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Investigation of resonance dynamics of near-Earth objects

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A brief summary of results obtained by authors when studying the influence of the secular resonances in the dynamics of near-Earth objects is presented. The orbital evolution of the near-Earth space objects in the range of the semi-axes from 8000 km to 60000 km are considered. The main attention is paid to the peculiarities arising in the orbital evolution of the objects which are subject to the action of multiple resonances: orbital and secular resonances or several secular resonances. It is shown that the overlapping of the resonances can lead to chaotic motion.

Keywords: Orbital and secular resonances, long-time orbital evolution

1 Introduction

A secular resonance is the presence of commensurability between the rates of change of longitudes of the ascending node and the pericentre of the satellite with one of the natural frequencies of the third body (the Moon or the Sun) motion, which include the mean motion of the third body, and the rate of change of the longitudes of the ascending node and pericentre.

The problem of the effect of secular resonances on the orbital evolution of an Earth artificial satellite (EAS) has been known for a long time. The first studies in this direction were made by M.L. Lidov (1962) within the doubly averaged problem and under the assumption that the effect of the third body is restricted by the Hill problem. Similar results for asteroids were obtained by Y. Kozai (1962) almost simultaneously. A more general classification of the main secular resonances in EAS motion was given by G.E. Cook (1962). See the review of the papers in (Bordovitsyna et al., 2012). This problem has become once again the subject of study and discussion in the literature (Chao, Gick, 2004; Rossi, 2008) in connection with the deployment of satellite radio navigation systems (SRNS) in the medium Earth orbits (MEO). The dynamics of SRNS objects is very complex after loss of control. The fact is that the secular lunisolar resonances produce an eccentricity growth for orbits with inclinations

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chosen for navigation constellations. This significantly changes the positions of these orbits in space and results in the ingress of uncontrolled objects into the area of operating satellites (Rossi, 2008; Bordovitsyna et al., 2012). As it has been first shown in (Bordovitsyna et al., 2014) the multiple action of orbital and secular resonances leads to the appearance of chaosity in the motion of the object.

The results of studies of the dynamics of navigation system uncontrolled objects have prompted us to investigate the problem of the influence of the secular resonances in the dynamics of the near-Earth objects in a wide range of orbital parameters. We have used the developed numerical-analytical method (Bordovitsyna et al., 2014) to study the secular resonances in the dynamics of the near-Earth objects and have realized a set of numerical-analytical experiments. A brief summary of the obtained results is presented in this paper.

2 Methods of study

We used for studying the effect of secular resonances on near-Earth space objects the methods which consist of: (i) identification of the secular resonances based on an analytical approach, (ii) modeling the long-term orbital evolution numerically using the Numerical Model of Motion of Satellite Systems software package (Bordovitsyna et al., 2009) implemented in a parallel programming environment at the Tomsk State University computing cluster; and (iii) conducting the MEGNO (Mean Exponential Growth of Nearby Orbit) analysis (Cincotta et al., 2003) of the dynamical evolution of objects. The method for identifying the secular resonances was described in detail in (Bordovitsyna et al., 2012). Let us recall briefly its principle. The arguments for a singly and doubly averaged perturbing function of the restricted three-body problem are in the following form:

$$\underline{\psi} = (l - 2p' + q')M' + (l - 2p)\omega + (l - 2p')\omega' + j(\Omega - \Omega'), \tag{1}$$

$$\underline{\underline{\psi}} = (l - 2p)\omega + (l - 2p')\omega' + j(\Omega - \Omega'), \tag{2}$$

where

$$\omega = \omega_0 + \dot{\omega}(t - t_0), \quad \Omega_0 + \dot{\Omega}(t - t_0)$$

are the longitudes of the pericentre and the ascending node of the satellite and

$$M' = M'_0 + n'(t - t_0), \quad \omega' = \omega'_0 + \dot{\omega}'(t - t_0), \quad \Omega' = \Omega'_0 + \dot{\Omega}'(t - t_0)$$

are the mean anomaly and the longitudes of the pericentre and the ascending node of the third body. The resonance condition may then be written down in the following form:

$$\dot{\underline{\psi}} \cong 0, \quad \dot{\underline{\underline{\psi}}} \cong 0. \tag{3}$$

Relations (3) mean that the object has repeatedly passed through the resonance during the orbital evolution.

The secular frequencies in the satellite motion due to the influence of the second zonal harmonic and a third body are calculated according to the well-known formulas. A complete set of low-order resonance relations is presented in Table 1.

Table 1 Types of low-order resonance relations.

No	Type of resonance relation	No	Type of resonance relation
1	$\dot{M}'_S - \dot{\omega} + (\dot{\Omega} - \dot{\Omega}'_S) + \dot{\omega}'_S$	16	$\dot{M}'_L - \dot{\omega} - (\dot{\Omega} - \dot{\Omega}'_L) + \dot{\omega}'_L$
2	$\dot{M}'_S - \dot{\omega} - (\dot{\Omega} - \dot{\Omega}'_S) + \dot{\omega}'_S$	17	$\dot{M}'_L - 2\dot{\omega} + (\dot{\Omega} - \dot{\Omega}'_L) - 2\dot{\omega}'_L$
3	$\dot{M}'_S - 2\dot{\omega} + (\dot{\Omega} - \dot{\Omega}'_S) - 2\dot{\omega}'_S$	18	$\dot{M}'_L + 2(\dot{\Omega} - \dot{\Omega}'_L)$
4	$\dot{M}'_S + 2(\dot{\Omega} - \dot{\Omega}'_S)$	19	$\dot{M}'_L - 2(\dot{\Omega} - \dot{\Omega}'_L)$
5	$\dot{M}'_S - 2(\dot{\Omega} - \dot{\Omega}'_S)$	20	$\dot{M}'_L + (\dot{\Omega} - \dot{\Omega}'_L)$
6	$\dot{M}'_S + (\dot{\Omega} - \dot{\Omega}'_S)$	21	$\dot{M}'_L - (\dot{\Omega} - \dot{\Omega}'_L)$
7	$\dot{M}'_S - (\dot{\Omega} - \dot{\Omega}'_S)$	22	$\dot{M}'_L - 2\dot{\omega} + 2\dot{\omega}'_L$
8	$\dot{M}'_S - 2\dot{\omega} + 2\dot{\omega}'_S$	23	$\dot{M}'_L - \dot{\omega} + \dot{\omega}'_L$
9	$\dot{M}'_S - \dot{\omega} + \dot{\omega}'_S$	24	$(\dot{\Omega} - \dot{\Omega}'_L) + \dot{\omega} - \dot{\omega}'_L$
10	$(\dot{\Omega} - \dot{\Omega}'_S) + \dot{\omega} - \dot{\omega}'_S$	25	$(\dot{\Omega} - \dot{\Omega}'_L) - \dot{\omega} + \dot{\omega}'_L$
11	$(\dot{\Omega} - \dot{\Omega}'_S) - \dot{\omega} + \dot{\omega}'_S$	26	$(\dot{\Omega} - \dot{\Omega}'_L) + 2\dot{\omega} - 2\dot{\omega}'_L$
12	$(\dot{\Omega} - \dot{\Omega}'_S) + 2\dot{\omega} - 2\dot{\omega}'_S$	27	$(\dot{\Omega} - \dot{\Omega}'_L) - 2\dot{\omega} + 2\dot{\omega}'_L$
13	$(\dot{\Omega} - \dot{\Omega}'_S) - 2\dot{\omega} + 2\dot{\omega}'_S$	28	$(\dot{\Omega} - \dot{\Omega}'_L)$
14	$(\dot{\Omega} - \dot{\Omega}'_S)$	29	$\dot{\omega}$
15	$\dot{M}'_L - \dot{\omega} + (\dot{\Omega} - \dot{\Omega}'_L) + \dot{\omega}'_L$		

We consider only the low-order resonances since it is known (Grebenikov, Riabov, 1978) that the higher the order of the resonance the less its influence on the orbital evolution of the object. Table 1 gives the resonance relations with the mean motions of the Moon and the Sun, the resonances of the nodal and apsidal secular frequencies of the satellite and perturbing bodies, and also a geometric resonance of Lidov–Kozai type $\dot{\omega} \approx 0$. The Lidov–Kozai resonance is a resonance of the lowermost order.

The time changes of relations (3) are considered along with the time changes of relations (1) and (2), the so-called critical arguments. This approach (Murrey, Dermott, 1999) allows us to determine whether this resonance is stable or not. If the critical argument shows libration oscillations in time, the resonant configuration is stable. If the critical argument circulates then the resonant configuration is unstable.

In the process of the study of orbital evolution the parameters of orbits are computed by using "The Numerical Model of Motion of Satellite Systems software package" (Bordovitsyna et al., 2009) which has been coupled with the MEGNO parameter calculation software (Bordovitsyna et al., 2010). The averaged MEGNO parameter (Cincotta et al., 2003) is a time-weighted integral form of the Lyapunov characteristic number. The time evolution of the averaged MEGNO value manifests certain features specific to different types of orbits. For example, it is known (Cincotta et al., 2003) that for quasi-periodic (regular) orbits the unaveraged MEGNO parameter oscillates around 2, and the averaged MEGNO value always tends towards 2; for stable orbits of a harmonic oscillator type the averaged MEGNO value equals zero and for chaotic orbits the averaged MEGNO value increases linearly.

The package has the following structure. Differential equations of motion together with MEGNO parameter equations are formulated in an inertial reference system and integrated numerically with the use of a high-order Gauss–Everhart integrator (Avdyushev, 2010). The potential of the gravitational field of the Earth is presented

in the form of an expansion in spherical functions in a reference system that is rigidly connected with the Earth. The spherical functions and their first and second partial derivatives with respect to coordinates are computed using the Cunningham recurrent algorithm (Cunningham, 1970). All the parameters in the expansion of the potential of the Earth are taken from the EGM96 geopotential model with a degree and an order of 360. The DE406\LE406 fund of coordinates of major planets is used for the calculation of the Moon and the Sun coordinates in the process of studying the long time orbital evolution of space object. The enhanced software package runs on the supercomputer "Skif Cyberia" that is classified (with respect to the short-term memory access scheme) as a cluster with distributed random-access memory and allows us to simulate the orbital evolution of more than 1000 objects simultaneously.

3 Results and conclusions

With the help of the above-mentioned methods we carried out three numerical-analytical experiments to estimate the abundance of secular resonances in near-Earth orbital space and to study the influence on the dynamics of the uncontrolled satellites.

The first experiment has been associated with the study of the dynamics of uncontrolled objects of the navigation satellite systems GLONASS and GPS and BEIDOU IGSO (Bordovitsyna et al., 2012, 2014; Bordovitsyna, Tomilova, 2015).

The second experiment has been devoted to study of the abundance of secular resonances in near-Earth orbital space for the following variations of the orbital parameters: $e = 0.01$, 0.1–0.8 with a step of 0.1, $i = 10$–90° with a step 5°, $a = 8000$–55000 km with a step of 1 km and the estimation of influence on the dynamics of the uncontrolled objects (Bordovitsyna, Tomilova, 2013).

The third experiment has been carried out to refine and extend the data obtained in the second experiment. The following range of orbital parameters: $e = 0.001$, $i = 0$–90° with a step of 1°, $a = 45000$–60000 km with a step of 200 m has been described in (Aleksandrova et al., 2014).

Analysis of obtained results leads to the following conclusions.

- Secular resonances are very common phenomena in the near-Earth orbital space, while sharp resonances are concentrated in the orbital space areas, where $a \geq 20000$ km and $i \geq 45^\circ$.
- Secular resonances with the mean motion of the Sun are more significant than the similar resonances with the mean motion of the Moon, and this has a good agreement with (Breiter, 2001).
- Secular lunisolar resonances produce the growth of e (for any i). This conclusion has a good agreement with other author results (Rossi, 2008; Bordovitsyna et al., 2012). Even a single sharp resonance can transform a circular orbit into a high-eccentric one.
- Objects from the region of the orbital space with semi-major axes 40000 km and higher and inclinations between 55° and 70° are subject to the simultaneous influence of a large number of orbital resonances; among them may be as stable resonances and instabilities in which the critical argument repeatedly changes the nature of the changes to the libration to circulation and back; the motion

of such objects is irregular and shows longperiod oscillations of the eccentricity and the inclination with large amplitudes; the chaotization of the motion may be considerable and the velocity of the chaotization increases with growth of the initial eccentricity (Bordovitsyna, Tomilova, 2013, 2014a).

- The evolution of the near-polar orbits with inclinations 80°11 and 90° is very complicated but degrees of the orbital destruction may be different.
- The long-time evolution of orbits with the inclination 90° has a catastrophic character for any eccentricities and the chaotization of the motion with rapidly growing of the Megno parameter takes place; as a case above the process of the chaotization is accelerated under the growth of the initial eccentricity but the destruction of orbits comes later than with the near zero value of the initial eccentricity (Bordovitsyna, Tomilova, 2014b).
- The superposition of several stable secular resonances causes no chaotization. At the same time the entry of the object in orbital resonance with the Earth's rotation in the presence of secular resonances may be accompanied by the appearance of chaotization in the motion of objects.
- Chaotization occurs also when the action of stable resonance combined with the influence of one or several unstable resonances, for which the critical arguments change the nature of libration oscillations into the circulation and back.

Next we present two interesting examples that illustrate the latest conclusions.

Figure 1 shows the orbital evolution of the Chinese navigation satellite BEIDOU IGSO 3 after losing control (graph a). Graph b presents the evolution of the critical arguments of secular resonances $\dot{\psi}_{11} = (\dot{\Omega} - \dot{\Omega}'_S) - \dot{\omega} + \dot{\omega}'_S$, $\dot{\psi}_{12} = (\dot{\Omega} - \dot{\Omega}'_S) + 2\dot{\omega} - 2\dot{\omega}'_S$, $\dot{\psi}_{13} = (\dot{\Omega} - \dot{\Omega}'_S) - 2\dot{\omega} + 2\dot{\omega}'_S$ and $\dot{\psi}_{29} = \dot{\omega}$, which act on the satellite. Graph c presents the time-evolution of the orbital period of the satellite and the critical argument associated with the 1:1 resonance of orbital period of the satellite to the Earth's rotation. Graph d gives a fragment of graph c. These data show that there is an overlap of orbital and secular resonances in the process of the orbital evolution of the satellite. As a result the motion of the object becomes chaotic.

The orbital evolution of the model near-polar satellite is presented in Fig. 2, graph a. Graphs b,c,d,e show the time evolution of resonance relations $\dot{\psi}_8 = \dot{M}'_S - 2\dot{\omega} + 2\dot{\omega}'_S$, $\dot{\psi}_{10} = (\dot{\Omega} - \dot{\Omega}'_S) + \dot{\omega} - \dot{\omega}'_S$, $\dot{\psi}_{28} = (\dot{\Omega} - \dot{\Omega}'_L)$ and $\dot{\psi}_{29} = \dot{\omega}$ and their critical arguments. These data indicate that the object exposed to several secular resonances, for which the critical arguments changes the libration oscillations into the circulation and back repeatedly during the evolution. The action of a such scope of resonances leads to the appearance of the chaosity in the motion of the object.

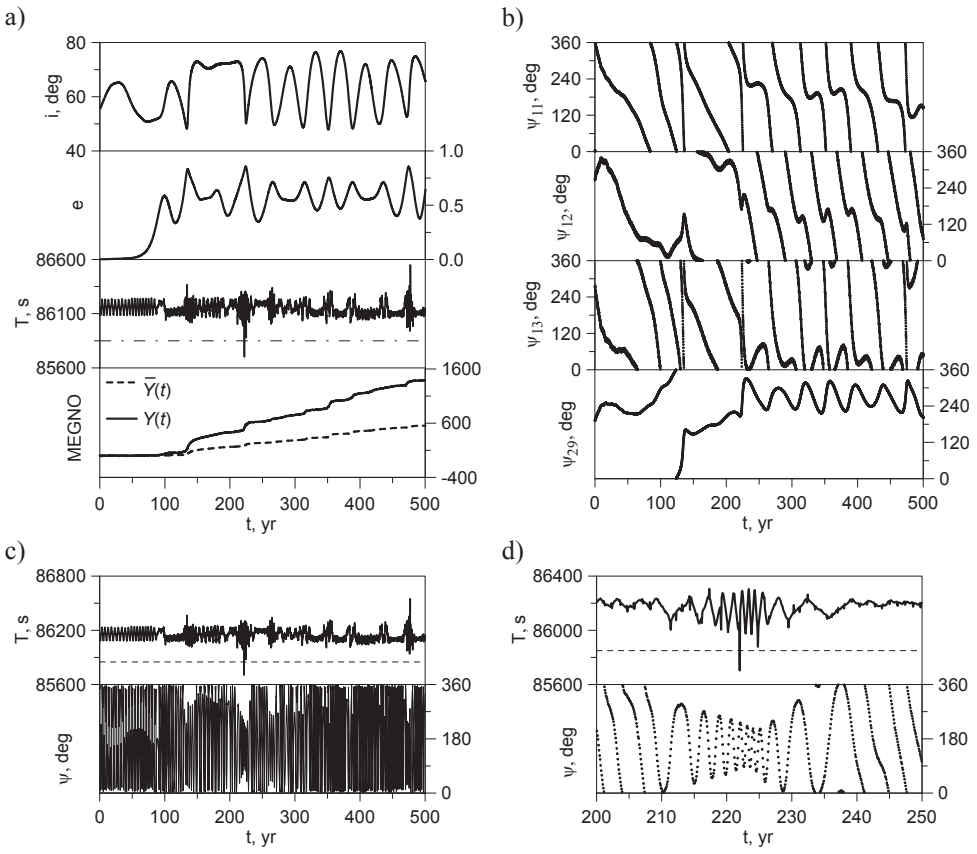


Figure 1 The orbital evolution of the artificial satellite Beidou IGSO 3 after losing control. The averaged parameter MEGNO is indicated by a dotted line and the unaveraged parameter MEGNO by a solid line.

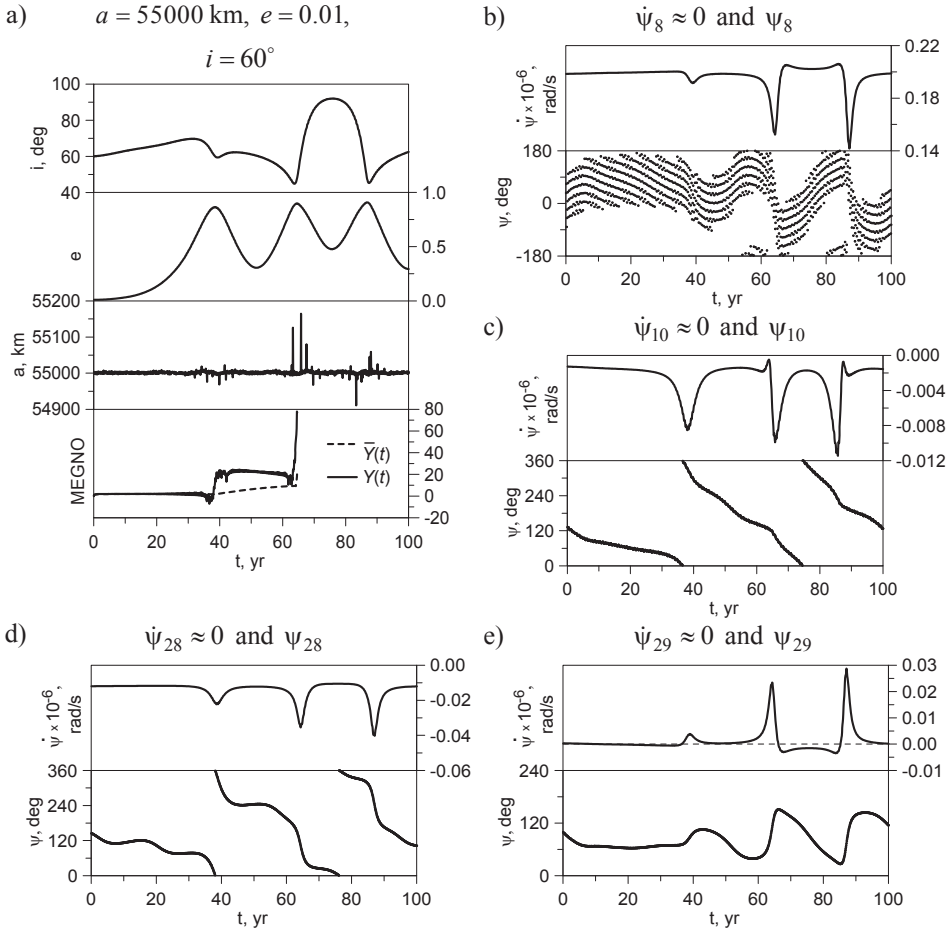


Figure 2 The orbital evolution of the near-polar satellite.

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