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Magnetic fields in the outer rings of spiral galaxies

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Now there is no doubt that at least in some spiral galaxies there are regular magnetic fields of several μG. Their generation is described by the so-called dynamo mechanism that is based on differential rotation and turbulent motions of the interstellar medium. Some galaxies have outer rings where there is some ionized gas. So the magnetic field can exist in the outer rings, too.

To model these magnetic fields we use the so-called no-z model that is based on the small thickness of the disc. z-Derivatives can be replaced by algebraic expressions and the z-component of the field can be neglected.

We have found that the magnetic field can be generated in rings that have half-width more than 1 kpc. However, even if the field generation *in situ* is impossible, the magnetic field can pass to the outer rings by a Kolmogorov–Petrovsky–Piskunov effect.

Keywords: Magnetic fields, outer rings, dynamo theory, spiral galaxies

1 Introduction

Magnetic fields of galaxies have been studied for a long time, and now there is no doubt that they exist at least in central parts of the objects that are quite similar to the Milky Way – for example, M 31, M 33, M 51 and some others (Beck et al. 1996; Arshakian et al. 2009). Some time ago there were studies to investigate the possibility of magnetic field appearance in the peripheral parts of spiral galaxies (Mikhailov et al. 2014). It has been shown that the magnetic fields in these regions can appear because of two effects: the field generation *in situ* and the dynamo-wave that is described by the Kolmogorov–Petrovsky–Piskunov effect (Kolmogorov et al. 1937; Moss et al. 1998; Mikhailov 2015).

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However, the observational data show us that a few galaxies have so-called outer rings (Sil'chenko et al. 2014). These rings have widths of some kiloparsecs and are situated at some distance from the main part of the galaxy. There is some amount of ionized hydrogen in these regions, so we can apply the standard methods of the magnetohydrodynamics with high magnetic Reynolds numbers for the outer rings. The magnetic field generation is described by the dynamo mechanism that is based on the differential rotation and the alpha-effect that is connected with the turbulent motions (Beck et al. 1996; Molchanov et al. 1985). Although the star formation surface density in these galaxies is quite high, it is lower than the critical value, and this makes impossible the large-scale structures of the magnetic field (Mikhailov et al. 2012; Mikhailov 2014). We can conclude that there is ample evidence that the magnetic field in the outer rings can be generated. So, the question about the possible magnetic field evolution in these regions should be studied in more detail.

To model the magnetic fields in the outer rings of galaxies we use the so-called no-z model. This model has shown itself to be suitable for research of the magnetic fields in the central parts of galaxies (Moss 1995; Phillips 2001). It takes into account that the galaxy disc is quite thin and the z-derivatives of the field can be replaced by algebraic expressions. The z-component of the field can be given by the solenoidality condition.

The dynamo mechanism is a threshold effect and it can work only for some values of such parameters as the velocity of the turbulent motions, half-thickness of the ionized gas layer and the rotation curve characteristics. For the magnetic fields in the outer rings we should add the ring width: the less is the width the weaker is the magnetic field generation. We have shown that if the ring width is less than about 1 kpc the magnetic field generation becomes impossible. So galaxies that have quite wide rings are the most interesting for us: for such galaxies the magnetic field can be supported.

However, even if the magnetic field generation *in situ* is impossible or if it is very weak, the magnetic field can appear in the ring because of the dynamo-wave, similarly as in the peripheral parts of the galaxies (Mikhailov et al. 2014). The magnetic field is generated in the central regions, and after that a wavefront appears there and carries the magnetic field over the gap between the main part of the galaxy and the outer ring. The main restriction that can weaken this mechanism is the interstellar medium density in this interval. If the density is lower by more than 3–4 orders of magnitude than one in the inner regions, the wave cannot transfer the magnetic field with satisfactory rate.

To describe the magnetic field front motion we use the asymptotic theory of contrast structures. This theory describes the motion of the transition layer that connects stationary solutions of the nonlinear differential equations. The velocity of the front motion is proportional to the square root of the turbulent diffusivity coefficient. This result can be obtained both numerically and asymptotically for some approximations. Then, this theory can describe some specific fronts that describe the field that can changes its sign.

We give estimates of the magnetic fields of some galaxies with the outer rings. These estimates can be checked observationally with such radio telescopes as LOFAR and VLA (Moss et al. 2016).

2 Galaxy dynamo

The galaxy magnetic field evolution is described by the dynamo mechanism. This mechanism is based on two effects: the first one is the differential rotation and the second one is the alpha-effect that characterizes the turbulent motions of the interstellar medium. Each of them, according to the Lenz's law can cause only the magnetic field decay, but their combined action can organize the positive feedback and for some values of the parameters the magnetic field generation can be observed.

The evolution of the regular component of the magnetic field **B** is described by Steenbeck–Krause–Raedler equation (Beck et al. 1996):

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}\,[\mathbf{V},\mathbf{B}] + \mathrm{rot}\,(\alpha\mathbf{B}) + \eta\Delta\mathbf{B}, \tag{1}$$

where **V** is the velocity of the large-scale motions (usually they are connected with the galaxy rotation), α is the characteristic of the alpha-effect. For thin galaxy disc, for which the half-thickness h is much less than typical lengths in the galaxy disc plane, we can use the so-called no-z model (Moss 1995; Phillips 2001). According to this model, the derivatives of the magnetic field components along the direction that is perpendicular to the disc plane can be replaced by algebraic expressions:

$$\frac{\partial^2 B_{r,\varphi}}{\partial z^2} \approx -\frac{\pi^2 B_{r,\varphi}}{4h^2},$$

and the z-component of the magnetic field can be reconstructed from the solenoidality condition. According to this approximation the vector equation (1) will transform into the system of two scalar equations for the plane components of the magnetic field:

$$\frac{\partial B_r}{\partial t} = -\frac{\Omega l^2}{h^2} B_\varphi + \eta\left\{-\frac{\pi^2 B_r}{4h^2} + \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_r)\right)\right\}; \tag{2}$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{\partial \Omega}{\partial r} B_r + \eta\left\{-\frac{\pi^2 B_\varphi}{4h^2} + \frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB_\varphi)\right)\right\}, \tag{3}$$

where Ω is the angular velocity of the galaxy rotation, l – scalelength of the turbulent motions, h – half-thickness of the galaxy disc, η – turbulent viscosity. Let us make some rough estimates for the outer rings of galaxies, for which $R < r < R + d$, where R is the inner ring radius, d is its width. The radius of the ring is quite large, so:

$$\frac{\partial}{\partial r}\left(\frac{\partial}{r\partial r}(rB)\right) \approx \frac{\partial^2 B}{\partial r^2}.$$

If the outer ring has half-width d, we can approximately assume that:

$$\frac{\partial^2 B_{r,\varphi}}{\partial r^2} \approx -\frac{\pi^2}{4d^2} B_{r,\varphi},$$

so the equations (2)–(3) can be rewritten as:

$$\frac{dB_r}{dt} = -\frac{\Omega l^2}{h^2} B_\varphi - \eta\frac{\pi^2}{4}\left(\frac{1}{h^2} + \frac{1}{d^2}\right) B_r; \tag{4}$$

$$\frac{dB_\varphi}{dt} = -r\frac{\partial\Omega}{\partial r}B_r - \eta\frac{\pi^2}{4}\left(\frac{1}{h^2}+\frac{1}{d^2}\right)B_\varphi. \quad (5)$$

The rate of the magnetic field growth can be described as

$$\gamma = \eta(-\frac{\pi^2}{4}(\frac{1}{h^2}+\frac{1}{d^2}) + \frac{1}{h^2}\sqrt{D}),$$

where $D = \dfrac{9h^2\Omega r\dfrac{\partial\Omega}{\partial r}}{v^2}$ is the dynamo number that characterizes the intensity of the dynamo action. As we can see, the magnetic field can grow if D is more than some threshold value. Then, the field can grow only for thick rings. For example, if the kinematic parameters of the interstellar medium are quite close to the ones in the Milky Way, the field will grow only if $d > 1$ kpc.

We should also take into account that the magnetic field evolution is connected with the turbulent motions. The energy of these motions transforms into the field energy. So the field growth should stop if the value of the field becomes close to so-called equipartition value (Beck et al. 1996; Arshakian et al. 2009):

$$B^* = 2v\sqrt{\pi\rho}.$$

It is quite possible to assume that in the inner part and in the ring $B^* = 5$ μG and in the gap between them $B^* = 0.5$ μG. The equations (4)–(5) can be rewritten as:

$$\frac{dB_r}{dt} = -\frac{\Omega l^2}{h^2}\frac{B_\varphi}{1+\dfrac{B_r^2+B_\varphi^2}{B^{*2}}} - \eta\frac{\pi^2}{4}\left(\frac{1}{h^2}+\frac{1}{d^2}\right)B_r; \quad (6)$$

$$\frac{dB_\varphi}{dt} = -r\frac{\partial\Omega}{\partial r}B_r - \eta\frac{\pi^2}{4}\left(\frac{1}{h^2}+\frac{1}{d^2}\right)B_\varphi \quad (7)$$

or in some similar nonlinear forms.

To check these approximate predictions we have calculated the field numerically for two different cases. At first we have studied the field generation that is connected only with the outer ring. After that we studied the question if the magnetic field can pass through the gap between the inner parts and the outer ring.

3 Numerical results

We have used the initial field of 10^{-3} μG. For different cases it was concentrated in the inner parts or in the ring.

As for initial field that is fully concentrated in the outer ring we have obtained the result that it can grow only for wide rings (Fig. 1 shows the field for $t = 10$ Gyr from the start of the field evolution). If the ring is thin, the field will grow very slowly ($d = 1000$ pc). For $d = 1500$ pc and for $d = 2000$ pc we obtain the field growing rapidly.

If we have some initial field in the inner parts, the field will be generated by another mechanism (Fig. 2). At first the field grows in the inner parts and then it passes

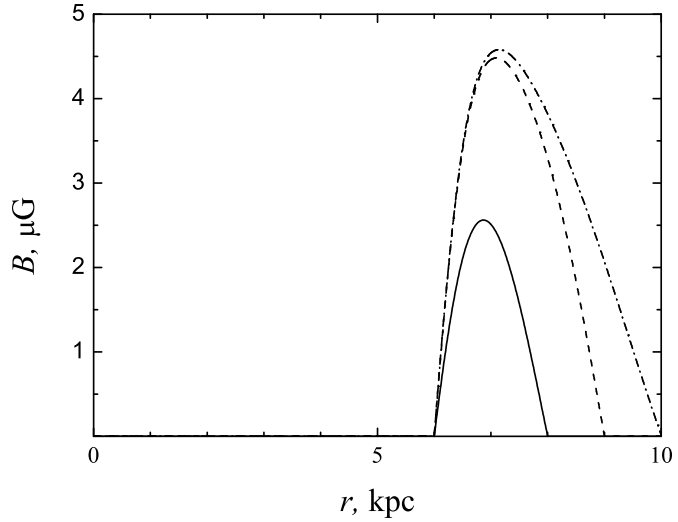


Figure 1 Evolution of the magnetic field that is fully concentrated in the outer ring. Solid line, $d = 1$ kpc; dashed line, $d = 1.5$ kpc; dot-dashed line, $d = 2$ kpc.

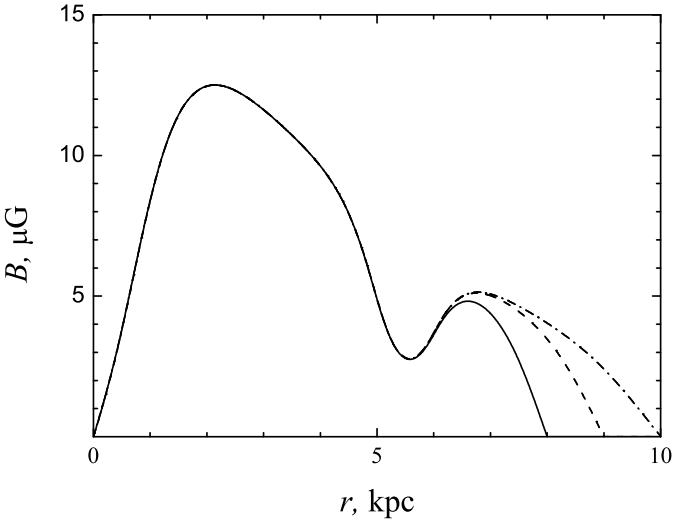


Figure 2 Evolution of the magnetic field, when the initial field is concentrated in the inner part. Solid line, $d = 1$ kpc; dashed line, $d = 1.5$ kpc; dot-dashed line, $d = 2$ kpc.

through the gap to the outer ring (Fig. 3). This is known in mathematical physics as the Kolmogorov–Petrovsky–Piskunov effect and is quite normal for parabolic differential equations.

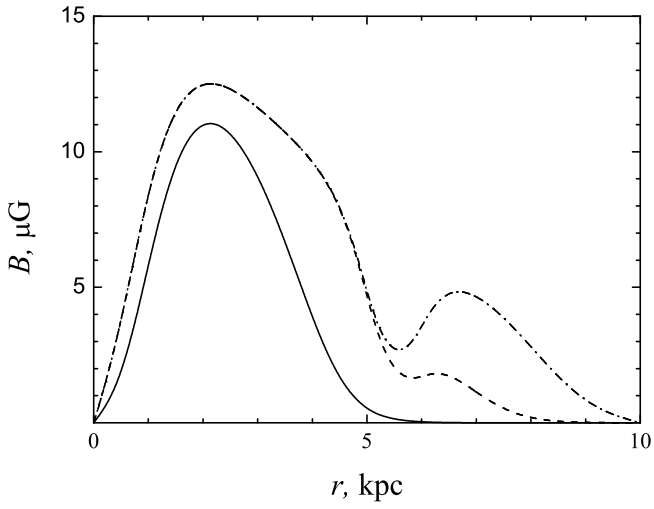


Figure 3 Magnetic field front motion. Solid line shows $t = 2$ Gyr, dashed line $t = 4$ Gyr, dot-dashed line $t = 6$ Gyr.

4 Kolmogorov–Petrovsky–Piskunov effect for galaxy dynamo

The Kolmogorov–Petrovsky–Piskunov effect can be described by the asymptotic theory of contrast structures (Volkov & Nefedov 2013). The problem for the two components of the galactic magnetic field is quite complicated. Some aspects of this problem for galaxy dynamo have been considered by Mikhailov (2015). Approximately the field evolution can be described by the equations:

$$\frac{\partial B_r}{\partial t} = \frac{\Omega l^2}{h^2}\left(1 - \frac{B_r^2 + B_\varphi^2}{B^{*2}}\right) B_r - \frac{\pi^2}{4}\left(\frac{1}{h^2} + \frac{1}{d^2}\right) B_\varphi - B_r + \eta \frac{\partial^2 B_r}{\partial r^2}; \tag{8}$$

$$\frac{\partial B_\varphi}{\partial t} = r\frac{\partial \Omega}{\partial r} B_r - \frac{\pi^2}{4}\left(\frac{1}{h^2} + \frac{1}{d^2}\right) B_\varphi + \eta \frac{\partial^2 B_\varphi}{\partial r^2}. \tag{9}$$

Here we omit the components of Laplace operator that do not contain second derivatives. This is quite possible because if we study outer rings, the distance from galactic center is quite large. At least for some values of the kinematic parameters the magnetic field will grow. But, as we can see, there is a nonlinearity in the first equation. So if the field becomes higher, the rate of the field growth will decrease. To obtain the stationary value of the magnetic field we should accept:

$$\frac{\partial B_r}{\partial t} = \frac{\partial B_\varphi}{\partial t} = 0.$$

If we neglect the second derivatives (as is usually done in the asymptotic theory of contrast structures), there will be such pairs of the field components (Mikhailov 2015):

$$B_{1r} = -B^* \frac{\pi^2 \eta}{4h^2 r \dfrac{\partial \Omega}{\partial r}} \sqrt{\frac{1 + \dfrac{\pi^2 \eta}{16 \Omega h^2 l^2 r \dfrac{\partial \Omega}{\partial r}}}{1 + \dfrac{\pi^4 \eta^2}{16 h^4 r^2 \left(\dfrac{\partial \Omega}{\partial r} \right)^2}}},$$

$$B_{1\varphi} = B^* \sqrt{\frac{1 + \dfrac{\pi^2 \eta}{16 \Omega h^2 l^2 r \dfrac{\partial \Omega}{\partial r}}}{1 + \dfrac{\pi^4 \eta^2}{16 h^4 r^2 \left(\dfrac{\partial \Omega}{\partial r} \right)^2}}};$$

$$B_{2r} = B_{2\varphi} = 0;$$

$$B_{3r} = B^* \frac{\pi^2 \eta}{4h^2 r \frac{\partial \Omega}{\partial r}} \sqrt{\frac{1 + \dfrac{\pi^2 \eta}{16 \Omega h^2 l^2 r \dfrac{\partial \Omega}{\partial r}}}{1 + \dfrac{\pi^4 \eta^2}{16 h^4 r^2 \left(\dfrac{\partial \Omega}{\partial r} \right)^2}}},$$

$$B_{3\varphi} = -B^* \sqrt{\frac{1 + \dfrac{\pi^2 \eta}{16 \Omega h^2 l^2 r \dfrac{\partial \Omega}{\partial r}}}{1 + \dfrac{\pi^4 \eta^2}{16 h^4 r^2 \left(\dfrac{\partial \Omega}{\partial r} \right)^2}}}.$$

If

$$1 + \frac{\pi^2 \eta}{16 \Omega h^2 l^2 r \dfrac{\partial \Omega}{\partial r}} > 0,$$

pairs "1" and "3" will be stable stationary solutions of (8)–(9). Pair "2" will be a non-stationary solution. If

$$1 + \frac{\pi^2 \eta}{16 \Omega h^2 l^2 r \dfrac{\partial \Omega}{\partial r}} < 0,$$

(8)–(9) will have the only solution "2", and it will be stationary.

If there is some initial magnetic field in the inner part (for small r) it will grow until it becomes close to pair "1" or "3". As for large r, where there is no initial field, we shall have a zero field. The asymptotic contrast structure field predicts that there

is a transition layer that connects the solution "1" in the inner part and the solution "2" in the outer part. This layer will move with velocity (Mikhailov 2015)

$$V_1 = C_1\sqrt{\eta},$$

where C_1 is the constant that depends on the kinematic parameters of the interstellar medium and can be calculated numerically.

Then, it is also interesting to describe the transition layer that connects the solutions "1" and "3" (it may be important for the rings of the magnetic fields, when the field has different signs). The velocity of motion of such value can be described by the formula (Mikhailov 2015)

$$V_2 = C_2\eta,$$

where C_2 is some constant.

However, we should have even approximate estimates of the magnetic field front motion. To study this process we can use some rough approximation that describes the evolution of some effective value of the field (Moss et al. 1998; Mikhailov 2015):

$$\frac{\partial B}{\partial t} = \gamma(r)B(1 - B^2) + \eta\frac{\partial^2 B}{\partial r^2},$$

where the magnetic field is normalized to equipartition values B^*, γ is the rate of the magnetic field exponential growth.

For the stationary solutions we have:

$$B_1 = 1,$$

$$B_2 = 0,$$

$$B_3 = -1,$$

where "1" and "3" will be stable stationary solutions (if $\gamma > 0$) and "2" will be unstable. If we describe the transition layer that connects the solutions "1" and "2" its velocity will be (Moss et al. 1998):

$$V_1 = \sqrt{2\gamma\eta}.$$

So, if $\gamma = 0.5$ Gyr^{-1}, $l = 100$ pc and $v = 10$ km s^{-1} the front velocity will be: $V_1 \approx 1$ kpc Gyr^{-1}. This value is quite close to the numerical results.

As for the layer that connects the solutions "1" and "3", the velocity will be (Mikhailov 2015)

$$V_1 = \frac{\gamma'(r)\eta}{2\gamma(r)}.$$

5 Conclusion

We have shown that the magnetic field can be generated in the outer rings. There are two possible effects. The first one is connected with the field generation *in situ* and the second one is connected with the Kolmogorov–Petrovsky–Piskunov effect. The field can pass through the gap between the inner part and the outer ring. Its velocity is proportional to the square root of the coefficient of turbulent diffusivity. The velocity of the wavefront is quite enough to reach the outer ring during several Gyr.

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