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Clusters of galaxies and a Zwicky's paradox

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In 1933 the Swiss astronomer Fritz Zwicky discovered that the gigantic cluster of galaxies in Coma Berenices constellation has the dynamical mass a few hundred times more than that obtained from a number of observed members of this cluster and its total luminosity. A few years ago such missing "hidden mass" has been called "dark matter". In spite of extensive research into the dynamical processes in clusters of galaxies it is incomprehensible a reason between the total mass found on the galaxy's luminosity in the cluster and those obtained due to the virial theorem (the so called "virial paradox") (Gorbatsky and Kritzuk, 1987). Supernova Cosmology project and the search group High-Z Supernova have shown that Einstein cosmological constant differs from zero and fills the outer space with an invisible "dark energy" the indication mark of which appears as a gravitational repulsion (Green, 2013). Thanks to that there is a possibility to make more precise the dynamical mass of the Coma cluster of galaxies with taking into account a cosmological repulsion of vacuum. It turns out to be that this brings to decrease the dynamical mass of the above mentioned cluster no more than 5%. This means that the Einstein cosmological constant does not remove the "virial paradox". Such insignificant influence of "dark energy" on the Coma cluster dynamical mass may be considered as an argument in favour of "hidden mass" or "dark matter" existence, the discovery of unbarion bearers of which being one of the most actual problems in physics and astronomy.

Both the space distribution and dispersia of velocities in the Coma cluster allow to consider this giant cluster of galaxies in Coma Berenices constellation as a dynamical system with spherical structure.

That is why the force field of such system can be described on the basis of Schwartzschild metric with additional cosmological member of Einstein (in the limit of slow motions and weak fields).

It is well known that for empty space around the sphere with substance belonging to the dynamical system the modified Schwartzschild metric is an external solution of Einstein's equations of gravitational field with cosmological member (Bogorodsky, 1971; Tolman, 1974).

Besides "the influence of this Lambda member on a force field of fixed relativistic centre of attraction becomes stronger with increase of concerned outer space region dimensions" (Tolman, 1974).

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In following designations

$$x^0 = ct, \quad x^1 = r, \quad x^2 = \theta, \quad x^3 = \phi,$$

where t, r, θ, ϕ are the time and spherical polar coordinates of the point of space, the square of linear element in four-dimensional pseudo-Riemannian space has the form:

$$ds^2 = g_{00}dx^{0^2} + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2,$$

$$g_{ik} = 0 \quad (i \neq k),$$

and non-zero components of fundamental metric tensor g_{ii} being expressed with formulas:

$$g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{1}{3}\Lambda r^2, \quad g_{11} = -\frac{1}{g_{00}},$$

$$g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2\theta.$$

Here M is the mass of attracting centre, Λ is the Einstein's cosmological constant, G is the universal gravitational constant, and c is the velocity of light in vacuum.

In the classic Newtonian limit (of weak fields and low motions in comparison with velocity of light) the components of fundamental metric tensor are:

$$g_{00} = 1 + \frac{2\psi}{c^2}, \quad g_{0\alpha} = 0, \quad g_{\alpha\beta} = -\delta_{\alpha\beta},$$

$$\delta_{\alpha\beta} = \begin{cases} 1 & (\alpha = \beta) \\ 0 & (\alpha \neq \beta) \end{cases} \quad (\alpha, \beta = 1, 2, 3).$$

From the known connection between potential ψ and temporal component of metric g_{00} (Bogorodsky, 1971):

$$\psi = \frac{c^2}{2}\left(g_{00} - 1\right),$$

the next dependence of effective potential $\psi(r)$ from distance r to the centre of attraction is happened to be:

$$\psi(r) = -\frac{GM}{r} - \frac{1}{6}\Lambda c^2 r^2.$$

The above mentioned effective potential $\psi(r)$ corresponds the superposition of static spherically symmetric force field of the fixed centre of attraction according to Newton's inverse square law and additional Einstein's cosmological repulsion which is proportional to the distance of coordinate system origin.

Let us suppose that all galaxies move around barycentre of spherically symmetric cluster of galaxies as massive material points acting on each other gravitationally at the phone of cosmic vacuum.

Consider now an individual galaxy at the cluster edge as a probe partical moving in the force field with effective potential $\psi(r)$, then its differential equations of motion in the form of Lagrange's second kind are such that:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i} \quad (i = 1, 2, 3),$$

where in the capacity of Lagrange's generalized coordinates q_i the spheric polar coordinates are chosen: $q_1 = r, q_2 = \theta, q_3 = \varphi$, while $\dot{q_i}$ mean corresponding generalized velocities, and L is Lagrange's function:

$$L = T + U.$$

Here T is the specific kinetic energy, U is the Lagrange's "force function" (that is the absolute value of effective potential $\psi(r)$ of the considered force field), so that

$$U = |\psi(r)| = \frac{GM}{r} + \frac{1}{6}\Lambda c^2 r^2,$$

but the specific kinetic energy T in spheric polar coordinates has the form:

$$T = \frac{1}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + r^2\sin^2\theta\left(\frac{d\varphi}{dt}\right)^2\right].$$

And so the individual galaxy motion as a probe particle motion in the force field with potential $\psi(r)$ is described by the next three ordinary differential equations of second order:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 - r\sin^2\theta\left(\frac{d\varphi}{dt}\right)^2 = \frac{\partial U}{\partial r},$$

$$\frac{d}{dt}\left[r^2\left(\frac{d\theta}{dt}\right)\right] = \frac{\partial U}{\partial \theta},$$

$$\frac{d}{dt}\left[r^2\sin^2\theta\left(\frac{d\varphi}{dt}\right)\right] = \frac{\partial U}{\partial \varphi}.$$

So as in the static spherically symmetric force field with effective potential $\psi(r)$ the following equalities take place:

$$\frac{\partial U}{\partial \theta} = 0, \quad \frac{\partial U}{\partial \varphi} = 0,$$

owing to undermentioned values of spheric polar coordinates

$$r = R = const,\ \theta = \frac{\pi}{2},\ 0 < \varphi < 2\pi,$$

the first of aforesaid differential equations of motion takes the form:

$$-R\left(\frac{d\varphi}{dt}\right)^2 = \frac{\partial U}{\partial r},$$

the second one is satisfied identically, while such equality results as the consequence of the third equation of motion:

$$R^2\left(\frac{d^2\varphi}{dt^2}\right) = 0.$$

Hence it follows at once that

$$\left(\frac{d\varphi}{dt}\right) = \omega = const.$$

As a consequence of the last three formulas the next equality is obtained:

$$R\omega^2 = -\frac{\partial U}{\partial r},$$

expressing the well-known connection between a probe mass centripetal acceleration and a force field (strainedness?).

The latter correlation is nothing else than a condition of the circular orbit existence for probe mass on the distance R from the coordinate system origin.

Introducing now instead of constant angular velocity ω a linear one V with the help of formula

$$V = \omega R,$$

we shall come to the conclusion on such evident expression of the circular orbit existence for a galaxy at the distance R from the centre of spherically symmetric cluster of galaxies in cosmic vacuum

$$\frac{V^2}{R} = \frac{GM}{R^2} - \frac{1}{3}\Lambda c^2 R.$$

If one takes into account the well known dependence of Hubble's constant H from Einstein's cosmological constant Λ:

$$H^2 = \frac{1}{3}\Lambda c^2,$$

then last two equations give rise to the next formula for calculation of Hubble's constant:

$$H = \sqrt{\frac{GM}{R^3} - \frac{V^2}{R^2}}.$$

That is why the age of Metagalaxy T_M as an inversely proportional value to the Hubble's constant will be determined by the equality:

$$T_M = \frac{1}{H},$$

whilst the value of cosmological constant Λ will be represented by means of formula:

$$\Lambda = 3\frac{H^2}{c^2}.$$

Finally in accordance with the well-known correlation

$$\Lambda c^2 = 8\pi G\rho_\Lambda$$

the cosmic vacuum density ρ_Λ is determined with the help of the following expression:

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}.$$

For the gigantic cluster of Coma galaxies in the Coma Berenices constellation

$$M = 2.6 \cdot 10^{15}\,(m_\odot); \quad R = 4.75\,(\mathrm{Mpc});$$

$$V = 1500\,(\mathrm{km/s})$$

corresponding values of above mentioned constants are:

$$H = 70\left(\frac{\mathrm{km/s}}{\mathrm{Mpc}}\right);\quad \Lambda = 2\cdot 10^{-56}\left(\frac{1}{\mathrm{cm}^2}\right);$$

$$\rho_\Lambda = 10^{-29}\left(\frac{\mathrm{g}}{\mathrm{cm}^3}\right);\ T_M = 13.6\cdot 10^9\ (\mathrm{yr}).$$

At last, from the already mentioned above equality:

$$\frac{V^2}{R} = \frac{GM}{R^2} - \frac{1}{3}\Lambda c^2 R$$

it follows that because of cosmological repulsion taking into account the dynamical mass decrease

$$\Delta M = \left(\frac{1}{3}\Lambda c^2\right)\frac{R^3}{G},$$

relative to the virial mass

$$M_v = \frac{V^2 R}{G}$$

may be represented with the aid of formula:

$$\frac{\Delta M}{M_v} = H^2\left(\frac{R^2}{V^2}\right),$$

where the already mentioned connection of Hubble's constant H, with Einstein's cosmological constant Λ and velocity of light c:

$$H^2 = \frac{1}{3}\Lambda c^2.$$

After substituting the value of Hubble's constant $H = 70\left(\frac{\mathrm{km/s}}{\mathrm{Mpc}}\right)$ and values both of R and V, corresponding to the examined super-cluster of galaxies:

$$R = 4.75\ (\mathrm{Mpc});\quad \mathrm{V} = 1500\ (\mathrm{km/s}),$$

the relative decrease of mass of the the super-cluster of galaxies in the Coma Berenices constellation is happened to be:

$$\frac{\Delta M}{M_v} = 0.049.$$

Therefore the relative decrease of dynamical mass of the above mentioned super-cluster of galaxies turns out to be no more than 5%.

The latter means that the Einstein's cosmological constant does not remove the "virial paradox". Such insignificant influence of "dark energy" on the Coma super-cluster dynamical mass may be considered as an argument in favour of "hidden mass" or "dark matter" existence, the discovery of unbarion bearers of which being one of the most actual problems in physics and astronomy.

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