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Towards the density of matter: From micro-mechanical (granular) to macro-mechanical (Universe) systems

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We discuss some visible analogies of mapping of the density of matter field which has been observed in Universe and in particular in micro-mechanical granular materials. In both of the mentioned global classes of physical systems the same known initial problem exists: the local density field is insufficiently defined. Several structural characters which are in common in those, quite differently scaled systems (like specific clusterisation), are outlined.

The observed external similarity of clustering, characterized by the formation of specific filamentous clusters, both in granular matter and in the distribution of matter in the Universe allows the model level to use the granular phase parameterization scenario (compaction parameter instead of density) in the study of distribution of matter in the Universe.

Keywords: Universe, micro-mechanical systems, granular materials, clustering, filaments, voids

1 Introduction

The external similarity of the different nature of stationary structures, such as, for example, a typical configuration of granular materials, arising from clustering, and the large-scale structure of the Universe, suggest that their formation is due to some General principles independent of physical details, the systematic study of which initiated the work of Turing (Turing, 1952).

A general study of a sufficiently large volume of Metagalaxy showed that with increasing scale (R) there are signs of thread-like structuring, that is, the distribution of the substance is anisotropic. With a further increase in R a complication of the morphology of the distribution of matter is observed in the form of a mesh or grid structure. Spaces in this kind of grid structure, i.e. interphase space, are called

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voids. Voids are one of the types of observed large-scale structuring of matter in the Universe. They are regions of density contrast observed with the increase in the scale of observation and have many sizes and shapes.

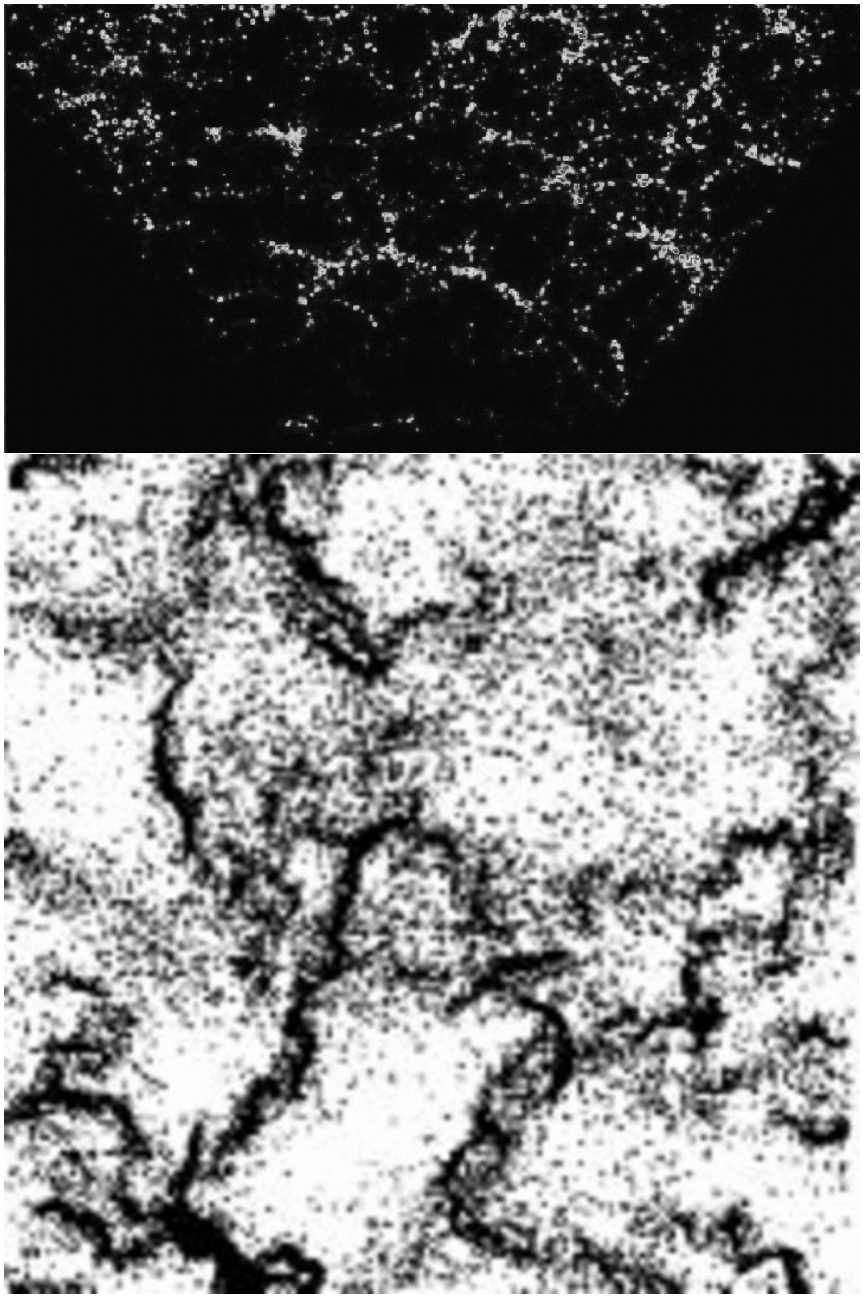


Figure 1 Large-scale structuring of matter in the Universe (top) (Einasto et al., 2007) and clustering in the granular system (bottom) (Jaeger et al., 1996).

Local void is the nearest giant area, almost free from galaxies, which consists of three separate sectors separated by filamentous bridge-thin threads. Located near the Local group of galaxies (LGG) and adjacent to it, removed from the LGG at a distance of 23 Mpc and limited to the Local Sheet (galactic filament containing LGG).

The cellular structure is observed at a distance of more than 1 billion light-years in any direction. As already mentioned, a similar structure is shown by granular materials, where friction exists in the system, and collisions of individual granules are inelastic. Granular materials are large agglomerations of discrete macroscopic particles. If they are not connected, the forces between the particles are only repulsive, so the shape of the material will be determined by the external boundaries and gravity. Granular material behaves differently from any familiar form of substance. There are two particularly important aspects that contribute to the unique properties of granular materials: 1) temperature in the conventional sense plays no role and 2) interactions between granules are dissipative due to static friction and inelastic collisions.

Under some conditions, it is possible to consider the granular flow as a dense gas. However, this is no ordinary gas! Within a system in which there are dissipative interactions, clustering begins to occur. Depending on the type of excitation, triggering the dynamics of the system, it leads to the appearance of not symmetrical seals, but to long thread-like configurations, resembling the density maps of the visible part of the Universe (Fig. 1). Perhaps nature offers us a way to explore the possibility of the existence of a unique scaling when scaling the structure of matter. Examples, which are known to us from everyday communication with granular materials in laboratory conditions and in the environment, allow us to consider such systems as unique laboratories for studying the nature of local ordering and equilibrium, as well as the formation of ordered structures in meso-, macro-, and maybe mega-scales (Gurbatov et al., 2012).

2 From micro-mechanical to macro-mechanical systems

Even a preliminary comparative analysis shows the commonality of the above phenomena in the following factors.

First, the instability with respect to clustering: in a granular system there are density inhomogeneities that do not disappear, but rather increase, which leads to the emergence of clusters. In the early Universe, there were also initial density inhomogeneities (small perturbations), which eventually evolved into strong perturbations and eventually formed a cellular structure. The question of the nature of these initial disturbances is one of the most difficult in cosmology. Can scenarios of the relevant phenomena in granular materials help to answer this question? Is there a theory that describes the behavior of granular materials as well as space matter similarly? It can be said that until now there are no generally accepted theoretical approaches to the description of many observed physical properties of either granular materials or space matter distribution and their evolution. However, to some extent, there is progress. For example, the authors of (Gerasymov et al., 2003) show that it is possible to derive some parameter of order, which is determined through the particle velocity field. The governing equations can be written as equations to this order parameter. Furthermore, the problem can be reduced to a kind of phase transition.

Cosmology also considers stars, galaxies, clusters and superclusters of galaxies as randomly moving points from which, like a gas from molecules, a meta-galactic medium is formed. Therefore, we can try to adapt these scenarios for the Universe, which is prone to self-organization at all levels and the creation of hierarchical structures.

Second, the determination of the density of matter: insufficiently correct determination of the density field in granular materials. This circumstance has led to the development of new specific methods of parametrization of their local and multiscale structure. Such as: the definition of the field of compaction instead of density; methods of random and deterministic stereology, whose task is to draw conclusions about the distribution of objects in the space R^3, based on incomplete observations of one-dimensional and two-dimensional varieties in this space. And these last methods can be considered as adequate for the filaments and voids.

For example, visual mathematically and physically correct methods of their parametrization are needed to compare the properties of real-world inputs with the results of cosmological models. For example, the parameter by which many properties of granular materials (both in static and dynamic forms) can be described is the so-called compaction parameter, which is the product of the volume of each granule on their density:

$$\eta = k\pi\sigma\rho. \tag{1}$$

Here ρ is the density, σ is a cross-section of the observable objects respectively, k is the scaling parameter (scale invariance).

In the case of compaction of granular matter of this kind, the parameter shows a tendency to saturation, which is characterized by its maximum value. Thus, it can be considered as an order parameter that determines not only the local structuring, but also the kinetics of the system.

Since the "packing" (compaction) no two objects can intersect with each other, and all objects are inside the placement area, the objects are sorted in order of decreasing volume. Then, as a packing parameter for the local area of the Universe, one can select the value in the form

$$\eta = \frac{S}{\sigma Z}, \tag{2}$$

where S is the square of the observable local cluster and Z is a number of clusters in the window. To describe different types of distribution of the objects, the use of the Voronoi–Delaunay method, as well as the method of structural invariants, are an effective approach to the description of different types of symmetry of the observed structured objects (Gerasymov and Khudyntsev, 2015).

Each interval of the values of packing corresponds to a definitive type of observable structures. Therefore (as well as in case of granular systems structure analysis (Gerasymov and Spivak, 2015), the packing can be considered as an adequate parameter in the parametrization of the distribution of space matter.

Let us consider together both definitions of compactization (1) and (2)

$$\frac{S(k)}{\sigma Z} = k\pi\sigma\rho. \tag{3}$$

From Equation (3) one can obtain the power-law for density:

$$\rho = \frac{1}{\pi Z}\frac{S(k)}{k}\sigma^{-2}. \tag{4}$$

Equation (4) can be used to control the scaling.

$$\frac{S(k)}{k} \sim k^{\alpha}, \tag{5}$$

where α is the scaling parameter.

In order to study also geometrical picture of the space matter translational and orientational order parameters can be introduced within the concept of structural invariants (Gerasymov et al., 2013).

3 Simple model for kinetics of the compaction field

In order to model the evolution of the compaction field, a free volume theory can be addressed. In this framework, one can describe the process where the solid particle (associated with the grain) with volume ω can jump into a hole of the appropriate size Ω distributed independently with a certain distribution function $f(\omega/\Omega)$. The simplest rate equation which describes such a free volume kinetics takes a form

$$\frac{d\eta}{dn} = kf, \tag{6}$$

where the variable η is the packing fraction, and k is a kinetic coefficient. Introducing the limiting maximum values for the compactivity η_m and estimating the simplest Poisson distribution for the free volume

$$\Omega = \omega\left(\frac{1}{\eta} - \frac{1}{\eta_m}\right), \tag{7}$$

one has

$$\frac{d\eta}{dn} = k\exp\left(-\frac{\eta_m\eta}{\eta_m - \eta}\right). \tag{8}$$

An analytical solution of this equation which connects the packing fraction η, say, with the number of taps n (or other parameters which are responsible for the systems perturbations) can be obtained in the following functional form

$$\begin{aligned}(\eta_m - \eta_1)\exp\left(\frac{\eta_m^2}{\eta_m - \eta_1}\right) + \eta^2 E_1\left(-\frac{\eta_m^2}{\eta_m - \eta_1}\right) - \\ -(\eta_m - \eta)\exp\left(\frac{\eta_m^2}{\eta_m - \eta}\right) + \eta_m^2 E_1\left(-\frac{\eta_m^2}{\eta_m - \eta}\right) = k\exp(\eta_m)n,\end{aligned} \tag{9}$$

where η_1 is the initial compactivity and $E_1(z)$ is an integral exponent. If the initial state is already densified, i.e. $\eta_1 \to \eta_m \left(\frac{\eta_m^2}{\eta_m - \eta_1} \gg 1, \frac{\eta_m^2}{\eta_m - \eta} \gg 1 \right)$, Equation (6) gives

$$\eta = \eta_m - \frac{\eta_m^2}{\ln \gamma + \ln\left(\frac{a}{\gamma} + n\right)}, \tag{10}$$

where

$$a = \left| (\eta_m - \eta_1) \exp\left(\frac{\eta_m^2}{\eta_m - \eta_1} \right) + \eta_m^2 E_1\left(-\frac{\eta_m^2}{\eta_m - \eta_1} \right) \right|$$

and $\gamma = k\exp(\eta_m)$.

This logarithmic law for granular compaction has been reported many times either by experimentalists or by theoreticians. When the system is initially diluted so that $\eta_m^2/(\eta_m - \eta) < 1$, from (6) we perform

$$\eta = \eta_1 + \gamma n, \tag{11}$$

i.e. the relevant dynamics developed much faster than a logarithmic behavior. One should note that when $\eta_m^2/(\eta_m - \eta_1) < 1$, but $\eta_m^2/(\eta_m - \eta) > 1$, from (6) we derive again the logarithmic law given by (7). Independently from how the system was initially prepared (diluted or densified), in the limit when η tends to η_m, the model gives the slow logarithm rate of η growth. This simple argumentation demonstrates the principal possibility of getting qualitative kinetic scenarios, i.e. logarithmic dynamics (or more even faster stages) of compaction already being in the framework of a mean-field arguments, included in the origin of free volume kinetic model. Different scenarios are found in laboratory experiments with granular systems and also observed in the visible cross-sections of the space matter and a mixture of those dynamics which has been described vide supra is more appropriate for interpretation of the experimental data.

4 Conclusions

To date, researchers are faced with a huge number of different types of structuring of the substance, in which the density field is not fully defined. These are space-time structures in systems of physical, chemical and biological nature. Due to the external similarity of the results (with completely different mechanisms) of the formation of these structures, it is hoped that the methods developed to consider/to study the systems of one type can be applied to other (of course, with relevant limitations), having a different nature as well as different physical properties.

The observed external similarity of clustering, characterized by the formation of specific filamentous clusters, both in granular matter and in the distribution of matter

in the Universe, inspire us to use the scenario of parametrization of the granular phase (in which the parameter of compaction is used instead of density) at the model level in the study of the distribution of matter in the Universe. Already simple estimates indicate the presence of the scenarios of slow saturation dynamics in both systems.

These facts open the prospects of application of methods developed for granular types of matter to the study of the structure and dynamics of the distribution of matter in the Universe and the establishment of deeper laws (e.g. static and dynamic scaling) linking the behavior of micro-mechanical and macro-mechanical systems.

Thus it is shown that the study of the density of space matter by means of the compaction field concept leads to the possibility of simple qualitative description of the behavior of this value especially under conditions of strong inhomogeneity (and even anisotropy) of those complex systems. The presented work has been done in the Department of General and Theoretical Physics in Odessa State Environmental University where a systematic international research program which focuses on the study of soft matter continues (Gerasimov and Schram, 2002; Gerasymov, 2005; Gerasymov, 2006; Vandewalle et al., 2007; Gerasymov, 2010; Gerasymov, 2013; Gerasymov, 2015).

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