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Cosmological redshift, recession velocities and acceleration measures in FRW cosmologies

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In this methodological note we discuss several topics related to interpretation of some basic cosmological principles. We demonstrate that one of the key points is the usage of synchronous reference frames. The Friedmann–Robertson–Walker one is the most well known example of them. We describe how different quantities behave in this frame. Special attention is paid to potentially observable parameters. We discuss different variants for choosing measures of velocity and acceleration representing the Hubble flow, and present illustrative calculations of apparent acceleration in flat ΛCDM model for various epochs. We generalize description of the "tethered" galaxies problem for different velocity measures and equations of state, and illustrate time behavior of velocities and redshifts in the ΛCDM model.

Keywords: FRW cosmology, Hubble flow, tethered galaxies, Universe acceleration

1 Introduction

There are many controversial issues related to interpretations of basic principles in cosmology. In addition, there are several widespread misconceptions (see discussions about many of them in Davis (2005) and references therein) and some examples of inappropriate choice of parameters to illustrate different aspects of cosmological phenomena. Most of these complications are due to the fact that cosmological observables are frame-dependent, which often is not fully acknowledged. The choice of a frame dictates which distances, velocities etc. fit better and do not mislead the discussion. Even disputes about the interpretation of the cosmological redshift (if it should be treated as some "third type" of redshift related to the expansion of space, or it can be reduced to the well-known types of redshift) can be, at least partly, reduced to the discussion about frames.

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Many papers have been dedicated to discussions of these misconceptions and difficulties. Without pretending to give a complete list we want to note several of them. The problems of superluminal velocities and many related issues have been discussed by Davis & Lineweaver (2004) (see also references therein for earlier results, among which we especially recommend Murdoch 1977). Interpretation of the cosmological redshift was the main topic of numerous studies. In particular, we want to mention Kaiser (2014) (references to earlier papers can be found there).

This paper is a continuation of the discussion started in Toporensky & Popov (2014). In the following sections we discuss the role of the synchronous nature of the Friedmann–Robertson–Walker (FRW) metric, discuss cosmological redshifts, describe four different measures for the Hubble flow, and discuss their advantages and disadvantages, then we discuss how time derivatives of the redshift and of the velocity at the moment of emission evolve in the ΛCDM cosmology, and finally we illustrate the behavior of so-called "tethered" (see Davis et al. 2003) galaxies in different cosmological models. All the main parameters used in our discussion are summarized in the Table at the end of the paper.

2 Light propagation and time intervals in the FRW metric

The metric in the FRW frame has the well-known form:

$$ds^2 = dt^2 - a^2(t)d\chi^2 - \chi^2 d\Omega^2,$$

where t is the cosmic time, a is the scale factor, and χ is the radial comoving coordinate. Ω includes all angular dependences which are not discussed below as we focus on radial motions.

As the coordinate system is synchronous, all points forming it share the same proper time. There is also an agreement about values of spatial proper distances which are equal to $a(t)(\chi_1 - \chi_2)$ for all observers despite their relative motion.

Note that points are moving with respect to each other — distances are changing with the rate $v = \dot{a}(\chi_1 - \chi_2)$. This represents the existence of the Hubble flow: points recede from each other due an increase in the scale factor a, while the comoving coordinate remains unchanged. As we will see soon, these velocities of the Hubble flow (recession velocities) have particular properties in the FRW frame, and should not be confused with velocities arising from changes of the comoving coordinate (peculiar velocities).

Due to synchronous nature of the FRW frame, motion in the Hubble flow does not lead to any change in the proper time and distance intervals. This situation is possible only in the presence of a gravitational field.* This is already enough to understand why this velocity, v, can exceed the speed of light – there is no Lorentz time dilation between points in the Hubble flow, so we can not expect any limiting velocity. This is not in any contradiction with Special Relativity (SR) because v is not a directly observable variable.

*Without gravity in a synchronous metric – i.e. in the Minkowski space – distances are not changing. Conversely, in the presense of a gravitational field a synchronous metric cannot be stationary. The FRW frame is the best example of such a situation.

The existence of superluminal motion in the Hubble flow is, as it is now well understood, a frame-dependent phenomenon. We can refer to Chodorowski (2007) where a detailed investigation of this problem is given. As the author says, "superluminarity of distant galaxies vanishes in suitable coordinate system". However, several paragraphs above this phrase Chodorowski recognizes that "the RW coordinate is more convenient for calculation" because it keeps the homogeneity of the Universe while the former coordinate (in which the superluminarity is absent) does not. So, the practical question is as follows: is this superluminarity so unpleasant that it is reasonable to avoid it at the price of more complicated calculations and losing spatial homogeneity? The answer from a practical cosmology point of view is definitely "no" – the FRW coordinates are used everywhere they can be used. The goal of this paper is to describe what we face in the FRW coordinates, and how to work with it correctly.

The fact that the time t is the proper time for each particle in the Hubble flow leads to even worse "blasphemy" than the superluminarity. Another strong deviation from a "normal" behavior of a system in SR appears. Suppose that the comoving coordinate of a galaxy is changing, i.e. the galaxy has a non-zero peculiar velocity $v_{\rm p}$. This velocity should be added to the Hubble flow velocity v using the Galilean rule: $v_{\rm tot} = v + v_{\rm p}$, independently of how large these velocities are. Indeed, we have:

$$d(a\chi)/dt = \chi da/dt + a d\chi/dt,$$

where the first term in the right hand side (r.h.s. hereafter) is the velocity of the Hubble flow, and the second term is the peculiar velocity. Obviously, expressions like $c/2 + c/2 = c$ may appear shocking for those who are familiar with SR (only!) from a kindergarten. However, this is a correct way of velocity summation *if one of velocities is caused by the Hubble flow* (two peculiar velocities evidently obey the relativistic law). This is true also for the speed of light which is equal to c only locally. Indeed, the equation of motion for light rays $ds^2 = 0$ leads to $d\chi/dt = \pm c/a$. Going from comoving to physical distance we get $d(a\chi)/dt = \chi da/dt + a d\chi/dt = v \pm c$, where two signs indicate two possible directions of light: towards an observer, and away from him. Again, this is not a danger for SR because the speed of light is constant only in inertial frame, which is not the case of FRW frame, being "a hybrid of distances measured in different inertial frames" (Chodorowski 2007b).

Therefore, a reader can treat the Hubble flow as a "gravitational wind" which drags objects with locally measured peculiar velocities. It is necessary to keep in mind, however, that this "dragging" is only kinematical – there are no additional forces acting as a result of the Hubble flow (see Davis et al. (2003); Davis & Lineweaver (2004) for a detailed description). This picture, being rather weird from the viewpoint of SR, may be rather comprehensive and helps to understand some unusual phenomena specific to cosmology. In the next sections we show how some known classical cosmological results can be incorporated in this picture.

3 Redshifts and distances

In this section we discuss cosmological redshifts, distances, and the necessity of the "expansion of space" as an additional concept to interpret cosmological data.

3.1 Interpretation of the cosmological redshift

Before considering redshifts in cosmology we briefly remind the reader what happens in SR. A Doppler shift appears due to, in fact, two reasons. The first reason is purely geometrical and exists in classical physics as well. When an emitter moves radially towards or from us, two light signals separated by a time interval $\Delta t_{\rm em}$ are also separated by the distance $v\,\Delta t_{\rm em}$. This means that an observer at rest would see them separated by a different time interval $\Delta t_{\rm obs} = \Delta t_{\rm em}(1 + v/c)$. This results in a classical (or kinematical) redshift $(1 + z_{\rm cl}) = 1 + v/c$.

The second reason is related to the Lorentz transformation linking time intervals in observer's and emitter's frames: $\Delta t_{\rm obs} = \Delta t_{\rm em}/(\sqrt{1 - v^2/c^2})$, resulting in the Lorentz part of redshift $(1 + z_{\rm L}) = 1/\sqrt{1 - v^2/c^2}$. For a tangent motion when the first effect is absent, the Lorentz shift is the only reason for a redshift: $z_{\rm t} = z_{\rm L}$. While for a radial motion the resulting redshift is a combination of these two effects: $(1 + z_{\rm r}) = (1 + z_{\rm L})(1 + z_{\rm cl})$. The latter formula usually is written in the form $z_{\rm r} = \sqrt{(1 + v/c)/(1 - v/c)}$ where classical and Lorentz parts are not separated. In observational astrophysics Lorentzian effects are important, e.g., in GRB physics: the time of variability in the observer's frame is much longer than in the frame of rapidly moving shells in the jet of the burst.

The classical redshift $z_{\rm cl}$ is responsible for another effect related to a non-zero emitter's velocity. Namely, it causes a difference between true and apparent velocities of the emitter seen by an observer. Usually this effect is illustrated in a science fiction style. Suppose, a spaceship is traveling with a subluminal velocity $v = (4/5)c$ from α Cen, located at $\sim$4 light years from the Earth. It starts at the moment $t_{\rm em}$, and reaches the Earth 5 years later according to the clocks at the terrestrial frame. However, a terrestrial observer would actually see the departure at $t = t_{\rm em} + 4$ yrs and arrival at $t = t_{\rm em} + 5$ yrs. So, the travel apparently has a duration of only one year, and the apparent velocity appears to be $4c$.

This situation is well-known in astrophysics in the case of so-called superluminal jets. The effect was predicted by Rees (1966), and few years later discovered by Gubbay et al. (1969) for a near-by quasar. A detailed explanation of jet properties (including superluminal motions in Galactic and extragalactic sources) can be found in the volume edited by Belloni (2010). The apparent velocity in this case can be calculated as:

$$v_{\rm app} = v \sin\theta/(1 + \beta\cos\theta), \tag{1}$$

where θ is the angle between the jet and the line of sight, and $\beta = v/c$, v is the real velocity in the jet.

As the true and apparent velocities are studied in the common frame – the frame of the observer, – the effect is caused only by the difference in the duration of the time intervals at the moments of emission and observation, so that only the classical part of the Doppler shift appears in the resulting formula. For a pure radial motion it takes a very simple form $v_{\rm app} = v/(1 + z_{\rm cl})$. So, the apparent velocity of an approaching emitter is larger than the true velocity, and actually can be arbitrarily large. While receding objects have a smaller apparent velocity which cannot exceed $c/2$ in special relativity.

Now having all this in mind, let us try to understand the nature of cosmological redshifts in the FRW frame. Suppose, an observer detects light from an object which recedes in the Hubble flow with the velocity $v_{\rm em}$ at the time of emission. As the Hubble flow "drags" the emitted light, so that its velocity with respect to an observer is $c - v$, the kinematical part of the Doppler effect is absent at $t_{\rm em}$: the distance between two light pulses is equal to $c\Delta t_{\rm em}$ instead of $(c + v_{\rm em})\Delta t_{\rm em}$ in SR. Also, as the time coordinate is the same for any object in the Hubble flow, no Lorentz shift is present either.

A difference in frequency appears due to the presence of a gravitational field, because metric is not stationary due to gravity. In the FRW frame two light pulses located at points with different coordinates χ would feel different Hubble flow "drag". Initially, they have radial coordinate difference, Δr, equal to $c\Delta t_{\rm em}$. This difference results in the velocity difference $\Delta v = H\Delta r$ which allows us to construct a differential equation $d\Delta r/dt = H\Delta r$. After obvious calculations using the definition of the Hubble parameter – $H = \dot{a}/a$, – we obtain that the ratio of time interval at $t_{\rm em}$ and the moment of observation, $t_{\rm obs}$, is equal to the inverse ratio of scale factors at these moments: $\Delta t_{\rm em}/\Delta t_{\rm obs} = a_{\rm obs}/a_{\rm em}$. This is the well-known result for a cosmological redshift. A common feature with the classical redshift $z_{\rm cl}$ is that both have their origin in the difference between $\Delta t_{\rm em}$ and $\Delta t_{\rm obs}$ which is of purely geometrical nature and is calculated in the same frame. An important difference is that a cosmological redshift is "formed" on the way of the light from an emitter to an observer, and so is not directly related to the velocity of the emitter (we can even construct a situation in an oscillating Universe where an object can be cosmologically blueshifted despite being receding at the time of emission, or vice versa). As an additional result, we obtain that the apparent velocity of an emitter in the Hubble flow is related to the velocity at the moment of emission as:

$$v_{\rm app} = v_{\rm em}(\Delta t_{\rm em}/\Delta t_{\rm obs}) = v_{\rm em}/(1 + z).$$

Now the question arises: how do we interpret the resulting formula for a cosmological redshift? A widespread opinion is that redshifts of objects in the Hubble flow represent some third type of redshift (i.e., it is different from kinematical and gravitational redshifts) appearing due to the stretching of space. The fact that this redshift can be defined as a ratio of scale factors at the moments of emission and observation seems to support this view. Sometimes, it is claimed that the derivation presented above (if re-interpreted accordingly) reduces a cosmological redshift to the Doppler effect integrated along the trajectory of the light (see, e.g., Zel'dovich & Novikov 1967). This might be rather puzzling because other types of redshift do not require any integration along the light traveling path. And again, a reasonable explanation invokes the concept of expanding space with a specified rate of expansion which is integrated over all points that the light passes through.

From our point of view, the concept of space expansion, formally, is not necessary to explain cosmological redshifts. First of all, it is worth noting that in an inhomogeneous Universe, in general, the value $1 + z$ does not coincide with the ratio of scale factors (see, e.g., Mustapha et al. 1998; Bassett et al. 2013). Second, in our considerations we never used the concept of expanding space. Moreover, we did not use any specific cosmological nature of elements of the picture under consideration. All we use is the fact that we work in a synchronous reference frame. Such a frame can

be used in absolutely different physical situations. For example, in describing a free fall into a black hole (in this situation this role is played by Lemaitre coordinates). Sometimes such a redshift is referred to as a tidal one. It is caused by a non-zero gravitational field, but in another way than the usual gravitational redshift caused by time dilation.

We mentioned the Lemaitre coordinates and the tidal redshift. It is well known that by using another set of coordinates (an obvious example is the Schwarzschild stationary system of coordinates) it is possible to express the tidal redshift with the standard kinematic and gravitational redshifts. Whether the same is possible for the case of the FRW space-time is still an open question. It is definitely possible for the Milne Universe with the scale factor linearly growing with time. This model was used many times to explain the frame dependence of superluminarity (see, e.g., Zel'dovich & Novikov 1967; Page 2009). In a recent paper (Melia 2012) six different cosmologies allowing such a decomposition are described. In all these models a transformation to a stationary metric exists. As synchronous coordinates in a non-zero gravitational field are necessarily non-stationary, the non-stationarity of the frame is the condition *sine qua non* for our description of the redshift. It is not surprising that the transformation to a stationary metric proposed by Melia (2012) indeed allows us to reduce the cosmological redshift to other forms of redshift.

However, these examples to date do not even cover all FRW single-fluid scenarios, let alone more complicated (though more realistic!) models like the ΛCDM model. So, we prefer to keep this "tidal" redshift as a separate one, at least for reasonable practical purposes.

We should also note that even in the situation when such coordinate sets which allow decomposition of the "tidal" redshift into other forms can be found for a particular FRW model, they include absolutely different physical values playing roles of "distances" and "velocities". As for these values in the FRW frame, despite the fact that they are formally not measurable, they still coincide (either directly or under a simple re-definition) with observable entities: the proper distance at the moment of emission coincides with the angular distance, the rate of change of the angular distance is equal to the apparent velocity at the moment of emission $v_{\rm em}/(1+z)$, the proper distance now coincides with the proper motion distance, etc. Ignoring peculiarities of the tidal redshift in a comoving frame, and using a common intuition originating from more obvious forms of redshift, we can come to "evident", but wrong conclusions. Some examples will be described below.

3.2 Radar ranging in an expanding Universe

According to the discussion above, a cosmological redshift is due to the nonstationarity of the metric used in the derivation, but not due to a mysterious "expansion of space". The "expansion of space" may be a good pedagogical concept, still, in general it is not necessary to use it. Consider, for example, another effect which is believed to support this idea. Namely, the difference in a radio signal travel time from an emitter to a reflector, and back from the reflector to the emitter in an expanding Universe. The return travel time is larger. Indeed, if we use notatation $t_{\rm e}$, $t_{\rm r}$, and $t_{\rm obs}$ for the time of emission, reflection and observation of a reflected signal, then using the equation for light propagation, we can see that the comoving distance travelled by light is:

$$\chi = \int_{t_e}^{t_r} \frac{dt}{a(t)} = \int_{t_r}^{t_{obs}} \frac{dt}{a(t)}. \quad (2)$$

As the comoving coordinate of a reflector does not change, two integrals are equal. This means that for a monotonically increasing $a(t)$ the return travel time $t_{obs} - t_r$ should be larger than the forward travel time $t_r - t_e$. This seems to support the expansion of space. Chodorowski (2007), though, argues that the moment of reflection (in contrast to the moments of emission and observation of the reflected signal) is not directly observable by the emitter (i.e. in the emitter's rest frame). So this moment is frame-dependent. Moreover, it is possible to find a frame in which this difference is completely explained by SR. This is correct, however, in the present paper we want to stay in the FRW frame. Does this mean that we are forced to accept the expansion of space paradigm to reach an agreement with the calculated time intervals? Not at all! Remembering that the velocity of light differs from c for a distant observer in the FRW frame, we immediately see that on the way from the emitter to the reflector the velocity of the Hubble flow is *added* to c, while on the way back it is *subtracted* from c, evidently resulting in a larger time interval on the way back. So, no additional concept (like stretching space) is necessary to explain the difference between these two time intervals.

3.3 Angular distance and its properties

In this subsection we discuss why *angular* distances in most of cosmological models starting from some χ decrease with increasing redshift.

We remind the reader that the angular distance D_θ for an object with a size S which have an angular diameter δ is by definition $D_\theta = S/\delta$, so it is equal to the distance in Euclidean geometry from which this object would have the same angular diameter. It is known that in the FRW Universe $D_\theta = a_{em}\chi$.

It is important that a proper distance, $D = a\chi$, being by definition an entity which can not be measured directly (such a measurement would require a chain of observers extending till the measured object, organized in such a way that they make measurements in their vicinities at the same cosmic time t, and after that all results should be summed, see Weinberg (1972)), if defined for the moment of emission, has the same form ($D_{em} = a_{em}\chi$ if we put $\chi = 0$ at the location of observer) as the angular distance. The reason is simple: as radial rays are light geodesics, the expansion of the Universe does not alter an angle between a pair of them. So, we see the object as it was at the time of emission (of course, disregarding other properties like spectrum and luminosity which are modified due to cosmic expansion).

We use the equation for the light propagation in order to get distances and velocities expressed through such a directly observable quantity as the redshift. Remembering that the redshift z obeys $(1 + z) = a_{obs}/a_{em}$ we can rewrite the comoving coordinate of the object seen now with the redshift z as (details can be found in most of standard textbooks on cosmology):

$$\chi = \frac{c}{a(t_0)H_0} \int_0^z \frac{dz}{H(z)}, \quad (3)$$

where t_0 and H_0 are the present values of cosmic time and of the Hubble parameter. For the FRW Universe filled with one type of matter (fluid) with the equation of

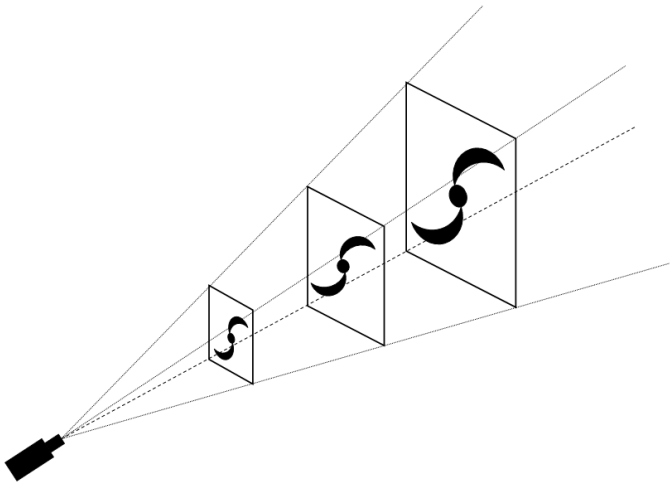


Figure 1 If in the flat universe we use a giant projector, then on all parallel screens images are non-distorted. Screens and the projector are moving relative to each other due to cosmic expansion. However, the size of an image on a given screen is determined at the moment of emission. If we fill the universe with similar projectors transmitting the same image, then on a given screen images of the same size can be produced by faraway projectors and by near-by, similar to the situation with angular sizes of galaxies at different distances in the sky.

state $p = w\epsilon$ (where ϵ is density of the fluid), the time evolution of the scale factor is $a \sim t^{2/(3+3\omega)}$, and correspondingly, $H(z) = H_0(1+z)^{3(\omega+1)/2}$. This gives the expressions:

$$\chi = \frac{c}{a(t_0)H_0}\frac{1}{1-\alpha}[(1+z)^{1-\alpha}-1], \tag{4}$$

where we denote $\alpha = 3(w+1)/2$.

Using this, we get the proper distance at $t_{\rm em}$:

$$D = D_\theta = \frac{c}{H_0(1-\alpha)(1+z)}[(1+z)^{1-\alpha}-1]. \tag{5}$$

We also can write down the expression for velocity:

$$v = v_{\rm em} = \frac{c}{1-\alpha}[1-(1+z)^{\alpha-1}]. \tag{6}$$

This is the well-known fact that $D_\theta(z)$ is not monotonic if $w > -1$, and has a maximum after which it decreases, so that angular sizes increase with z. How can it happen? As radial rays are light geodesics in the FRW coordinates any explanation based on the gravitational focusing of light rays (see, e.g., Zel'dovich & Novikov 1967) fails. Also, as our universe is flat, we cannot use a popular explanation (see, e.g., Mukhanov 2005) via light propagation on a two-dimensional sphere, where due to curvature an observer on a pole determines for objects behind the equator that D_θ is decreasing while the physical (proper) distance is increasing. The failure of this

"explanation" can be seen also from the fact that it should be applicable to the de Sitter model as well, however, $D_\theta(z)$ increases monotonically for $\alpha = 0$.

The true reason of non-monotonic behavior of D_θ is that two objects with the same angular distances (which coincide with the proper distance at $t_{\rm em}$) and different redshifts were indeed at the same distance from an observer when they emitted light visible now. In a flat universe angles between light rays are invariant, i.e., a figure formed by simultaneously emitted light pulses is transformed in a homothetic way during light propagation in the expanding flat universe (see Fig. 1). This means that the angular distance is determined at the moment of emission, and does not change by the cosmic expansion.

In Fig. 2 (reprinted from Toporensky & Popov 2014) we depicted schematically what happens. When the more redshifted object was emitting, it receded superluminally, so the light emitted towards us has been actually receding from us. After some time when its v diminishes due to decreasing H, and became smaller than c, the light started to approach us. At some moment the proper distance of the light again becomes equal to the distance at which it had been emitted.

This illustration explains the pecularities of $D_\theta(z)$. It also tells us why the situation in the de Sitter world (it corresponds to $w = -1$) is different. From eq. (6)

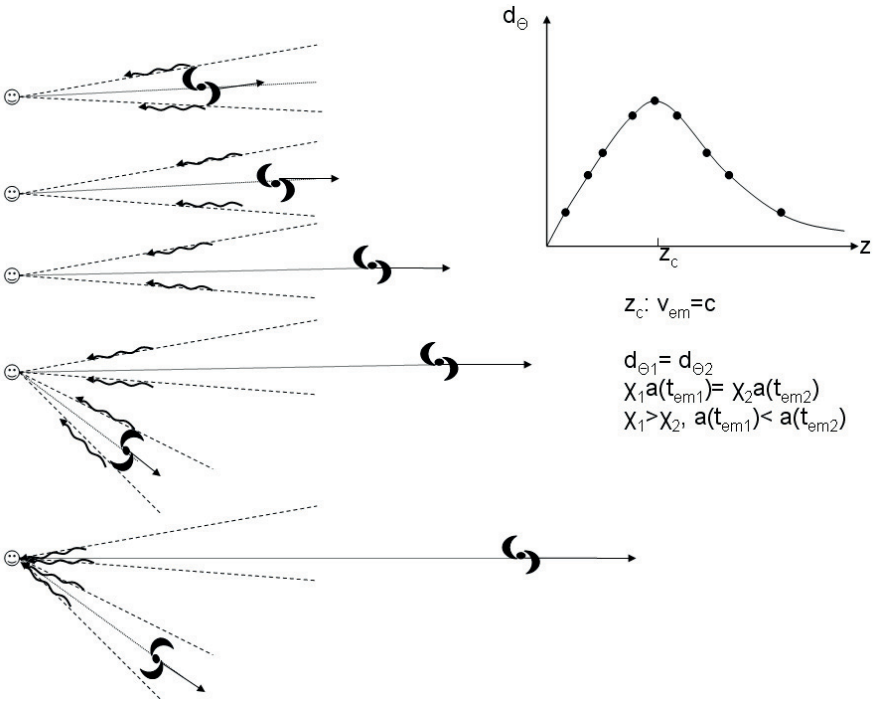


Figure 2 A cartoon illustrating the effect of decreasing of D_θ at large redshifts, and the fact that $D_{\rm em}$ is equal to the angular distance. Two galaxies have the same D_θ despite the fact that they have different z. Reprinted from "Khablovski potok v kartine nablyudatelya", *Usp. Fiz. Nauk* 184:767–774 (2014) ["The Hubble flow: an observer's perspective", *Phys. Uspekhi* 57:708–713 (2014)] by A.V. Toporensky and S.B. Popov, ©2014 Uspekhi Fizicheskikh Nauk. Reprinted with permission.

we can see that v is always subluminal there, so this effect cannot appear. Also, it is evident that $D_\theta(z)$ has its maximum (when it exists) exactly when $v = c$ – the fact that can be formally derived from eqs. (5–6).

Finally, we would like to mention a curious fact that for the radiation-dominated Universe ($w = 1/3$, and so $\alpha = 2$) eq. (6) tells us that the relation between recession velocities and cosmological redshifts has the "naive" form $v = cz$. We will discuss other peculiarities of the $w = 1/3$ (radiation-dominated) Universe below, in Sec. 4.

4 Velocities

This section is devoted to a discussion of several possible definitions of the Hubble flow velocity. We also demonstrate how some of these velocities evolve with cosmic time.

4.1 Four types of velocities

It is known that the Hubble flow can be characterized by several physical parameters having the meaning of "velocity" of some kind. In the present section we compare four possible definitions.

First of all, there are different measures of "distance" used in different situations. In the preceding sections we considered the proper distance at the time of emission – $D_{\rm em}$. Similarly, we can consider the proper distance at the present time – $D_{\rm now}$:

$$D_{\rm now} = a_{\rm now}\chi = a_{\rm em}(1+z)\chi = (1+z)D_{\rm em}. \tag{7}$$

This value is not directly measurable *per se*, though it is easy to show that in the FRW Universe this value formally coincides with the proper motion distance and, thus, in principle can be measured.

Sometimes the third distance is used (especially in popular literature where it is usually expressed in light years). It is the light travel distance $D_{\rm light} = ct$, where t is the time during which the signal was propagating. It is determined along the light cone (see Fig. 3).

Obviously, in an expanding Universe, $D_{\rm em} < D_{\rm light} < D_{\rm now}$. Taking time derivatives we obtain three possible definitions of velocity with different properties. The velocity "now" is useful to form our mental image of the Universe seen simultaneously as a whole (so-called "God's view"): $v_{\rm now} = \dot{D}_{\rm now} = \dot{a}_{\rm now}\chi$. Oppositely, in the picture of the Universe seen by an observer, the velocity at the time of emission $v_{\rm em} = \dot{D}_{\rm em} = \dot{a}_{\rm em}\chi$ is more reliable.

It is necessary to note, however, that the apparent velocity measured by an observer differs from $v_{\rm em}$ due to the difference in the march of time: $v_{\rm app} = v_{\rm em}/(1+z)$. It is interesting that this velocity in the FRW Universe filled with matter with $w \le 1/3$ is always subluminal (see a recent duscussion and references in Toporensky & Popov 2014). This, however, is a purely kinematical effect and is not related to a rather special role of an ultra-relativistic matter (having $w = 1/3$) in modern physics. Indeed, the fact that this velocity is not restricted near the Big Bang singularity for $w > 1/3$ is a simple consequence of the fact that the value of $\dot{a}/(1+z) \sim \dot{a}a$ for the power law

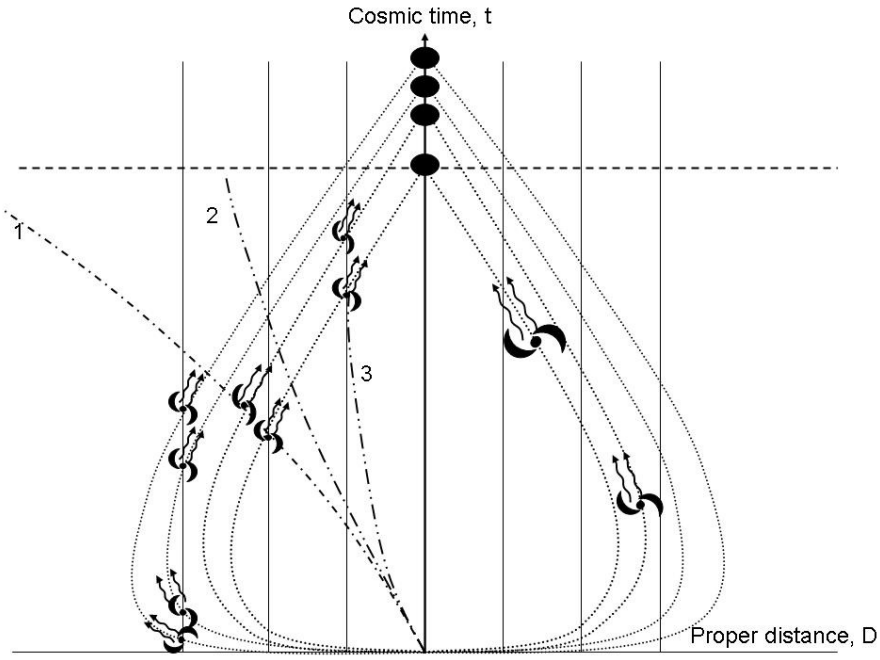


Figure 3 Dotted lines of the onion shape are light trajectories. We observe objects on the light cone. Note that $D_{\rm light}$ is determined along the light cone. The plot illustrates observations in several consequent moments of time. Dashed-dotted curve 1 is the world line of a galaxy in the Hubble flow. Vertical lines of constant proper distance correspond to world lines of "tethered" galaxies. Dashed-dot-dotted lines 2 and 3 are world lines of galaxies with constant $D_{\rm light}$.

evolution $a \sim t^{1/\alpha}$ is not restricted near $t = 0$ for $\alpha > 2$. The law $a \sim t^{1/2}$ is related to the equation of state $p = \epsilon/3$ only for the four-dimensional General Relativity (GR), and corresponds to other equations of state in modifications of GR, as well as in GR with a larger number of spatial dimensions.

Finally, the time derivative of the light travel distance gives us the velocity $v_{\rm l} = dD_{\rm light}/dt$. Simple geometric considerations show that the velocity $v_{\rm l}$ is fully determined by the redshift independently of the particular cosmological model. Indeed, the difference in light travel distance is obviously $dD_{\rm light} = c(dt_{\rm obs} - dt_{\rm em})$. This equation, after we substitute $1 + z = dt_{\rm obs}/dt_{\rm em}$, immediately gives:

$$v_{\rm l} = dD_{\rm light}/dt_{\rm obs} = cz/(1 + z).$$

In some papers and textbooks this value is referred to as an "effective velocity of the Hubble flow". Indeed, we can formally define velocity using a redshift by a non-relativistic formula $v = cz$, and apply the correction factor $(1 + z)^{-1}$ to obtain the apparent velocity measured by an observer. Then we derive the above mentioned equation. On the one hand, the definition of such an effective velocity uses formulae beyond their range of application. On the other hand, the velocity $v_{\rm l}$ is meaningful and represents the time derivative of an appropriate distance. It is also evident that

this velocity is always subluminal. So, why don't we consider $v_{\rm l}$ as a "natural" characteristic of the Hubble flow? Formally this is possible, however, there are significant drawbacks.

Remember that the cosmological redshift is not determined at the moment of emission, but is a cumulative effect, and, hence, the value of $v_{\rm l}$ is determined by the whole evolution history of the Universe since the time of emission of the signal. To show how uncomfortable this property is with respect to our intuitive notion of "velocity", let us consider a loitering Universe (see Sahni et al. (1992) and references therein). Such models received some attention and popularity some time ago, though they require a rather exotic matter content. Independently of the exact reason to introduce loitering models, they are useful for our thought experiment. Hence, consider a Universe with a scale factor which does not grow (at least significantly) at some transitional epoch after which it continues to grow rather rapidly (here we do not need to specify this rate in more detail). What is the velocity of an object which emitted light in the loitering epoch and is observed in the epoch of a rapid expansion? The velocity $v_{\rm em}$ is evidently very small (since it is proportional to $\dot{a}$ which is small during loitering). The velocity $v_{\rm l}$ is, on the contrary, in general not small, because the loitering epoch is characterized by objects with almost equal but not small, in general, redshifts. This discrepancy is even more pronounced if we allow the Universe to contract a little during the loitering stage (though it is even harder to achieve). In such situation $v_{\rm em}$ is negative (since the scale factor decreases at the moment of emission), though $v_{\rm l}$ is positive (since this contraction is compensated in the following epoch).

More formally, suppose we compare two different Universes so that each has two phases in its history. The first phase in one Universe is equivalent to the first phase in the other. But the second phase of each Universe are drastically different. An obvious

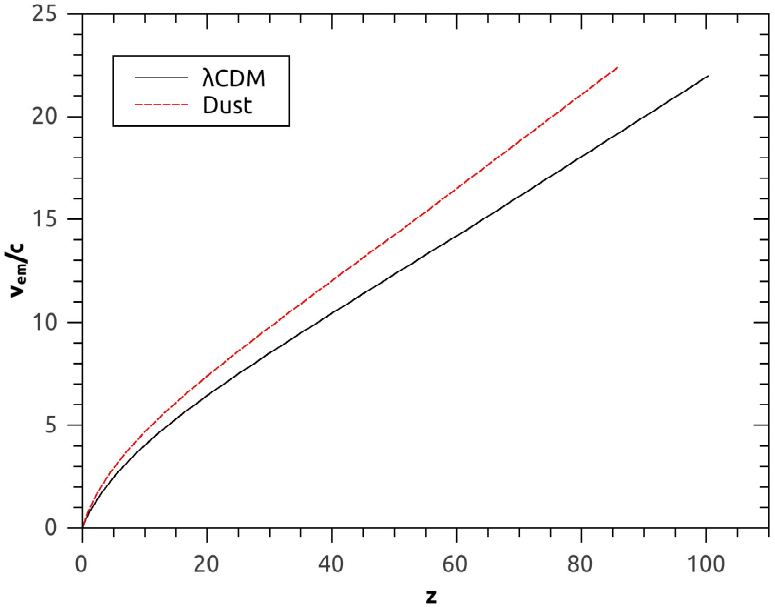


Figure 4 $v_{\rm em}$ vs. z in the dust and ΛCDM models.

example are dust and ΛCDM Universes, which share common early dust dominated stages, but then they follow different evolutionary paths. Suppose that two identical objects (one in each Universe, both have the same age since the Big Bang) in the dust phases are observed by two observers one of which lives in a Λ-dominated phase of one Universe, and the second observer lives in the dust Universe. Would they agree about the recession velocity of the observed objects (we ignore practical impossibility for astronomers from different Universes to communicate and compare their results)? It is clear that they would determine the same $v_{\rm em}$ which is "recorded" at the time of emission, and disagree about $v_{\rm l}$ which "encodes" all the cosmic evolution from the time of emission to the time of observation.

That is why we consider the velocity $v_{\rm em}$ (and its observable "twin" $v_{\rm app}$) as the most suitable parameter to describe the Hubble flow at the moment of emission.

There is, however, one important detail. If distant objects are formally marked with their comoving coorinate χ, the observers in two universes indeed determine the same $v_{\rm em}$ for sources emitting in the dust stage. However, real galaxies observed in a sky are not marked with χ (which should be calculated using a correct model of the Universe), but instead marked with directly observable redshifts. As redshifts depend on the whole evolution, two "identical" objects at the dust phase observed much later would have different redshifts. This means that the values of $v_{\rm em}$ as a function of z are not the same in the common epoch of these two universes (see Fig. 4). The same is true if objects are marked (by some clear evolution effects) with the time of emission: in a given time of observation $t_{\rm obs}$ the corresponding comoving coordinate of an object emitting at $t_{\rm em}$ depends on the expansion history. So, only $v_{\rm em}$ as a function of χ has the same values in the Universes with common evolution if observed later when the evolution histories are different. Nevertheless, as other functions ($v_{\rm em}(z)$ or $v_{\rm em}(t_{\rm em})$) are obtained from $v_{\rm em}(\chi)$ by an argument re-definition, a receding object would always have a positive $v_{\rm em}$, and vice versa. As we have seen above, this is not true for $v_{\rm l}$.

Finally, it is also worth noting that velocities defined directly through the time derivative of scale factor obey the Hubble law since we have:

$$v = \dot{a}\chi = \frac{\dot{a}}{a} a\chi = HD.$$

Here we can take scale factors (and, correspondingly, the Hubble parameter) either at the time of emission, or at the time of observation (i.e., "now"). On the other hand, Hubble law is not valid for the apparent velocity and $v_{\rm l}$.

4.2 "Tethered" objects and the Hubble flow

If we allow non-zero peculiar velocities for distant galaxies, the striking difference between the sign of z and the rate of proper distance changing can be found in the Universe without a non-trivial expansion history. Namely, as it has been pointed out by Davis et al. (2003) (see also Clavering (2006)), it is possible to have blueshifted receding and redshifted approaching objects. The reason is that the resulting redshift of an object with a peculiar velocity $v_{\rm p}$ and recession velocity v is given by $(1+z) = (1+z_{\rm r})(1+z_{\rm p})$ where $z_{\rm r}$ is redshift of the Hubble flow at the point where the observed galaxy is located (and is related to the recession velocity at the emission by eq. (6)),

and $z_{\rm p}$ is the Doppler shift due to the peculiar velocity which is calculated using the standard relativistic formula

$$v_{\rm p} = c\left(\frac{(1+z_{\rm p})^2-1}{(1+z_{\rm p})^2+1}\right). \tag{8}$$

The difference in the presentations of eqs. (6) and (8) results, in particular, in a curious fact that so-called "tethered" galaxy with $v_{\rm p} = -v$ (so that its proper distance does not change in time) usually has a non-zero z. It can be very easily understood in the case of peculiar velocities close to the speed of light. Since in most cosmological models the redshift for a recession velocity equal to c is finite, while the blueshift for a peculiar velocity equal to c obviously diverges, a "tethered" galaxy (with a constant proper distance) in the region of near-luminal recession velocities will be blueshifted. For a one-component perfect fluid FRW model it is simple to show using eq. (6) that three different cases are possible. For the de Sitter Universe any "tethered" galaxy is redshifted (remember that in contrast to other cases, in the de Sitter model $z \to \infty$, while $v_{\rm em} \to c$). For other accelerated Universes nearby "tethered" objects are redshifted, while distant ones are blueshifted. Finally, in decelerating models all "tethered" objects are blueshifted (see Fig. 5).

The same question can be considered for other definitions of velocity. What happens if we take the velocity $v_{\rm l}$? The situation appears to be somewhat different from the one described above. If we calculate the resulting redshift of an object, which has light travel time unchanged due to its non-zero peculiar velocity, we obtain $(1+z) = (1+z_{\rm cosm})(1+z_{\rm cl})(1+z_{\rm L})$. Here we denote by $z_{\rm cosm}$ the cosmological redshift of an object in the Hubble flow with zero $v_{\rm p}$. One can calculate that $v_{\rm p} = cz/(1+z)$.

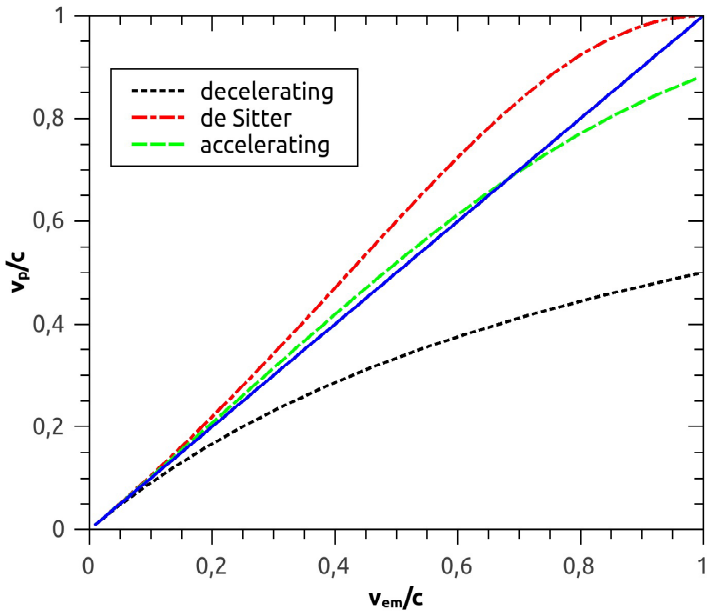


Figure 5 Relation between $v_{\rm p}$ and $v_{\rm em}$ for "tethered" galaxies in different cosmologies. The straight solid line indicating $v_{\rm em} = v_{\rm p}$ is shown for convenience.

The first term in the r.h.s. (the cosmological redshift) and the second term (the classical part of the redshift caused by peculiar velocity) represent time delay in the observer's frame, while the third term is caused by the Lorentzian time dilation in the emitter's frame. This means that the first and the second terms for the object under consideration cancel out (leaving the observed light travel time unchanged), and the object is always redshifted due to the Lorentzian part of the peculiar velocity redshift.

5 Accelerations

5.1 Different acceleration measures

In this Section we discuss how an observer can see the Universe acceleration. As in the case of velocities, there are several possible measures. In the present paper we consider only measures connected with time derivatives of proper distance. Accelerations are denoted by letter $\mathfrak{A}$.

Similar to the case of velocities, in the "God's view" approach we can distinguish between different measures – an acceleration "now" and an acceleration at emission. Both are expressed by the same simple formula $\mathfrak{A}_1 = \ddot{a}\chi$ with the present-day scale factor for the former case (acceleration "now"), and the scale factor during emission for the latter.

For the important case of a Universe filled by a single perfect fluid the acceleration "now" is equal to $cH_0[(1+z)^{1-\alpha} - 1]$ while the acceleration at emission is $cH_0(1+z)^{\alpha}[1-(1+z)^{\alpha-1}]$. Evidently, $\mathfrak{A}_1 < 0$ for $\alpha > 1$ (decelerating Universe) and $\mathfrak{A}_1 > 0$ for $\alpha < 1$ (accelerating Universe). The acceleration at emission clearly diverges at a horizon because $\ddot{a}$ diverges at the Big Bang. An observer inside the Universe evidently has no access to these two types of acceleration.

What combination of cosmological variables is more reliable to describe what can an observer see "from inside"? If we consider a rather clever observer who can calculate as well as to observe, we can imagine that such an observer uses observational data in combination with an adequate model of expansion history of the Universe in order to calculate $v_{\rm em} = \dot{a}(t_{\rm em})\chi$. Repeating this procedure after a short time interval the clever observer can get the difference in velocities at emission with time, and calculate an acceleration $\mathfrak{A}_2 = dv_{\rm em}/\Delta t_{\rm obs}$, where $\Delta t_{\rm obs}$ is the time interval measured by the observer. As usual, the ratio of time intervals during emission and observation is equal to $1+z$, and we obtain $\mathfrak{A}_2 = \ddot{a}(t_{\rm em})\chi/(1+z)$. For a one-component Universe $\mathfrak{A}_2 = cH_0(1+z)^{\alpha-1}[1-(1+z)^{\alpha-1}]$. It is interesting that this acceleration tends to zero at the event horizon, while diverges at the particle horizon.

It is possible also to consider $\mathfrak{A}_2$ from a different approach. Knowing the functional dependence of $v_{\rm em}(z)$ and time dependence of z (Balbi & Quercellini 2007):

$$\dot{z} = H_0[1 + z - H(z)/H_0] \tag{9}$$

we easily get the observable rate of change of the velocity at emission:

$$dv_{\rm em}/dt = \frac{dv}{dz}\,\frac{dz}{dt} = \mathfrak{A}_2. \tag{10}$$

The second equality can be checked by straightforward calculation.

However, for a "not so clever observer" who can only measure what is directly seen, $\mathfrak{A}_2$ is also unobservable. A rate of Universe expansion directly seen "from inside" is $v_{\rm app} = \dot{a}(t_{\rm em})\chi/(1+z)$, and its time derivative $\mathfrak{A}_3 = dv_{\rm app}/\Delta t_{\rm obs}$ is not equal to $\mathfrak{A}_2$. Moreover, the sign of $\mathfrak{A}_3$ can be different from the sign of $\mathfrak{A}_1$ and $\mathfrak{A}_2$ (obviously, as $\mathfrak{A}_1$ and $\mathfrak{A}_2$ differ by the always positive multiplier $1+z$, they have the same sign). For a one-component Universe with a perfect fluid this can be easily seen from eq. (6), which after dividing by $1+z$ gives us a functional dependence of $v_{\rm app}(z)$. Again, we can write:

$$\frac{dv_{\rm app}}{dt} = \frac{dv_{\rm app}}{dz}\frac{dz}{dt}. \tag{11}$$

As $\dot{z} > 0$ for accelerating (in the usual sense) models, and remembering that $v_{\rm app}$ grows for small z and decreases starting from some $z_{\rm cr}$, we get that $\mathfrak{A}_3 > 0$ for $z < z_{\rm cr}$ and $\mathfrak{A}_3 < 0$ for $z > z_{\rm cr}$ in accelerating (from the "God's view") Universe. Similarly, we get that $\mathfrak{A}_3 < 0$ for $z < z_{\rm cr}$, and $\mathfrak{A}_3 > 0$ for $z > z_{\rm cr}$ for decelerating models with $\alpha < 2$. As for $\alpha > 2$ $v_{\rm app}(z)$ is always increasing, $\mathfrak{A}_3 < 0$ for all redshifts in this case.

It can be easily seen also that $\mathfrak{A}_3$ always vanishes at the event horizon. As for its behavior at the particle horizon, it depends on the equation of state. For $1 < \alpha < 3/2$ it tends to zero, for $\alpha > 3/2$ it diverges (in the boundary case of $\alpha = 3/2$ it tends to the constant positive value equal to cH_0 while z tends to infinity). We see that $\mathfrak{A}_3$ has a more complicated behavior in comparison with $\mathfrak{A}_1$ and $\mathfrak{A}_2$.

Using the definition $1+z = a(t_{\rm obs})/a(t_{\rm em})$ we can write down a general expression for $\mathfrak{A}_3$ through scale factors (and their time derivatives) at the present time and at emission. Indeed, we can write $v_{\rm app} = \chi a(t_{\rm em})\dot{a}(t_{\rm em})/a(t_{\rm obs})$. Taking time derivatives and remembering that $dt_{\rm obs} = (1+z)dt_{\rm em}$ we obtain:

$$\mathfrak{A}_3 = \frac{\chi}{a(t_{\rm obs})^2}\left[\ddot{a}(t_{\rm em})a^2 + \dot{a}(t_{\rm em})^2 a - \dot{a}(t_{\rm em})a(t_{\rm em})\dot{a}(t_0)\right].$$

Summarizing, we show that apart from the true (i.e. defined with respect to cosmic time intervals) acceleration "now" (which clearly belongs to the "God's view" picture), it is reasonable to introduce three different measures of cosmic acceleration: the true rate of change of recession velocity at emission, $\mathfrak{A}_1$; the apparent rate of change of velocity at emission, $\mathfrak{A}_2$; and the rate of change of apparent velocity, $\mathfrak{A}_3$.

5.2 Velocity evolution in realistic models

In this subsection we compare time evolution of velocities with time evolution of the redshift in models more complicated than a one-fluid model. We will see that an interesting discrepancy between these two rates appears in the ΛCDM model, which is currently believed to be the most appropriate to describe the Universe we live in. Let us recall what happens in a one-component model. The formula for time evolution of redshift (9) immediately gives that redshifts become smaller in time in decelerating Universes, and larger in accelerating Universes. As for velocities, the situation is generally the same, and can be illustrated even more easily. Since the comoving coordinate does not change in time, time evolution of both $v_{\rm em} = \dot{a}_{\rm em}\chi$ and $v_{\rm now} = \dot{a}_{\rm now}\chi$ is determined only by the second derivative of the scale factor (at $t_{\rm em}$ or "now"). The observed rate of change of $v_{\rm em}$ can be also found as:

$$\mathfrak{A}_2 = \frac{dv_{\rm em}}{dz}\frac{dz}{dt},$$

where the corresponding derivatives are calculated following eqs. (6) and (9). Since $dv_{\rm em}/dz$ is always positive in a one-component Universe, the sign of $\mathfrak{A}_2$ is determined by the sign of $\dot{z}$.

The situation changes for more general models including the important case of the two-component ΛCDM. It is evident that $v_{\rm em} = \dot{a}_{\rm em}\chi$ decreases for objects which were emitting when $\ddot{a}_{\rm em} < 0$ (they have current redshifts larger than $z \sim 0.6$), and increases for closer observed objects which were emitting when the Universe was already accelerating. On the other hand, using the exact formula for the time evolution of the scale factor in the flat ΛCDM Universe (see, e.g., Sahni & Starobinsky 2000):

$$a = \left(\sinh\left(3/2\right)\sqrt{\Lambda/3}ct\right)^{2/3}, \tag{12}$$

where $\Lambda = 8\pi G\rho_{\rm vac}/c^2$ is the cosmological constant, we can see that $\dot{z} > 0$ for z less than approximately 2. So that, in our Universe (if the ΛCDM model is a good approximation to reality) for $0.6 < z < 2$ redshifts increase while corresponding velocities at emission decrease.

Here we again come across the situation when the value of recession velocity (with its time derivative) is determined at the time of emission, while the corresponding rate of redshift changes depends on the whole expansion history till the moment of observation (this manifests itself in an explicit dependence in the r.h.s. of eq. (9) on the *present* value of the Hubble parameter).

A question can arise: what is wrong with equation (10) in the ΛCDM case? Does this mean that $dv_{\rm em}/dz$ changes its sign for some range of z (which would be rather unusual)? No, it can be checked that $dv_{\rm em}/dz$ in the ΛCDM model is always positive.[†] The true reason is that an analogue of formula (6) for a multi-component models does not exist in the following sense: in eq. (6) the velocity is related to the corresponding value of the redshift independently of the time of observation. Obviously, we cannot expect such a property in a two-component model. In, say, ΛCDM an observer living in the dust-dominated epoch should use eq. (6) with $\alpha = 3/2$, while a "later" observer in the epoch of Λ-domination would use eq. (6) with $\alpha = 0$. Using the exact formula (12) it is possible to get a relation between v and z (at least numerically). However, this result should include the time of observation (or, equivalently, the parameter Ω_Λ which is *currently* close to 0.7 and is changing with time). For example, the parametrization $H(z) = H_0(1+z)^\alpha$ now takes the form:

$$H(z) = H_0(1+z)\left(1 + \Omega_M z + \Omega_\Lambda\left(\frac{1}{(1+z)^2} - 1\right)\right)^{1/2}.$$

This means that eq. (10) is not applicable in this case. Some numerical results are shown in Figs. 6–9. We can see that v and z both decrease for an observer living at the epoch of deceleration. After acceleration starts, nearby objects have increasing v and z. However, for a certain range of intermediate redshifts we have $\dot{v} < 0$ and $\dot{z} > 0$. This is again because of the "cumulative" nature of redshifts and their time

[†]In principle, it is possible to obtain $dv_{\rm em}/dz < 0$. However, it requires exotic cosmological dynamics. For example, it happens for the solution $H \sim t$ in $f(R)$-gravity for $f(R) = R + R^2$.

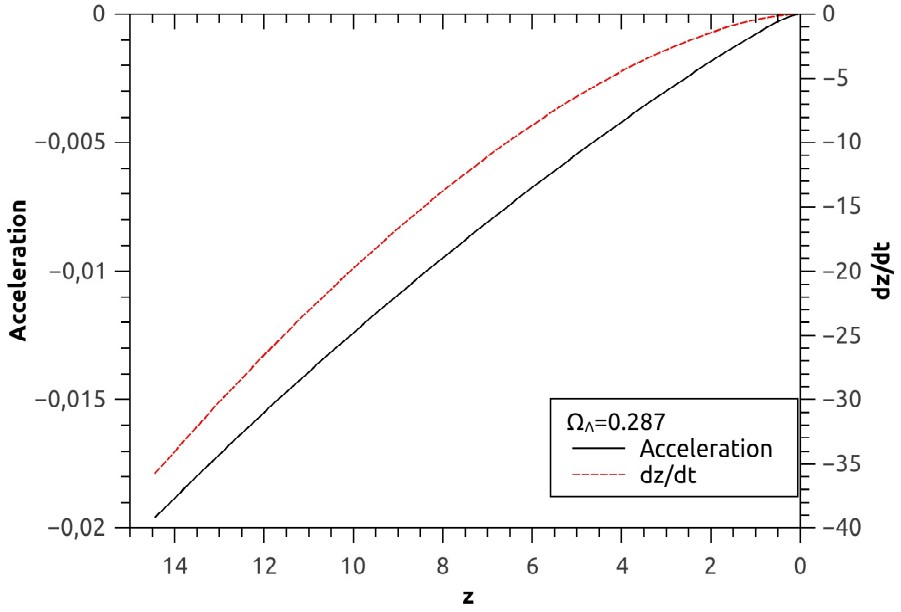


Figure 6 Evolution of apparent acceleration $\mathfrak{A}_2 = (\dot{v}_{\rm em}/c)/(1+z)$ (solid line) and $\dot{z}$ (dashed line). This plot corresponds to an observer at the epoch when $\Omega_\Lambda = 0.287$. We see that both functions are always below zero.

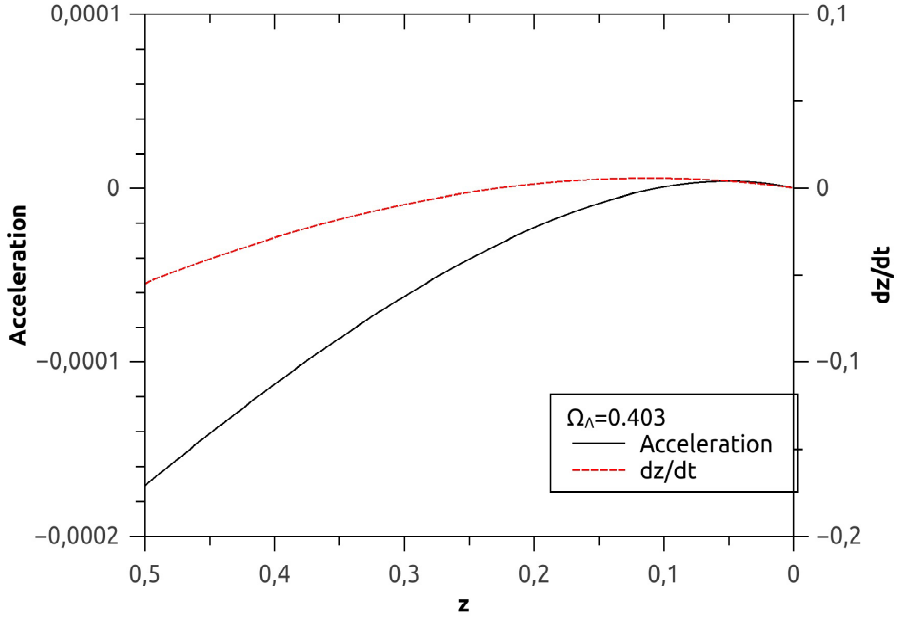


Figure 7 Evolution of apparent acceleration $\mathfrak{A}_2 = (\dot{v}_{\rm em}/c)/(1+z)$ (solid line) and $\dot{z}$ (dashed line). This plot corresponds to an observer at the epoch when $\Omega_\Lambda = 0.403$.

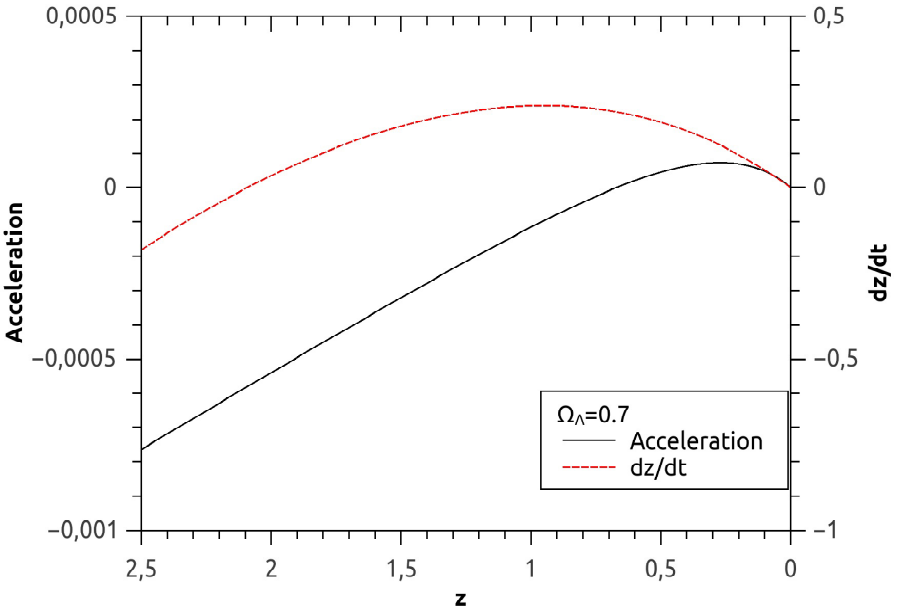


Figure 8 Evolution of apparent acceleration $\mathfrak{A}_2 = (\dot{v}_{\rm em}/c)/(1+z)$ (solid line) and $\dot{z}$ (dashed line). This plot corresponds to an observer at the epoch when $\Omega_\Lambda = 0.7$. We see that at $z \sim 2$ $\dot{z}$ becomes positive.

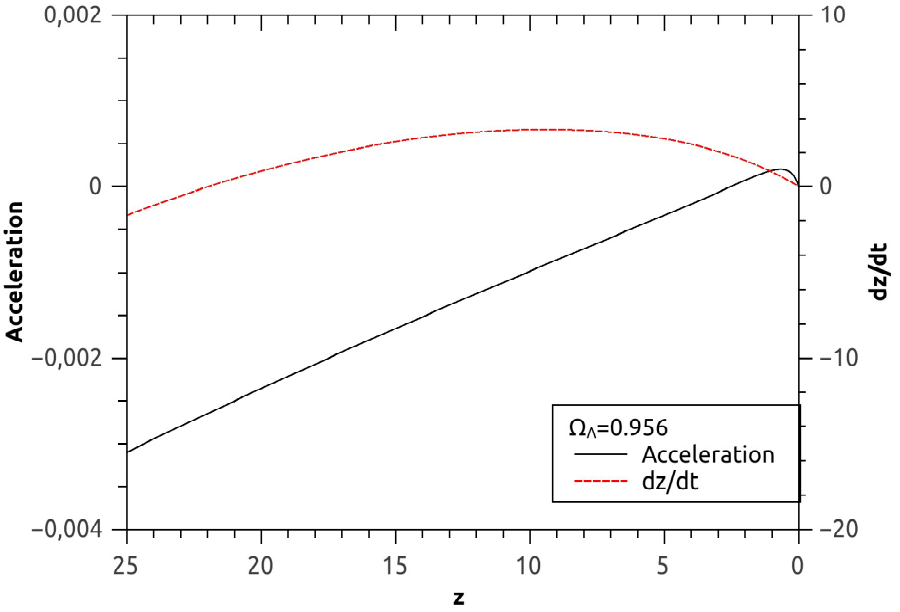


Figure 9 Evolution of apparent acceleration $(\dot{v}_{\rm em}/c)/(1+z)$ (solid line) and $\dot{z}$ (dashed line). This plot corresponds to an observer at the epoch when $\Omega_\Lambda = 0.956$.

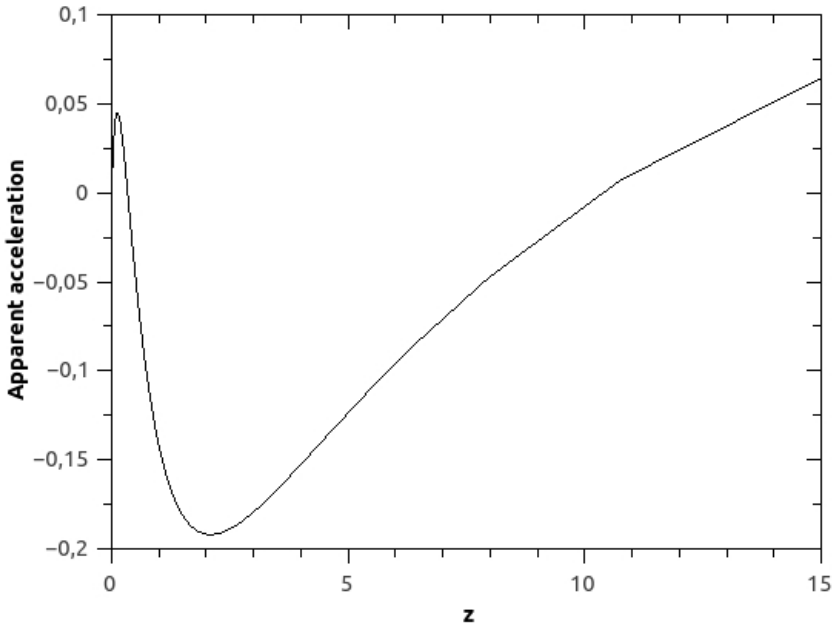


Figure 10 Apparent acceleration $\mathfrak{A}_3(z)$ in the ΛCDM model for the present-day observer.

derivatives along a light trajectory. As for the rate of change of the apparent velocity, it has more complicated behavior with the redshift. It is clear that $\mathfrak{A}_3$ is positive for very small redshifts (at least in the present epoch when Λ dominates) and tends to cH_0 for very large redshifts (sharing this asymptotic with the pure dust model). The plot of $\mathfrak{A}_3(z)$ for the epoch when $\Omega_\Lambda = 0.7$ is shown in Fig. 10. It is visible in the figure that the function changes sign twice, and it is negative for intermediate redshifts (approximately from 1 to 10).

For the realistic cosmological model $\dot{z}$ for different z are calculated in Davis & Lineweaver (2004). Observational prospects are discussed in Quercellini et al. (2012). There is a hope that thanks to new large ground-based telescopes and spectrographs it will be possible to measure $\dot{z}$ in the next few decades. Variations of $v_{\rm em}$ can be more elusive. At the moment the most precise method to determine angular distances is related to maser measurements (see, e.g., Kuo et al. 2013; Humphreys et al. 2013 and references therein). However, there is not much hope that changes in D_θ can be detected in the near future.

6 Conclusions

In this methodological note we presented a discussion of several issues important in the explanation of cosmological phenomena.

In particular, we underline that in cosmology we mostly work in a synchronous frame (typically, in the FRW frame), and properties of such frames can be used to

Table 1 Basic parameters used in the paper.

$D_{\rm now}$	Proper distance now	$a(t_{\rm obs})\chi$	$\frac{c}{H_0(1-\alpha)}[(1+z)^{1-\alpha}-1]$
$D_{\rm em}$	Proper distance at emission	$a(t_{\rm em})\chi$	$\frac{c}{H_0(1-\alpha)(1+z)}[(1+z)^{1-\alpha}-1]$
$D_{\rm light}$	Light travel distance	$c(t_{\rm obs}-t_{\rm em})$	$\frac{c}{\alpha H_0}[1-(1+z)^{-\alpha}]$
$v_{\rm now}$	Rate of change of the proper distance now	$\dot{a}(t_{\rm obs})\chi$	$\frac{c}{1-\alpha}[(1+z)^{1-\alpha}-1]$
$v_{\rm em}$	Rate of change of the proper distance at emission with respect to cosmic time	$\dot{a}(t_{\rm em})\chi$	$\frac{c}{1+\alpha}[1-(1+z)^{\alpha-1}]$
$v_{\rm app}$	Apparent rate of change of proper distance at emission	$\dot{a}(t_{\rm em})a(t_{\rm em})\chi/a(t_{\rm obs})$	$\frac{c}{(1+\alpha)(1+z)}[1-(1+z)^{\alpha-1}]$
$v_{\rm l}$	Rate of change of $D_{\rm light}$	$c\frac{a(t_{\rm obs})-a(t_{\rm em})}{a(t_{\rm obs})}$	$cz/(1+z)$
$\mathfrak{A}_{\rm now}$	Acceleration now	$\ddot{a}(t_{\rm obs})\chi$	$cH_0[(1+z)^{1-\alpha}-1]$
$\mathfrak{A}_1$	Rate of change of velocity at emission with respect to cosmic time	$\ddot{a}(t_{\rm em})\chi$	$cH_0(1+z)^{\alpha}[1-(1+z)^{\alpha-1}]$
$\mathfrak{A}_2$	Apparent rate of change of velocity at emission	$\ddot{a}(t_{\rm em})a(t_{\rm em})\chi/a(t_{\rm obs})$	$cH_0(1+z)^{\alpha-1}[1-(1+z)^{\alpha-1}]$
$\mathfrak{A}_3$	Rate of change of apparent velocity	$\frac{\chi}{a(t_{\rm obs})^2}[\ddot{a}(t_{\rm em})a^2 + \dot{a}(t_{\rm em})^2 a - \dot{a}(t_{\rm em})a(t_{\rm em})\dot{a}(t_{\rm obs})]$	$\frac{cH_0}{1-\alpha}[-(1+z)^{-1}-(\alpha-3)(1+z)^{\alpha-2}+(\alpha-2)(1+z)^{2\alpha-3}]$

explain some non-trivial facts, such as superluminal velocities in the Hubble flow, or Galilean summation of velocities in cosmology. Interpretation of the cosmological redshift also can be based on the properties of synchronous systems, and so, formally, a very illustrative concept of "stretching space" is not necessary to explain the origin of the redshift, if one intends to go deeper into the nature of this phenomenon. Moreover,

a similar scheme (though more technically complex due to spatial inhomogenity) can be used in other situations in GR where "stretching of space" cannot be considered as a useful concept (all we need is a synchronous coordinate system). The cosmological redshift can be reduced to gravitational effects in the FRW frame, however, not just to the well-known gravitational time dilation.

In addition, we provided some illustrations related to different velocities used in cosmology. We revisit the "tethered galaxy" problem and generalize results obtained by Davis et al. (2003) for other definitions of recession velocities and other matter contents (equation of state) of the Universe.

Finally, we discuss several possible measures of cosmic acceleration. Using them we compare time evolution of velocities and redshift in the ΛCDM cosmology, and show why they evolve differently in this model.

We hope that these notes can be useful for better understanding of some basic cosmological concepts by non-specialists. Especially, we address this discussion to college lecturers who are teaching elements of cosmology in their courses.

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