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Evolution of baryonic-matter density perturbations in the real Universe

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The evolution of baryonic-matter density perturbations against the background of oscillating dark matter is presented. The cases of coherence fluctuations, of a correlation between contributions to the kinetic energy of baryonic matter oscillations and to the potential energies of baryonic and dark matter at generalized frequency are examined. The mutual influence of baryonic oscillations and oscillating dark matter is also discussed. A system of Jeans equations is generalized for the case of the occurrence of external background dark matter.

Keywords: Dark matter, gravitational instability, theory of small perturbations, formation of galaxies

1 Introduction

One of the topical problems of modern cosmology is search for the physical properties of dark matter (Copeland et al. 2006; Orlov and Raikov 2014; Read 2014). Such properties, in particular, can be understood within the framework of examining the influence of dark matter (DM) on the dynamical processes that take place in large-scale cosmic objects (see, e.g., Bohmer et al. (2008)). Processes of the influence of dark matter on galaxies and evolution of their clusters are also related to this problem.

The problem of galaxy formation was first developed by Jeans based on ordinary (baryonic) matter perturbations (Zel'dovich and Novikov 1975). However, from the modern viewpoint on the substantial structure of the Universe (Chernin 2001), there is the necessity of generalizing Jeans equations to the case of the occurrence of dark matter background.

Various aspects of this problem were discussed, for example, in Yoshida et al. (2000, 2003) and Ahn and Shapiro (2005). Thus, Yoshida et al. (2000) study the formation of dark halos within the standard Universe model, assuming that DM particles in this model have a finite cross section at elastic collisions. They argue that each particle collision during the Hubble time can essentially affect the halo profile.

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Moreover, it is shown that the model of self-interacting dark matter does not allow us to construct the likelihood rotation curves for dwarf galaxies.

In Yoshida et al. (2003), the authors investigate the hierarchical growing of coupling cosmic structures that formed at redshifts $z \approx 25$–30 with masses within the range of $(10^5$–$10^6)\ M_\odot$. Based on numerical methods they propose a semi-analytic model for the formation of first stars that describes well the rate of star formation in the early Universe. In Ahn and Shapiro (2005) the authors study the formation of a cold dark matter halo in the presence of non-gravitational collisions and propose a theory that predicts the likelihood of halo profiles.

Unlike these and other papers, in this article we consider the evolution of baryonic-matter density perturbations in the background of oscillating dark matter that gives the chance for the most correct description of the galaxy formation process. Note that in closed formulation the evolution of baryonic matter density was recently discussed in Magana (2012).

2 Generalization of the Jeans equations for the external background of dark matter

As we said above the problem of homogeneous distribution matter stability was first formulated and solved within the framework of small perturbations theory by Jeans. He took into account two factors: gravity that tends to conserve the substance's division into separate clusters, and pressure that has the tendency to smoothing appeared nonhomogeneities.

Recall the self-consistent equations of hydrodynamics and gravitation in the Newtonian approximation for an ideal gas

$$\left.\begin{aligned} \frac{\partial p}{\partial t} + \operatorname{div}(\rho\vec{u}) = 0, \\ \frac{\partial \vec{u}}{\partial t} + (\vec{u}\operatorname{grad})\vec{u} + \frac{1}{\rho}\operatorname{grad} P + \operatorname{grad}\phi = 0, \\ \delta\phi = 4\pi G\rho. \end{aligned}\right\} \tag{1}$$

where $\rho = \rho_{\mathrm{BM}}$ is density, $\vec{u}$ is velocity, ϕ is gravitational potential. Consider that the unperturbed state is the stationary gas $\vec{u}_0 = 0$, that is uniformly distributed in space ($\rho = \rho_0 = const$). Its pressure is constant everywhere ($P = P(\rho_0) = const$). And the last assumption is the following: $\operatorname{grad}\phi = 0$.

To obtain a solution for perturbations it is necessary to use the method of decomposing any perturbed function over the system of orthogonal functions and their time evolution searching. Following Zel'dovich and Novikov (1975), the perturbed solution is searched for in the form of a plane wave with wave vector $\vec{k}$

$$\left.\begin{aligned} \rho_{BM}(x,t) &= \rho_0\left[1+\delta(t)e^{ikx}\right], \\ \vec{u}(x,t) &= 0+\vec{v}(x,t)=\vec{\omega}e^{ikx}, \\ \phi(x,t) &= \phi_0+f(t)e^{ikx}, \\ P &= P_0+\frac{\partial P}{\partial\rho}(\rho-\rho_0)=P_0+b^2\rho_0\delta e^{ikx}, \end{aligned}\right\} \tag{2}$$

where $\vec{b}$ is adiabatic speed of sound. Therefore from (1) and (2) we get the standard set of Jeans equations

$$\left.\begin{aligned} \dot{\delta}+ik\omega &= 0, \\ \dot{\omega}+ikf+ikb^2\delta &= 0, \\ k^2 f &= -4\pi G\rho_0\delta. \end{aligned}\right\} \tag{3}$$

Our next goal is generalization (3) for the case of the presence of dark matter in the total structure of external background*. We will describe dark matter by the potential Φ that satisfies the Poisson equation

$$\triangle\Phi = 4\pi G\bar{\rho}_{\mathrm{DM}}, \tag{4}$$

where $\bar{\rho}_{\mathrm{DM}}$ is the density of dark matter. Note that mostly the static spherically symmetric profiles of dark matter are discussed in the literature (Bullock et al. 2001). However, to achieve the goal of our article we must pass to the nonstationary profile of dark matter. These types of study are also known (see, e.g., Dolag et al. (2004), Prada et al. (2012)).

So, we will look for the nonstationary profile in the form

$$\bar{\rho}_{\mathrm{DM}} = \bar{\rho}_0\delta(t)e^{iKx}, \tag{5}$$

that corresponds to (2). Here, in accordance with the problem organization, the following condition $\bar{\rho}_0 < \rho_0$ takes place, and $\vec{K}$ is the wave vector of dark matter oscillations.

In accordance with (2) assume that $\Phi(x,t) = F(t)e^{iKx}$. Then the system of equations (3) takes the form

*Note that earlier Jeans equation generalization for the expanding Universe (for dark energy) is given in the above cited monograph, and in the case of a rotating Universe in Chechin and Ibraimova (2014).

$$\left.\begin{aligned} \dot{\delta} + ik\omega = 0, \\ \omega + ikb^2\delta + ikf - iKFe^{i(K-k)x} = 0, \\ k^2 f = -4\pi G\rho_0\delta, \\ K^2 F = -4\pi G\bar{\rho}_0\delta. \end{aligned}\right\} \tag{6}$$

It can be easily transformed to a differential equation of the second order

$$\ddot{\delta} + \left[k^2b^2 - 4\pi G\rho_0 - \frac{k}{K}4\pi G\bar{\rho}_0 e^{(K-k)x}\right]\delta = 0. \tag{7}$$

The expression in square brackets is a value constant in time; it has the meaning of the square of total frequency. By introducing the designation

$$\Omega^2 = k^2b^2 - 4\pi G\rho_0 + 4\pi G\frac{k}{K}\bar{\rho}_0 e^{i(K-k)x}, \tag{8}$$

expression (7) reduces to the standard form

$$\ddot{\delta} + \Omega^2\delta = 0, \tag{9}$$

with the solution

$$\delta(t) = \delta_0 e^{i\Omega t}. \tag{10}$$

Equation (9) describes the harmonic oscillations of baryonic matter with frequency

$$\Omega = \left(k^2b^2 - 4\pi G\rho_0 + 4\pi G\frac{k}{K}\bar{\rho}_0 e^{(K-k)x}\right)^{\frac{1}{2}}. \tag{11}$$

This frequency, as is seen, contains a constant part

$$\Omega_0^2 = k^2b^2 - 4\pi G\rho_0 \tag{12}$$

and a periodic component

$$\Omega_I^2 = 4\pi G\frac{k}{K}\bar{\rho}_0 \exp i(K-k)x = \tilde{\Omega}^2 \cdot \exp[i(K-k)x], \tag{13}$$

where $\tilde{\Omega}^2 = 4\pi G\frac{k}{K}\bar{\rho}_0$. Further we will examine the relations between these two components.

2.1 Case I

Assuming $\Omega_0 \gg \Omega_I$ we get the standard expression for baryonic-matter density perturbation

$$\delta(t) = \delta_0 \exp i\Omega_0 t. \tag{14}$$

Recall that this solution describes the harmonic oscillation with amplitude δ_0 and frequency Ω_0. Therefore, from the physical viewpoint the solution (14) means the emergence of stable perturbations at distances proportional to the integer number of Jeans wavelengths. Its value is easily obtained from (12) assuming $\Omega_0 = 0$:

$$\lambda_{\text{J}} = \frac{2\pi}{k_J} = \frac{\sqrt{\pi} \cdot b}{\sqrt{G\rho_{\text{BM}}}}. \tag{15}$$

2.2 Case II

Assuming $\Omega_I \gg \Omega_0$ we get the expression for baryonic-matter density perturbation similarly to (14)

$$\delta(t) = \delta_0 \exp i\Omega_{\text{I}} t, \tag{16}$$

where the frequency of oscillation, according to (13), depends not only on the ratio between wave vectors $\vec{K}$ and $\vec{k}$, but is also the periodic function of distance x. From (13) we have

$$\Omega_{\text{I}} = \tilde{\Omega} \exp i\frac{1}{2}(K-k)x. \tag{17}$$

After the real part of (17) choosing we get the following oscillating frequency

$$\Omega_{\text{I}} = \tilde{\Omega} \cos \frac{1}{2}(K-k)x. \tag{18}$$

If $\Omega_{\text{I}} \to 0$ it is easy to see that this condition leads to the following relationship

$$\frac{1}{2}(K-k)x = n\pi, \tag{19}$$

where n is an arbitrary integer. Hence, we find the new critical wavelength that describes by difference between wave vectors of baryonic and dark matter

$$\lambda_{\text{I}} = \frac{2\pi n}{(K-k)}. \tag{20}$$

It is clear that if $\vec{K} \to \vec{k}$ then critical wavelength λ_{I} tends to infinity. Physically this means the absence of dark matter density perturbations influence on baryonic substrate's evolution.

If $K \gg k$ then the minimal critical wavelength is $\lambda_{\text{I}} = 2\pi/K$, and it is completely determined by the wave vector $\vec{K}$.

2.3 Case III

Consider the most important case when (11) has the form

$$4\pi G\rho_0 = 4\pi G\frac{k}{K}\bar{\rho}_0 e^{i(K-k)x}. \tag{21}$$

It means that the total potential energy of baryonic matter dominates over its kinetic energy. Hence, taking the real part of (21), we get

$$\rho_0 = \frac{k}{K}\bar{\rho}_0 \cos[(K-k)x]. \tag{22}$$

As the periodic function is bounded above by the value of the unit, then (22) allows us to find the following relation between wave vectors $\vec{K}$ and $\vec{k}$:

$$\frac{k}{K} = \frac{\bar{\rho}_0}{\rho_0}, \tag{23}$$

and the corresponding relation for the wavelengths:

$$\frac{\lambda_I}{\lambda_J} = \frac{\bar{\rho}_0}{\rho_0}. \tag{24}$$

For further calculations we use the following considerations. It is known (Zel'dovich and Novikov 1975) that due to the expansion of the Universe the baryonic matter density decreases with time according to the law $\rho(t) = 1/6\pi G t^2$. As the formation of galaxies started from the hydrogen recombination epoch, $t_{\rm rec} \cong 10^{13}$ sec (Byrd et al. 2007), the corresponding maximal density equals $\rho_{\rm rec} \cong 10^{-20}$ g $\cdot$ cm^{-3}, hence.

During the further expansion it continues to decrease and at a transitive time (equality of baryonic matter and vacuum densities) $t_{\rm rec} \cong 10^{17}$ sec its minimal density is $\rho_{0\min} \cong 10^{-28}$ g $\cdot$ cm^{-3}. So, the rough averaging density of baryonic matter for the period of active formation of galaxies will be $\rho_0 \cong 10^{-24}$ g $\cdot$ cm^{-3}, and we will use this value for subsequent numerical estimations.

Concerning the density of dark matter, its value was estimated in Byrd et al. (2007): $\bar{\rho}_0 \sim 10^{-29}$ g $\cdot$ cm^{-3}. Then we have the following numerical relation between two critical wavelengths: $\lambda_{\rm I}/\lambda_{\rm J} \propto 10^5$. It means that fragments of stable baryonic matter can appear on the (most) larger distances from each other than before.

2.4 Case IV

Finally, let the potential energy of baryonic matter be negligible as compared with its kinetic energy and the potential energy of dark matter. Then

$$k^2 b^2 + 4\pi G\bar{\rho}_0 \frac{k}{K} e^{i(K-k)x} = 0. \tag{25}$$

Hence, the maximal value of the wave vector is

$$K \leq \pm 4\pi G \frac{\bar{\rho}_0}{k b^2} \tag{26}$$

and the corresponding wavelength is

$$\Lambda \geq \pm \frac{b^2}{4\pi G\bar{\rho}_0 \lambda}. \tag{27}$$

Therefore, from (27) we have the relationship between the corresponding wavelengths:

$$\frac{\Lambda}{\lambda} \geq \pm \frac{b^2}{2G\bar{\rho}_0 \lambda^2}, \tag{28}$$

which depends essentially on the wavelength of baryonic matter oscillations.

For preliminary estimations, we assume that at the matter dominated epoch the speed of sound is $b \cong 6,4 \cdot 10^5$ cm/sec, and the wavelength is approximately equal to the corresponding Jeans length, i.e., $\lambda \approx \lambda_{\text{J}} \cong 0.75 \cdot 10^{20}$ cm (Byrd et al. 2007). Then we have the ratio $\Lambda/\lambda_{\text{J}} \cong 5.0 \cdot 10^{-2}$. Hence, we conclude that in this case the relationship between wavelengths becomes in favor of baryonic substrate wavelength, and its fluctuations can suppress the oscillations of dark matter. But if $\lambda < \lambda_{\text{J}}$ or $\lambda \ll \lambda_{\text{J}}$, the situation is the opposite – dark matter suppresses oscillations of baryonic matter.

3 Conclusion

The search for the evolution of baryonic-matter density perturbations against the background of oscillating dark matter has shown that:

- the contribution of dark matter to the oscillating frequency of baryonic substrate radically depends on the difference between the wave vectors of oscillated dark and baryonic matters;
- the coherent oscillation of baryonic matter and dark matter leads to the emergence of a new critical wavelength: $\lambda_I = 2\pi n/(K - k)$;
- damping of baryonic substrate oscillations by oscillating dark matter leads to a stable formation of baryonic matter at larger distances from each other;
- the opposite case is possible when baryonic substrate oscillations suppress dark matter oscillations and therefore do not play a significant role in galactic-type structure formation.

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