe prior to the Big Bang could have the potential energy density $\Delta U = 6.75\mu^4/g^2$, which is 64 orders of magnitude larger than that value for the Higgs field, once more emphasizing the lack of usability of the latter in the inflation theories of the early Universe.

In the model of "chaotic inflation" (Linde (1982)), with the restriction on the scalar field potential $V(\phi) \leq M_P^4$, for the size of the "inflated" Universe to exceed by many orders of magnitude the size of observable Universe it is necessary to have large initial amplitude of the scalar field $\phi \gg M_P$ and anomalously small values of either the effective mass of the scalar field $m \ll M_P$ for the quadratic potential $V(\phi) = m^2\phi^2/2$, or the nonlinearity coefficient $\gamma \ll 1$ for the potential $V(\phi) = \gamma\phi^4$. On the other hand, it should be stressed that in the framework of the proposed model with $\xi \to \xi_{\min} = 0.04$ the Universe's inflation to arbitrary large sizes in the point of the phase transition is possible for $\phi_0 < M_P$ and $\Delta U \ll \varepsilon_P$.

Consequently, the presently proposed scenario of evolution of the early Universe towards the point of phase transition with unrestricted "inflation" for $\xi \to \xi_{\min}$, may be called "hyperinflation".

Let us estimate the total energy of the scalar field E_c, released in the first-order phase transition for $(\tilde{\xi} - \tilde{\xi}_{\min}) = 10^{-4}$, with account for the parameters' values $a_c \approx 20 l_P$ and $\Delta U \approx 1.45 \cdot 10^{-3} \cdot M_P^4$, obtained above:

$$E_c = 2\pi^2 a_c^3 \cdot \Delta U \approx 2.3 \cdot 10^2 \cdot M_P \approx 2.76 \cdot 10^{21} \text{ GeV}. \quad (16)$$

The discharge of such enormous energy during phase transition should lead to the birth of a huge amount of particle-antiparticle pairs of various kinds, and to the rapid heating of the matter to high temperature of the order of the Planck one $T_P \approx M_P/k_B \approx 1.2 \cdot 10^{32}$ K (k_B is the Boltzmann constant), thus causing the "Big Bang" starting the hot phase of our Universe.

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