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A new general normal mode approach to dynamic tides in rotating stars with realistic structure and its applications

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We review our recent results on a unified normal model approach to dynamic tides proposed recently in Ivanov, Papaloizou & Chernov (2013) and Chernov, Papaloizou & Ivanov (2013). Our formalism can be used whenever the tidal interactions are mainly determined by normal modes of a star with identifiable regular spectrum of low frequency modes. We provide in the text basic expressions for tidal energy and angular momentum transfer valid both for periodic and parabolic orbits, and different assumptions about efficiency of normal mode damping due to viscosity and/or non-linear effects and discuss applications to binary stars and close orbiting extrasolar planets.

Keywords: Hydrodynamics – celestial mechanics – planetary systems: formation, planet–star interactions, stars: binaries: close, rotation, oscillations, solar-type

1 Introduction

In this contribution we briefly discuss our results on a unified normal model approach to dynamic tides and its applications published in Ivanov, Papaloizou & Chernov (2013) and Chernov, Papaloizou & Ivanov (2013) hereafter Paper 1 and 2, respectively. Our formalism is applicable when the tidal interactions are mainly determined by normal modes of a star with identifiable regular spectrum of low frequency modes, such as rotationally modified gravity modes. In this case it allows one to consider the well known theory of dynamic tides (e.g. Zahn 1977, Goodman & Dickson 1998) operating in binaries with periodic orbits as well as the excitation of normal modes

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as the result of a flyby around a perturbing body by a rotating star moving on a parabolic orbit as discussed by Press & Teukolsky 1977 (see also, e.g. Lai 1997, Ivanov & Papaloizou 2004, 2007, hereafter IP7). These are considered in a general way in parallel developments.

It is shown that in general, energy and angular momentum transfer due to dynamic tides is determined by the so-called overlap integrals provided that a mechanism of dissipation of energy stored in normal modes is specified and orbital parameters together with the state of rotation of the star are given. It is stressed that in certain important cases, such as dynamic tides operating in the Press & Teukolsky sense, or dynamic tides operating in the Zahn sense in the regime of moderately large dissipation, for which internally excited waves travelling with their group velocity do not reach either the centre or boundary of the configuration on account of damping, the resulting expressions for either the energy and angular momentum transfer rates, or the total amount transferred in each case, do not depend on details of energy dissipation, see Section 2.

In Section 3 we briefly present the overlap integrals obtained for Sun-like stars and stars with mass $M_* = 1.5M_\odot$ referring the reader to Paper 1 and Paper 2, where these quantities are calculated numerically for a range of rotating Population I stellar models of different masses and ages. These results are obtained for the slow rotation regime where centrifugal distortion is neglected in the equilibrium and the traditional approximation is made, and an analytical WKBJ theory of their evaluation is developed for the case of Sun-like rotating stars.

In Section 4 we use our formalism to determine the tidal evolution time scales for an object, in an orbit of small eccentricity, around a Sun-like star in which the tidal response is assumed to occur. This is taken to have a perturbing companion, either with the same mass, or with a mass typical of exoplanets. We consider the case of synchronous rotation and in addition a non rotating star when the companion is an exoplanet. We assume that wave dissipation occurs close to the stellar centre such that the moderately large dissipation regime applies. Additionally, we apply the formalism to the problem of tidal excitation of normal modes after a periaston flyby showing that realistic stars having an extended convective envelope are more susceptible to the action of tides than a frequently used reference model consisting of a polytrope, with index $n = 3$, and with the same mean density, see Section 5.

2 General expressions for the energy and angular momentum transfer

2.1 Basic definitions

We assume below that when the orbital motion may be considered to be periodic a discrete Fourier series in time is appropriate. For a real quantity, Q, we write

$$Q = \sum_{m,k} \left(Q_{m,k} \exp(-\mathrm{i}\omega_{m,k} t + im\phi) + cc\right), \tag{1}$$

where cc denotes the complex conjugate, $\omega_{m,k} = k\Omega_{\rm orb} - m\Omega$, ϕ is the azimuthal angle in the spherical coordinate system (r, θ, ϕ) associated with a frame rotating with the angular frequency Ω with respect to an inertial frame with the rotation axis being

directed along $z = r\cos\theta$. The orbital frequency is $\Omega_{\rm orb}$, and m and k are integers, with only positive values of m included in the summation as in IP7. When the orbital motion may be treated as parabolic we employ a Fourier transform instead of a Fourier series. The stellar perturbations are described by the Lagrangian displacement vector $\boldsymbol{\xi}$. The dominant quadrupole component of the tidal potential, Ψ, is represented in the form

$$\Psi = r^2 \sum_{m,k} \left(A_{m,k} e^{-i\omega_{m,k}t} Y_2^m + cc \right) \tag{2}$$

where Y_2^m are the spherical functions. The coefficients $A_{m,k}$ are given in Appendix A of Paper 1 for the case of coplanar orbit with small eccentricity. For orbits having an arbitrary value for the eccentricity, these coefficients can be expressed in terms of so-called Hansen coefficients discussed eg. in Witte & Savonije (1999). We use a parallel development formulated in terms of the Fourier transform of the tidal potential in the case of parabolic orbits.

2.2 The energy and angular momentum transfer rates in the case of periodic orbits

In this case the general expressions are obtained in Paper 1. The rate of transfer of energy in the rotating frame, $\dot{E}_c$, the rate of transfer of angular momentum, $\dot{L}_c$, and the rate of transfer of energy in the inertial frame, $\dot{E}_I$, read as

$$\begin{aligned} \dot{E}_c &= -\sum_{m,k,j} \omega_{\nu,kj} \frac{|A_{m,k}\hat{Q}_j|^2}{D_{k,j,j}}, \\ \dot{L}_c &= -\sum_{m,k,j} m \frac{\omega_{\nu,kj}}{\omega_{m,k}} \frac{|A_{m,k}\hat{Q}_j|^2}{D_{k,j}} \qquad \text{and} \\ \dot{E}_I &= -\sum_{m,k,j} \frac{\omega_{\nu,kj}(\omega_{m,k} + m\Omega)}{\omega_{m,k}} \frac{|A_{m,k}\hat{Q}_j|^2}{D_{k,j}}. \end{aligned} \tag{3}$$

Here

$$D_{k,j} = (\omega_{m,k} - \omega_j)^2 + \omega_{\nu,kj}^2, \tag{4}$$

where ω_j are eigenfrequencies of normal modes with the index (j) designating particular modes, and $\omega_{\nu,kj}$ are their damping rates.

The overlap integrals are

$$\hat{Q}_j = Q_j/\sqrt{n_j}, \tag{5}$$

where

$$Q_j = \int d^3x \rho e^{-im\phi} \boldsymbol{\xi}_j^* \cdot \nabla(r^2 Y_2^m) \tag{6}$$

and

$$n_j = \pi((\boldsymbol{\xi}_j|\boldsymbol{\xi}_j) + (\boldsymbol{\xi}_j|\mathcal{C}\boldsymbol{\xi}_j)/\omega_j^2), \tag{7}$$

and $\mathcal{C}$ is an integro-differential operator describing the action of pressure and gravity forces on stellar perturbations. Here dot and asterisk stand for the scalar product and complex conjugate, respectively.

2.2.1 A dense spectrum of modes

Let us assume that the difference between neighboring values of eigenfrequencies is much smaller than their typical absolute values and that for a particular forcing frequency, $\omega_{m,k}$, the resonance condition is most closely satisfied for a particular mode with index $j = j_0$, (which we recall will vary with m and k). Accordingly, we have

$$\omega_{j_0} = \omega_{m,k} + \Delta\omega_{j_0}, \tag{8}$$

where the offset, $\Delta\omega_{j_0}$, is such that $|\Delta\omega_{j_0}| < |d\omega_{j_0}/dj_0|$. In this case the general expressions (3) can be simplified. In general, their form depends significantly on two dimensionless parameters.

They take an especially simple form depending on whether the parameter $\kappa = |\omega_{\nu,kj_0}/d\omega_{j0}/dj_0|$ that measures the effectiveness of dissipation is large or small. As long as $|\omega_{\nu,kj_0}/\omega_{j0}| \ll 1$, the former case corresponds to the so-called regime of moderately large dissipation, for which the decay time due to viscosity is much smaller than the time for waves to propagate across the star with their group velocity, see e.g. Goodman & Dickson (1998). In this case we have

$$\begin{aligned}
\dot{E}_c &= -\pi \sum_{m,k} \frac{|A_{m,k}\hat{Q}_{j_0}|^2}{|d\omega_{j_0}/dj_0|}, \\
\dot{L}_c &= -\pi \sum_{m,k} \frac{m}{\omega_{m,k}} \frac{|A_{m,k}\hat{Q}_{j_0}|^2}{|d\omega_{j_0}/dj_0|}, \qquad \text{and} \\
\dot{E}_I &= -\pi \sum_{m,k} \left(1 + \frac{m\Omega}{\omega_{m,k}}\right) \frac{|A_{m,k}\hat{Q}_{j_0}|^2}{|d\omega_{j_0}/dj_0|}.
\end{aligned} \tag{9}$$

In the latter case when viscosity is very small, and, accordingly, $\kappa \ll 1$, we obtain

$$\begin{aligned}
\dot{E}_c &= -\sum_{m,k} \frac{\omega_{\nu,kj_0}}{D_*} |A_{m,k}Q_{j_0}|^2, \\
\dot{L}_c &= -\sum_{m,k} \frac{m\omega_{\nu,kj_0}}{\omega_{m,k}D_*} |A_{m,k}Q_{j_0}|^2, \qquad \text{and} \\
\dot{E}_I &= -\sum_{m,k} \left(1 + \frac{m\Omega}{\omega_{m,k}}\right) \frac{\omega_{\nu,kj_0}}{D_*} |A_{m,k}Q_{j_0}|^2,
\end{aligned} \tag{10}$$

where $D_* = (\omega^2_{\nu,kj_0} + (d\omega_{j_0}/dj_0)^2 \sin^2 \pi\delta/\pi^2)$, and $\delta = |\Delta\omega_{j_0}/d\omega_{j0}/dj_0|$.

2.3 Energy and angular momentum transfer in the case of a prescribed parabolic orbit

The transfers of energy, ΔE_c, ΔE_I and angular momentum, ΔL_c, between the orbital motion and normal modes of the star during a periastron flyby on a prescribed parabolic orbit can also be expressed in terms of overlap integrals. For completeness we give expressions for these here. As mentioned above, in this case we should represent the results in terms of the Fourier transform of the tidal potential instead of the Fourier coefficients used in the previous sections.

Expressions for ΔE_c, ΔE_I and ΔL_c in terms of this were derived by IP7*. As in the case discussed above they also can be represented as sums over quantities determined by the eigenmodes, which for a given m have eigenfrequency ω_k, where, for convenience, m is here suppressed as a suffix. They take the form

$$\begin{aligned}
\Delta E_c &= 2\pi^2 \sum_{m,k} |A_m(\omega_k)\hat{Q}_k|^2, \\
\Delta L_c &= 2\pi^2 \sum_{m,k} \frac{m}{\omega_k} |A_m(\omega_k)\hat{Q}_k|^2, \\
\Delta E_I &= 2\pi^2 \sum_{m,k} \left(1 + \frac{m\Omega}{\omega_k}\right) |A_m(\omega_k)\hat{Q}_k|^2. \qquad (11)
\end{aligned}$$

Note that the quantities $A_m(\sigma)$ entering (11) are discussed in e.g. Ivanov & Papaloizou (2011) for the general case of a parabolic orbit inclined with respect to equatorial plane of a rotating star.†

3 Overlap integrals

In this Section we present the overlap integrals for two models of Sun-like stars with mass $1\,M_\odot$, model 1a and 1b having ages 1.67×10^8 yr and 4.41×10^9 yr, respectively, and three models of a star with mass $1.5\,M_\odot$ with ages 1.27×10^7 yr (model 1.5a), 5.96×10^7 yr (model 1.5b) and 1.58×10^9 yr (model 1.5c) (see Figs 1 and 2). The stellar models were calculated by Christensen-Dalsgaard (1996) and Roxburgh (2008). We also consider a polytrope with index $n = 3$ as a reference model, denoting it as model 1p. The overlap integrals Q are expressed in the natural units $\sqrt{M_*}R_*$, where M_* and R_* are the stellar mass and radius, respectively, and the eigenfrequencies are expressed in terms of $\Omega_* = \sqrt{GM_*/R_*^3}$, where G is the gravitational constant. All stars are assumed to be non-rotating. A discussion of effects due to rotation, the calculation of overlap integrals for stars with larger masses can be found in Paper 2 and the comparison of the numerical results shown here with the corresponding analytical expressions for Sun-like stars can be found in Paper 1. We only mention here that the comparison shows a good agreement between the two approaches, even for fast rotators.

One can see from Fig. 1 the Sun-like models have much larger values of the overlap integrals as compared to those for the polytropic model at small eigenfrequencies, ω. As discussed in Paper 1 it is due to the presence of extended convective envelopes in the solar models, which result in a power law decay of $\hat{Q}$ with ω in comparison to an exponential decay found for the polytropic model. This means that tidal interactions

*Note that in equation (5) of IP7 there should be a $(-)$ sign in front of the expression on the right hand side and in equation (28) the imaginary unit should be inside the summation.

†The analogous expressions of Ivanov & Papaloizou (2011) differ by factor of two from those given in (11). This is due to the fact that in Ivanov & Papaloizou (2011), the summation over the azimuthal mode number is formally performed over positive and negative values of m, while in this paper we consider only positive azimuthal mode numbers. Note also that in this paper we take all quantities related to eigenmodes to be defined in the rotating frame, while in Ivanov & Papaloizou (2011) the inertial frame was used.

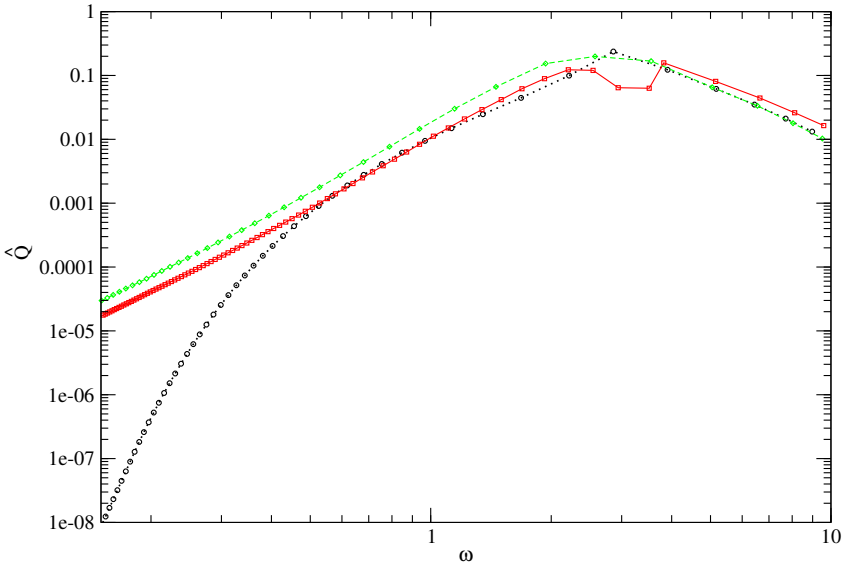


Figure 1 The overlap integrals $\hat{Q}$ as functions of mode eigenfrequency ω for models 1a and 1b plotted with dashed and solid curves, respectively. The dotted curve plots the overlap integrals for a polytrope with $n = 3$. Note that for low frequencies, these are much smaller than the ones corresponding to stellar models with realistic structure. Symbols show the positions of eigenfrequencies. The smooth curves are interpolated through these.

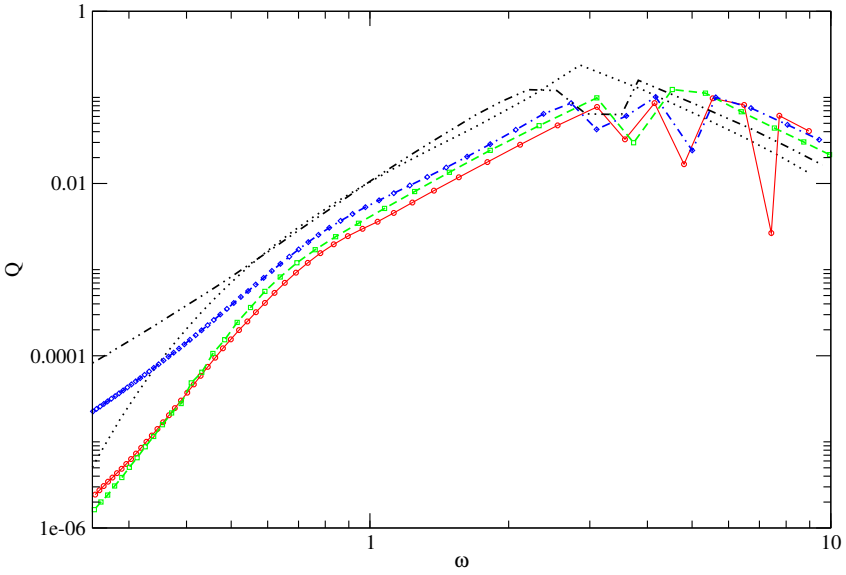


Figure 2 As for Fig. 1 but for models with $M_* = 1.5\,M_\odot$. Dot dashed, dashed and solid curves are for models 1.5a, 1.5b and 1.5c, respectively. We show the results for models 1p and 1b, plotted as dotted and dot dot dashed curves respectively, for comparison.

of Sun-like stars at sufficiently low forcing frequencies are stronger than those corresponding to a polytropic star with the same mean density, see also Fig. 6 below. The $\hat{Q}$ corresponding to models with $M_* = 1.5\,M_\odot$ are smaller than those of Sun-like stars. This is because the convective envelopes are much less pronounced in the models of more massive stars discussed in the text. For the shown range of eigenfrequencies, values of $\hat{Q}$ for more massive models are even smaller than those corresponding to the polytropic model with the exception of the youngest model 1.5a, which has indeed the most extended convective envelope among the models with $M_* = 1.5\,M_\odot$ discussed here.

4 Application of the formalism to the case of a binary orbit with small eccentricity

In order to illustrate applications of our formalism let use discuss two simple problems assuming that the regime of moderately large viscosity operates in the star where tides are raised. The first concerns a binary consisting of a Sun-like star in a state of synchronous rotation and a point-like object of the same mass in an orbit of small eccentricity, e. We plot the corresponding time scale for the decay of eccentricity in Fig. 3.

The short dashed and dotted curves are for model 1b with curves of different types corresponding to different fits for the Brunt–Väisälä frequency close to the base of

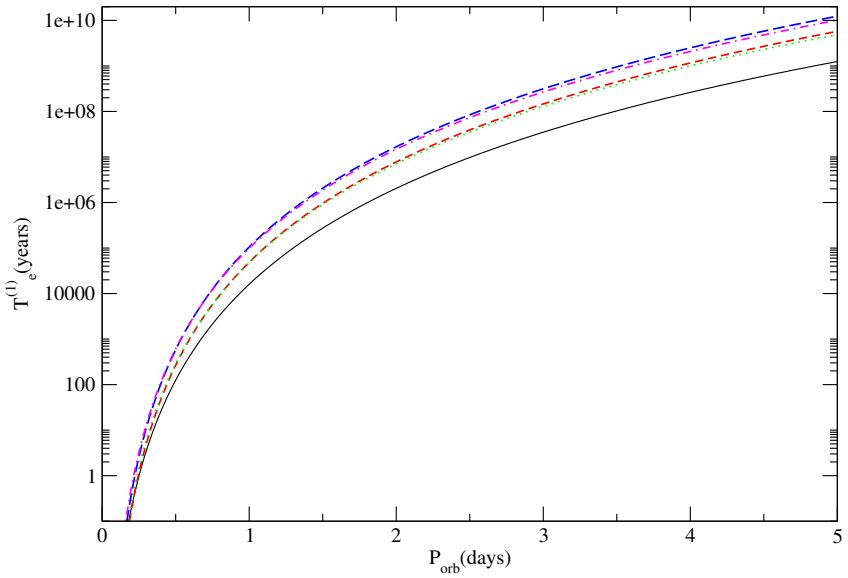


Figure 3 The time scale for evolution of eccentricity, $T_e^{(1)}$, calculated for the 1a and 1b models compared to the result of Goodman & Dickson (1998), for $\mu = 1$. See the text for the description of different curves.

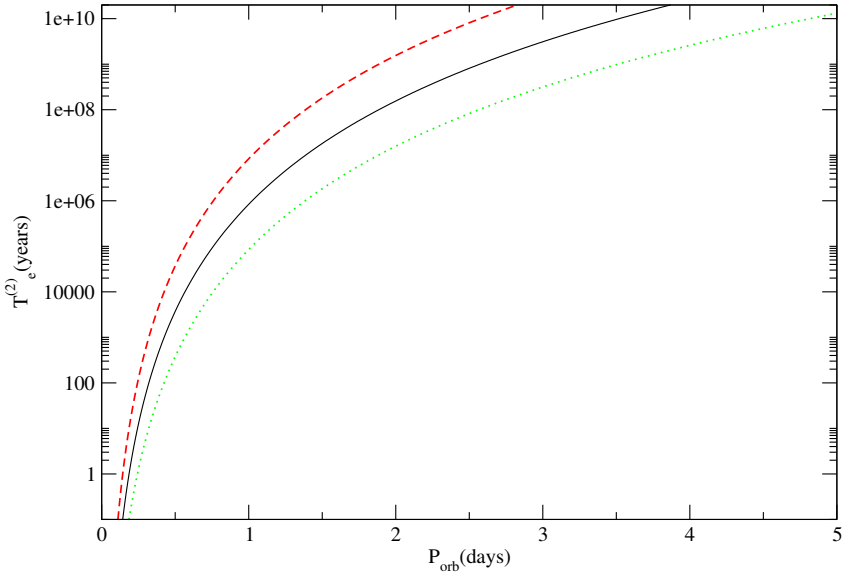


Figure 4 The evolution time scales for orbital eccentricity, $T_e^{(2)}$, plotted for the mass of perturbing object, $m_p = M_J$, $0.1M_J$ and $10M_J$ using the solid, dashed and dotted curves, respectively.

convection zone while the long dashed and dot dashed curves are for model 1a. One can see from Fig. 3 that there is little difference between the curves corresponding to different fits. The solid line plots a minor modification of the expression of Goodman & Dickson (1998) who give $T_e^{GD} = 8 \cdot 10^3 P_{\rm orb}^7$, with $P_{\rm orb}$ in days, for a system of two tidally interacting Sun-like stars of equal mass. Since we assume that the tides are raised only on one component, we plot $T_e^{GD} = 1.6 \cdot 10^4 P_{\rm orb}^7$ in Fig. 3. Our results are seen to give larger values of T_e for a given $P_{\rm orb}$, e.g. for $P_{\rm orb} = 4$ days, the results differ by a factor of five to ten. Thus we have $T_e \approx 2.7 \cdot 10^8$ yr, $1.15 \cdot 10^9$ yr and $2.35 \cdot 10^9$ yr from the modified Goodman & Dickson (1998) expression and for our calculations corresponding to models 1b and 1a, respectively.

The second problem deals with orbital evolution of a point-like object with a mass appropriate for a massive exoplanet orbiting around a non-rotating Sun-like star. The corresponding time scales of the evolution of eccentricity and semi-major axis are shown in Figs 4 and 5, respectively.

The solid, dashed and dotted curves there correspond to the perturbing mass $m_p = M_J$, $0.1M_J$ and $10M_J$. As seen from these plots, dynamical tides in the regime of moderately large viscosity are potentially efficient only when orbital periods are rather small. Thus, the eccentricity is damped in a time less than 10^9 yrs only for $P_{\rm orb} < 1.85$, 2.56 and 3.48 days, for $m_p = 0.1M_J$, M_J and $10M_J$, respectively. Similarly the semi-major axis can decay in a time less than 10^9 yrs only for $P_{\rm orb} <$ 1.24, 1.69 and 2.3 days respectively.

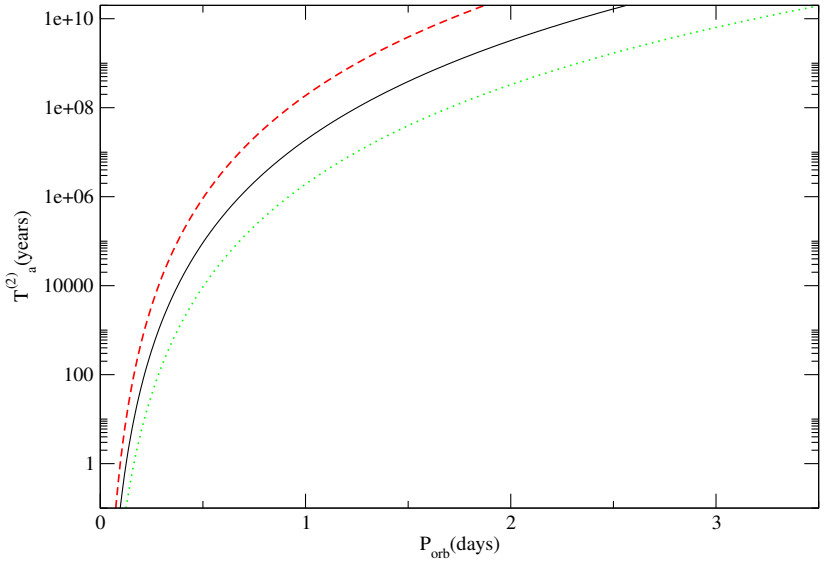


Figure 5 Same as Fig. 4 but the evolution time scale of the semi-major axis, $T_a^{(2)}$, is shown.

5 Energy transferred as the result of a flyby around a centrally condensed mass on parabolic orbit

Now let us consider the problem of parabolic flyby of a star around a point-like source of gravity. One can show that once the ratio of the rotational frequency to the characteristic stellar frequency, Ω/Ω_*, is specified, the energy and angular momentum transferred, ΔE and ΔL, expressed in units of $E_* = Gm_p^2/((1+q)^2 R_*)$ and $L_* = q^2(1+q)^{-2} M_* \sqrt{GM_* R_*}$, where $q = m_p/M_*$, respectively, are functions of only one parameter (see eg. Press & Teukolsky 1977, Ivanov & Papaloizou 2004, 2007)

$$\eta = \sqrt{\frac{1}{1+q}\left(\frac{R_p}{R_*}\right)^3} = 3.05\sqrt{\bar{\rho}} P_{\rm orb}, \tag{12}$$

where R_p is the periastron distance. The quantity $\bar{\rho}$ is the ratio of the mean stellar density to the solar value, $\bar{\rho} = R_\odot^3 M_*/(R_*^3 M_\odot)$, and $P_{\rm orb}$ is orbital period of a circular orbit which has the same value of the orbital angular momentum as the parabolic orbit under consideration, expressed in units of one day. Here we consider only the case of non-rotating Sun-like star referring the reader to Paper 2 and Ivanov & Papaloizou 2011, where more general cases are discussed.

The energy transferred as a result of tidal encounter is shown in Fig. 6. One can see that the realistic stellar models give much larger values of ΔE at sufficiently large values of η as compared with the reference polytropic model. This effect is due to the power law dependence of the overlap integrals in the limit $\omega \to 0$ mentioned above, which is, in turn, is due to the presence of convective regions in realistic stellar models.

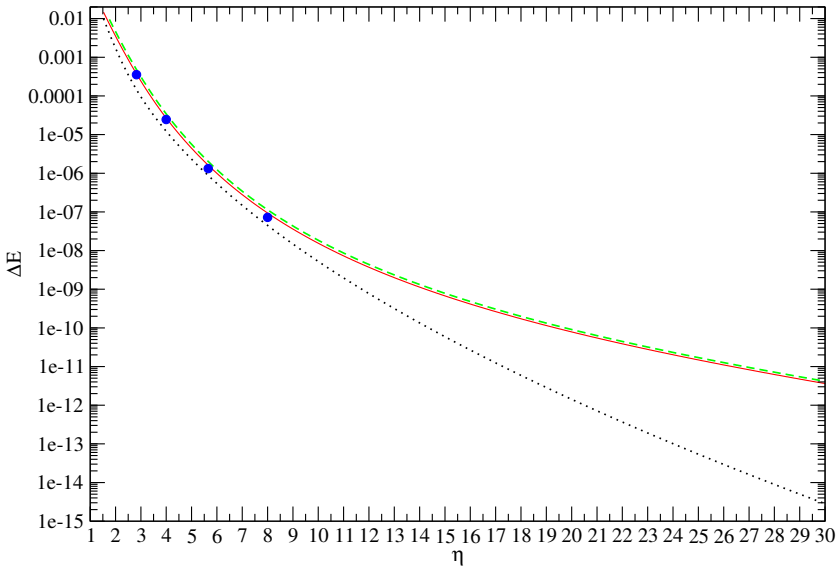


Figure 6 Energy transferred to the normal modes of a non-rotating star with $M_* = M_\odot$ as the result of a parabolic flyby of a perturber of mass $m_p = M_J$, expressed in units of E_*, as a function of the parameter η. The solid, dashed, and dotted curves are for models 1b, 1a and 1p, respectively. The dot dashed curve shows results calculated using the purely analytic expressions for the mode eigenfrequencies and overlap integrals obtained in Paper 1 for model 1b. Circles show the energy transfer calculated for model 1b using the direct numerical approach.

6 Conclusions

In this contribution we present a new general approach to the problem of dynamic tides. It allows one, in principal, to calculate all quantities of interest both for a periodic orbit of any eccentricity and for a prescribed parabolic orbit. It generalizes the well known approaches of Zahn and Press & Teukolsky. It is shown that provided the rate of decay of free stellar oscillations due to either linear dissipation or non-linear effects, as well as the orbital parameters, are specified, all information about tidal interactions is encoded in the overlap integrals. It is stressed that when either a system is evolving in the so-called regime of moderately large viscosity or undergoes a flyby around a gravitating body, the expressions governing energy and angular momentum exchange do not depend on the quantities describing the decay of free modes. The overlap integrals are calculated numerically for a set of Population I stellar models of different masses and ages, and, additionally, analytically for Sun-like stars with convective envelope and radiative core. The formalism is applied to the orbital evolution of systems having periodic orbits with small eccentricity evolving in the regime of moderately large viscosity and the parabolic flyby problem. In the former case, it is shown that a Hot Jupiter can spiral in a host Sun-like star in a time smaller than or of the order of 10^9 when its orbital period is smaller than $\sim 1.7d$. In the latter case it is shown that, in general, stars with realistic structure have much

stronger tidal response at sufficiently large values of impact parameters than the reference polytropic model provided that they have an extended convective envelope. One unresolved issue is to provide an extension of the analytic methods of calculation of the overlap integrals in the limit $\omega \to 0$ to the case of stars with convective cores and radiative exteriors. This is left for a possible future work.

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