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Receiver gain and system temperature instability during the calibration of spectral data at radio telescopes in C-band

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The errors arising at the calibration of spectral data from single-dish observations at the radio telescopes of C-band are considered. It was assumed that errors arise as a result of the instability of the receiver gain and system temperature. Theoretical formula for the relative error of the antenna temperature of the source is obtained. Simulation of the calibration process was carried out to verify this formula. The process of observation of methanol masers at the frequency of 6.7 GHz at the Irbene RT-32 radio telescope (Ventspils) was reproduced during simulation. The errors of the antenna temperature of the source arising from observations are estimated.

Keywords: Radio telescope, calibration, single-dish, spectral observation

1 Introduction

The calibration process during single-dish observations on the radio telescopes of the C-band is described in many articles (see Winkel et al. (2012), Maddalena (2002), Braatz (2009)). Typically, the authors focus on the calculation of antenna temperature of the source or flux density. In the C-band, the calibration signal is usually formed by a noise diode. To organize spectral observations, it is necessary to set the temperature of noise diode T_{cal} and the on/off frequency $\frac{1}{2t_q}$ of the noise diode. In our previous work (Antyufeyev et al. (2020)), the ranges of permissible values for T_{cal}

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and t_q are discussed. T_{cal} and t_q limits are determined by the errors of the source antenna temperature. With an increase in T_{cal}, the system temperature T_{sys} and the standard deviation of the source antenna temperature σ_{T_s} increase. With a decrease in T_{cal}, σ_{T_s} decreases, but the error of the source antenna temperature δ_{T_s} increases. This error occurs due to the instability of the atmosphere and the receiver parameters.

In our previous article (Antyufeyev et al. (2020)) we considered errors that occur at the instability of the system temperature. This article is its continuation and development. It discusses errors of the antenna temperature of source T_s arising from the simultaneous instability of the system temperature T_{sys} and receiver gain g. In order to confirm the theoretical formulas, simulation of the calibration process was carried out.

2 Error of T_s at instability of T_{sys} and g

It is assumed that observations are carried out in the frequency switching mode. As in Antyufeyev et al. (2020) a noise diode is used as a calibration source. Observation time is divided into short periods which are called HS (high stability) periods. During the HS period for each of the spectrometer channels there is registration of signals P^{on}_{sig}, P^{on}_{ref}, P^{off}_{sig} and P^{off}_{ref} in different modes of the noise diode and shifts of oscillator frequency (Fig. 1). Definitions "sig" and "ref" correspond to the shifts of the oscillator frequency to $\pm\Delta f_m$. Definitions "on" and "off" correspond to the on and off mode of the noise diode. Each P signal accumulates for t_q time. The total number of HS periods, when observing one source, is N.

Under the assumption that T_{sys} and g are constant during an HS period, the antenna temperature of the source T_s is determined by the formula (Winkel et al. (2012))

$$
\begin{aligned}
&T^j_s(f) = T^j_{sou}(f) = \\
&\frac{1}{4}\left(T_{cal}(f-2\Delta f_m)\,k^j_{ref}(f-\Delta f_m)\,\frac{P^{off}_{sig}(f-\Delta f_m,j)-P^{off}_{ref}(f-\Delta f_m,j)}{P^{off}_{ref}(f-\Delta f_m,j)}+\right.\\
&+T_{cal}(f-2\Delta f_m)\left[k^j_{ref}(f-\Delta f_m)+1\right]\frac{P^{on}_{sig}(f-\Delta f_m,j)-P^{on}_{ref}(f-\Delta f_m,j)}{P^{on}_{ref}(f-\Delta f_m,j)}+\\
&+T_{cal}(f+2\Delta f_m)\,k^j_{sig}(f+\Delta f_m)\,\frac{P^{off}_{ref}(f+\Delta f_m,j)-P^{off}_{sig}(f+\Delta f_m,j)}{P^{off}_{sig}(f+\Delta f_m,j)}+\\
&\left.+T_{cal}(f+2\Delta f_m)\left[k^j_{sig}(f+\Delta f_m)+1\right]\frac{P^{on}_{ref}(f+\Delta f_m,j)-P^{on}_{sig}(f+\Delta f_m,j)}{P^{on}_{sig}(f+\Delta f_m,j)}\right)
\end{aligned}
\tag{1}
$$

Here

$$
k^j_{ref}(f-\Delta f_m) = \left(fitting\left(\frac{P^{on}_{ref}(f-\Delta f_m,j)}{P^{off}_{ref}(f-\Delta f_m,j)}\right)-1\right)^{-1}
$$

$$
k^j_{sig}(f+\Delta f_m) = \left(fitting\left(\frac{P^{on}_{sig}(f+\Delta f_m,j)}{P^{off}_{sig}(f+\Delta f_m,j)}\right)-1\right)^{-1}
$$

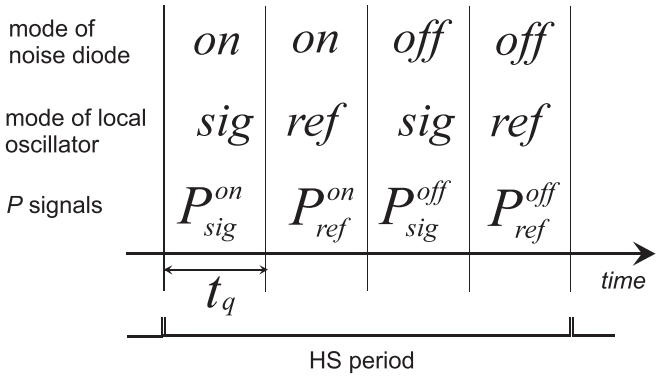


Figure 1 Time structure of the HS period.

T_{cal} is the temperature of the noise diode. T_{sou} is the antenna temperature of the source defined without error with stable T_{sys} and g. j is the HS period number ($j = 1...N$). "Fitting" in the formulas for k_{sig} and k_{ref} means that instead of the relations of the P signals, polynomials are used (Winkel et al. (2012)). We used the method of least squares to fit $\frac{P^{on}_{ref}(f-\Delta f_m,j)}{P^{off}_{ref}(f-\Delta f_m,j)}$ and $\frac{P^{on}_{sig}(f+\Delta f_m,j)}{P^{off}_{sig}(f+\Delta f_m,j)}$ by low degree polynomial.

In this article, we investigate the error of the antenna temperature of the source δ_{T_s}. We assume that its source is the instability of the receiver gain g and the system temperature T_{sys}. That is, when the noise diode is turned on and off, the T_{sys} may have different values, since the diode is turned on and off at different points in time (similar to g). This effect influences the value of T_s directly and does not depend on the noise which is visible in the T_s spectrum. Since we are searching for the error of the source antenna temperature, which does not depend on noise, we can neglect the noise component of T_s. Then it is not necessary to do fitting of k_{sig} and k_{ref}, and formula (1) will take the following form

$$T^j_s(f) = T^j_{sou}(f) = \frac{1}{4}T_{cal}(f-2\Delta f_m)\cdot \frac{P^{on}_{sig}(f-\Delta f_m,j) - P^{on}_{ref}(f-\Delta f_m,j) + P^{off}_{sig}(f-\Delta f_m,j) - P^{off}_{ref}(f-\Delta f_m,j)}{P^{on}_{ref}(f-\Delta f_m) - P^{off}_{ref}(f-\Delta f_m)} + \\ + \frac{1}{4}T_{cal}(f+2\Delta f_m)\cdot \frac{P^{on}_{ref}(f+\Delta f_m,j) - P^{on}_{sig}(f+\Delta f_m,j) + P^{off}_{ref}(f+\Delta f_m,j) - P^{off}_{sig}(f+\Delta f_m,j)}{P^{on}_{sig}(f+\Delta f_m,j) - P^{off}_{sig}(f+\Delta f_m,j)} \tag{2}$$

In Antyufeyev et al. (2020), it is shown that the greatest errors of T_s occur when T_{sys} and g change for different modes of the noise diode. Suppose that in the on mode of noise diode, the system temperature and the receiver gain are T_{sys} and g, and in the off mode they are $T_{sys} + \Delta T_{sys}$ and $g + \Delta g$ respectively. Source antenna temperature will also contain error: $T_s = T_{sou} + \Delta T_s$. P signals will be equal to

$$\begin{array}{l} P^{on}_{sig}(f,j) = g^j(f+\Delta f_m)\cdot \\ \quad \left(T^j_{sys}(f+\Delta f_m) + T_{cal}(f+\Delta f_m) + T^j_{sou}(f+\Delta f_m)\right) \\ P^{off}_{sig}(f,j) = \left(g^j(f+\Delta f_m) + \Delta g^j\right)\cdot \\ \quad \left(T^j_{sys}(f+\Delta f_m) + \Delta T^j_{sys} + T^j_{sou}(f+\Delta f_m)\right) \\ P^{on}_{ref}(f,j) = g^j(f-\Delta f_m)\cdot \\ \quad \left(T^j_{sys}(f-\Delta f_m) + T_{cal}(f-\Delta f_m) + T^j_{sou}(f-\Delta f_m)\right) \\ P^{off}_{ref}(f,j) = \left(g^j(f-\Delta f_m) + \Delta g^j\right)\cdot \\ \quad \left(T^j_{sys}(f-\Delta f_m) + \Delta T^j_{sys} + T^j_{sou}(f-\Delta f_m)\right) \end{array} \quad (3)$$

Since the studied maser lines have a small frequency band, during observations we use smaller spectrometer bandwidth and smaller Δf_m. This gives a reason to assume that $g^j(f+\Delta f_m) = g^j(f-\Delta f_m) = g^j$, $T^j_{sys}(f+\Delta f_m) = T^j_{sys}(f-\Delta f_m) = T^j_{sys}$ and $T_{cal}(f) = const$. Substituting P signals from formula (3) to formula (2) and replacing T_s with $T_s = T_{sou} + \Delta T_s$, we obtain the formula for T_s at frequency f (hereinafter the f symbol is omitted for simplicity)

$$T^j_s = T^j_{sou} + \Delta T^j_s = \frac{T_{cal}}{2}\frac{g^j T^j_{sou} + \left(g^j + \Delta g^j\right)T^j_{sou}}{g^j T_{cal} - \Delta g^j T^j_{sys} - g^j \Delta T^j_{sys} - \Delta g^j \Delta T^j_{sys}}$$

Since $\Delta g^j << g^j$, $\Delta T^j_{sys} << T^j_{sys}$, so $\Delta g^j \Delta T^j_{sys}$ can be neglected. Defining relative errors of the receiver gain, system temperature and source antenna temperature as $\delta g^j = \frac{\Delta g^j}{g^j}$, $\delta T^j_{sys} = \frac{\Delta T^j_{sys}}{T^j_{sys}}$ and $\delta T^j_s = \frac{\Delta T^j_s}{T^j_{sou}}$ we obtain the formula for δT^j_s

$$\delta T^j_s = \frac{\frac{\delta g^j}{2} + \delta g^j \frac{T^j_{sys}}{T_{cal}} + \delta T^j_{sys}\frac{T^j_{sys}}{T_{cal}}}{1 - \delta g^j \frac{T^j_{sys}}{T_{cal}} - \delta T^j_{sys}\frac{T^j_{sys}}{T_{cal}}}$$

In practice, t_q time is short. Therefore, during $2t_q$ the changes of T_{sys} and g are less than 1%. On the other hand, the selected $\frac{T_{sys}}{T_{cal}}$ ratio is ~ 10. Hence, $\frac{\delta g^j}{2} << \delta g^j \frac{T^j_{sys}}{T_{cal}}$, $\frac{\delta g^j}{2} << \delta T^j_{sys}\frac{T^j_{sys}}{T_{cal}}$, $1 >> \delta g^j \frac{T^j_{sys}}{T_{cal}}$ and $1 >> \delta T^j_{sys}\frac{T^j_{sys}}{T_{cal}}$ and

$$\delta T^j_s \approx \left(\delta g^j + \delta T^j_{sys}\right)\frac{T^j_{sys}}{T_{cal}} \quad (4)$$

As we know from the probability theory, $\delta g^j + \delta T^j_{sys} \approx \delta\left(g^j T^j_{sys}\right)$. $g^j T^j_{sys}$ is the P^{off}_j signal. Note that δT^j_s is calculated for the whole HS period ($4t_q$). And formulas (3) are written with the assumption that T^j_{sys}, g^j and P^{off}_j have changed during $2t_q$. That is, signals P follow in the order P^{on}_{sig}, P^{on}_{ref}, P^{off}_{sig}, P^{off}_{ref}. Then from (4) it follows:

$$\delta T^j_s \approx \frac{1}{2}\frac{T^j_{sys}}{T_{cal}}\delta P^{off}_j \quad (5)$$

The resulting source antenna temperature is averaged over all j

$$T_s = \frac{1}{N}\sum_{j=1}^{N} T^j_s = \frac{1}{N}\sum_{j=1}^{N} T^j_{sou} + \frac{1}{N}\sum_{j=1}^{N}\Delta T^j_s = \overline{T_{sou}} + \frac{1}{N}\sum_{j=1}^{N}\Delta T^j_s$$

The relative error of δT_s is determined using the formula

$$\delta T_s = \frac{T_s - \overline{T_{sou}}}{\overline{T_{sou}}} \approx \frac{1}{N}\sum_{j=1}^{N} \delta T_s^j \tag{6}$$

We take into account that the total observation time is $ObsTime = N \cdot 4t_q$; the average system temperature, $\overline{T_{sys}} = \frac{1}{N}\sum_{j=1}^{N} T_{sys}^j$ as well as $\delta P_j^{off} = \frac{\Delta P_j^{off}}{P_j^{off}}$, $\sum_{j=1}^{N} \Delta P_j^{off}$ $= P_N^{off} - P_1^{off}$, $\overline{P^{off}} = \frac{1}{N}\sum_{j=1}^{N} P_j^{off}$. It is also assumed that T_{sys} during observations changes weakly and therefore $\sum_{j=1}^{N} T_{sys}^j \Delta P_j^{off} \approx \overline{T_{sys}} \sum_{j=1}^{N} \Delta P_j^{off}$. Considering this, as well as formulas (5) and (6) we obtain the final formula to estimate the relative error of the source antenna temperature δT_s

$$\delta T_s \approx \frac{\overline{T_{sys}}}{T_{cal}} \frac{P_N^{off} - P_1^{off}}{\overline{P^{off}}} \frac{2t_q}{ObsTime} \tag{7}$$

3 Simulation of the calibration process

Errors calculated in the previous section are much less than the errors arising from antenna pointing and it is difficult to check the δT_s obtained by formula (7) using observations. Therefore, for verification of δT_s, we simulated the calibration process during observations on the RT-32 radio telescope of the methanol maser at the frequency 6.7 GHz. The following equipment parameters were used:

- number of spectrometer spectral channels $Nch = 8000$
- channel separation of the spectrometer $\Delta f = 390$ Hz
- frequency shift of local oscillator $\Delta f_m = 390$ kHz
- system temperature T_{sys} varied from 30 to 40 K
- total observation time of one source $ObsTime = 1$ hour
- temperature of the noise diode T_{cal} varied from 2.2 to 6.2 K
- t_q time varied and we used t_q of 10 sec, 20 sec and 30 sec

The receiver gain consisted of three parts $g(f,t) = G_{IF}(f)G_{RF}(f)G_t(t)$. $G_{IF}(f)$ is the coefficient which does not depend on the frequency of the oscillator and is usually determined by the amplitude-frequency characteristic of the filter before the Fourier-spectrometer. $G_{RF}(f)$ is the coefficient that depends on the oscillator frequency (by analogy with Winkel et al. (2012)). $G_t(t)$ is the coefficient describing the dependence of gain on time. In our case, $G_{IF}(f)$ was determined from the spectrum of P^{off} signals. We presented this G_{IF} as a function of the spectrometer channel number i: $G_{IF}(i) = 0.03\left(1/3 - \left(\frac{i - Nch/2}{Nch/2}\right)^2\right) + 1$. Since in our case Δf_m is very small, it was assumed that $G_{RF}(f) = const$. To determine $G_t(t)$, an experiment was carried out. The receiver's horn was closed by the absorber whose temperature practically did not change within an hour. During this time, the P^{off} signal was registered, being averaged throughout the spectrum. We considered the changes of the P^{off} signal as

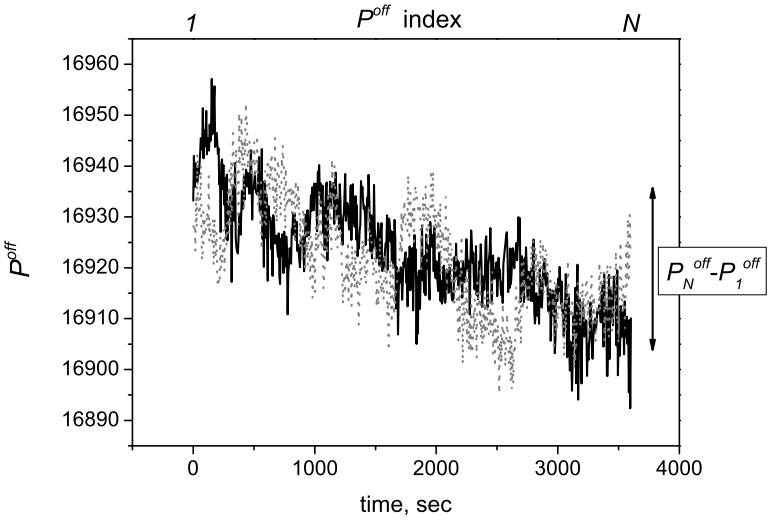


Figure 2 The black line shows the averaged P^{off} signal, measured within 1 hour with the horn, which is closed with the absorber. Gray dots show one of the mathematically simulated P^{off} signals for the case when the horn is closed with the absorber.

$G_t(t)$ instability, since the receiver noise temperature within 1 hour should be stable. We calculated the spectral power density of the P^{off} signal and generated a random $G_t(t)$ signal with this spectral power density (except for several first harmonics). Figure 2 shows two signals. The black line shows the averaged P^{off} signal, measured within 1 hour with the horn, which is closed with the absorber. On this basis, receiver gains $G_t(t)$ are generated, which are used when modeling the calibration process. Based on $G_t(t)$, P^{on} and P^{off}signals are simulated for different local oscillator and noise diode modes. Gray dots in Fig. 2 show one of the mathematically simulated P^{off}signals for the case when the horn is closed with the absorber.

Since the bandwidth of the spectrometer is small and amounts to 3.12 MHz, we believed that the frequency dependence of T_{sys} and T_{cal} can be neglected.

P^{off} signals were the sum of T_{sys} and Gaussian noise with a standard deviation $\frac{T_{sys}}{\sqrt{t_q \Delta f}}$, which was multiplied by the gain $g(f,t)$. P^{on} signals were formed similarly, only T_{sys} was used instead of $T_{sys} + T_{cal}$.

During the simulation process a small test signal (Gauss function with the maximum height A) was added to P^{off} and P^{on} signals. Then the $T_s(f)$ was calculated by formula (1). The T_s base line was aligned using fitting by polynomial function. Then it was fitted by the Gauss function and the maximum height of the Gauss function A_{fit} was determined. The relative error δT_s was calculated by formula $\delta T_s = \frac{A_{fit} - A}{A}$.

Checking the simulation of the calibration process and formula (7) was carried out as follows. At first, we set T_{sys}= 30 K throughout the *ObsTime* and calculated the standard deviation of the source antenna temperature σ_{T_s}. The temperature of the noise diode T_{cal} varied from 2.2 to 6.2 K. Figure 3 shows dependences of σ_{T_s} on T_{sys}/T_{cal} marked by points, crosses and triangles for t_q= 10 sec, 20 sec and 30

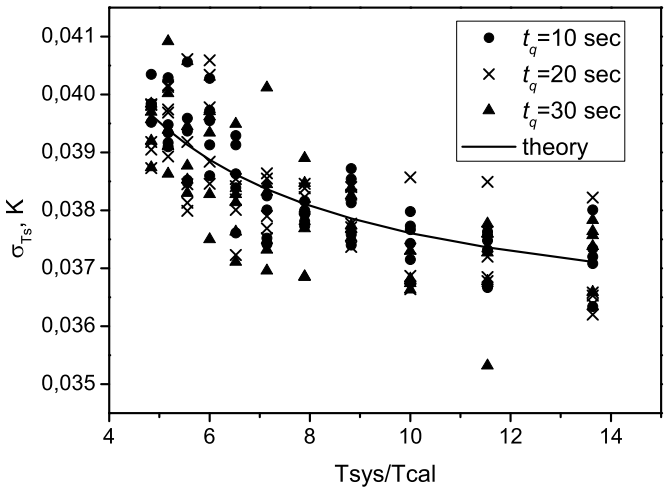


Figure 3 Dependences of σ_{T_s} on T_{sys} marked by points, crosses and triangles for t_q= 10 sec, 20 sec and 30 sec respectively.

sec respectively. A theoretical formula for σ_{T_s} is given in Winkel et al. (2012) (for frequency-dependent T_{sys}) or in Antyufeyev et al. (2020):

$$\sigma_{T_s} = \sqrt{\frac{T_{sys}^2 + \left(T_{sys} + T_{cal}\right)^2}{\Delta f \cdot ObsTime}} \tag{8}$$

Considering Fig. 3 we can see that the theoretical σ_{T_s} corresponds to the results of simulations and does not depend on t_q. This indicates the correctness of formula (8) and simulations of the calibration process.

We performed calculations of the relative error of the source antenna temperature δT_s at various time dependences of T_{sys} and g. The theoretical formula (7) always gave the result close to the simulation. Figure 4 shows the results of one of these calculations. At this T_{sys} consistently increased from 30 to 35 K during $ObsTime$. It corresponds to cloudy weather. $G_t(t)$ was determined as described at the beginning of this section. When calculating T_{sys}/T_{cal} (to display data along the X axis), the time-averaged system temperature was used. In Fig. 4 points, crosses and triangles show the results of alculations of simulation for t_q= 10 sec, 20 sec and 30 sec, respectively. Solid, dash and dot lines show the corresponding δT_s calculated by formula (7).

4 Discussion

We could not find in the literature papers devoted to the effect of instability of receiver gain and system temperature during the calibration of spectral data at radio telescopes in C-band. Therefore, in this section, we only provide our results for the RT-32 radio telescope of the Irbene observatory. The test with a closed horn indicates that the receiver gain g is sufficiently stable. Therefore, formula (7) can be simplified. This was done as follows.

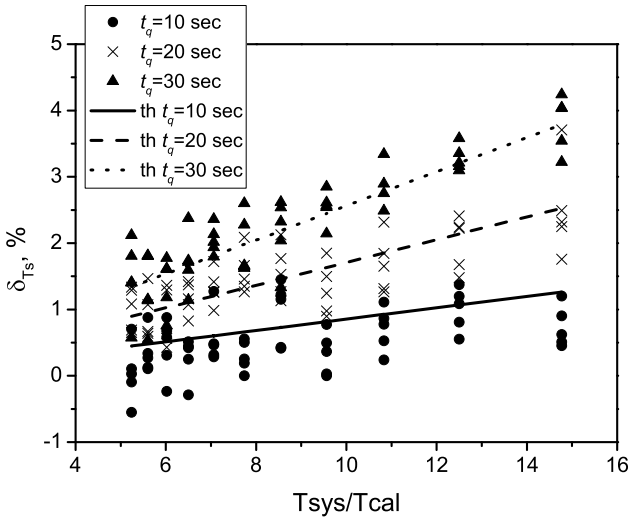


Figure 4 The relative error of the source antenna temperature δT_s vs T_{sys}/T_{cal}. Points, crosses and triangles show the results of calculations of simulations for t_q= 10 sec, 20 sec and 30 sec respectively. Solid, dash and dot lines show the corresponding δT_s calculated by formula (7).

Assuming that T_{sys} and g values are quite stable, formula (7) can be written as

$$\delta T_s \approx \frac{\overline{T_{sys}}}{T_{cal}} \left(\frac{T_{sys}^N - T_{sys}^1}{\overline{T_{sys}}} + \frac{g^N - g^1}{\overline{g}} \right) \frac{2t_q}{ObsTime}$$

From the experiment with a closed horn, we estimated the stability of $g(t)$ for an hour. Since the temperature of the absorber during the experiment did not change, $P^{off} \sim g(t)$. Therefore, in this case $\frac{P_N^{off} - P_1^{off}}{\overline{P^{off}}} = \frac{g^N - g^1}{\overline{g}}$. Analysis of P_{off} (Fig. 2) shows that $P_N^{off} - P_1^{off} \approx 30$ and $\overline{P^{off}} = 16923$. Hence $\frac{P_N^{off} - P_1^{off}}{\overline{P^{off}}} \approx 0.2\%$. We made several observations at the RT-32 radio telescope and determined that within 1 hour $\frac{T_{sys}^N - T_{sys}^1}{\overline{T_{sys}}}$ can vary from 2% to 10% depending on the elevation and weather conditions. Consequently, $\frac{T_{sys}^N - T_{sys}^1}{\overline{T_{sys}}} >> \frac{g^N - g^1}{\overline{g}}$ and the $\frac{g^N - g^1}{\overline{g}}$ can be neglected. In this case, g_t can be considered constant and formula (7) can be written as:

$$\delta T_s \approx \frac{T_{sys}^N - T_{sys}^1}{T_{cal}} \frac{2t_q}{ObsTime} \tag{9}$$

Using formulas (7) and (9), the δT_s error can be calculated accurately. So, the antenna temperature of the source T_s can be refined to compensate for this error.

The process of observation of methanol masers at the frequency of 6.7 GHz at the

Irbene RT-32 radio telescope was reproduced during simulation. We carried out numerous T_{sys} measurements at different elevations, under different weather conditions. It is known that within 1 hour T_{sys} can change by several Kelvins in cloudy weather and no more than 1–2 K in good weather. For RT-32 we selected the following parameters: $t_q = 20$ sec, $T_{cal} = 3.8$ K at the frequency of 6.7 GHz. T_{sys} depended on weather conditions and elevation and was $30 \ldots 40$ K. From formula (9) and Fig. 4 it can be seen that with these values δT_s is 1%–3% in bad weather and less than 1% in good weather. This is consistent with the results of our previous article (Antyufeyev et al. (2020)).

5 Conclusions

The article is devoted to the calibration of spectral data obtained from radio telescope observations using a noise diode as a calibration source. The frequency switch mode was considered. The errors of the source antenna temperature arising from the instability of the receiver gain and the system temperature were discussed. Theoretical formula for determination of the relative error of the antenna temperature of the source was obtained. Simulation of the calibration process was carried out taking into account the instability of the receiver gain and the system temperature. It is shown that the results of simulation and theoretical formula for the relative error are close. The parameters characteristic of RT-32 radio telescope were reproduced in this simulation. It is shown that the relative error of the source antenna temperature at RT-32 is 1%–3% in bad weather and less than 1% in good weather.

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