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# Black hole shadows: A new possibility to constrain extended gravity F

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We continue the discussion of the idea how to use the new results on black hole shadows from the Event Horizon Telescope with the aim to constrain the extended gravity models. As the limit for the ground-based radio telescopes is almost reached, in order to be able to test the extended theories of gravity it is necessary to project spacecraft observations. Further the inclusion of a rotation (especially a strong one) decreases by itself the required resolution by an order.

*Keywords*: General relativity, black hole shadows

## 1 Introduction

The general theory of relativity (GR) was tested many times in experiments and observations. This theory predicts the existence of black holes (BH): objects with event horizons. There was a set of intimations on the existence of BH in astrophysics: the dynamics of closed binary systems [1], the results of gravitational wave astronomy [2], direct images of the BH shadow within the framework of the Event Horizon Telescope project [3]. It is important to note that GR describes astrophysical phenomena with great accuracy. However, there are some problems in GR, for example, dark matter, dark energy, evolution of the early universe, quantization of the gravitational field, ... The development and improvement of experimental astronomy methods requires the further development of the theoretical basis. Therefore, new theories of gravity modifying GR are being developed. They are, for example, models of $f(R)$ gravity [4], scalar–tensor theories with their most general case providing the second-order field

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equations (Horndeski model) [5, 6, 7, 8], teleparallel gravity models [9], conformal gravity [10], quantum-loop gravity [11, 12], etc. During the development of new theories one would like to assess the limits of their applicability. The study of the recorded (at least potentially) astronomical effects of the BH proximity acts as one of these tests.

Due to the limited accuracy of the experimental possibilities alternative (competing) explanations of the same phenomenon from different theories appear to be possible [13]. Usually the simplest and the most logical of them is selected. Therefore, to identify the effects of a particular theory a larger number of experimentally measured characteristic quantities are required. In this case, a step towards more subtle effects appears to be possible (for example, discussions of the BH birth at the Large Hadron Collider [14]). The size of the BH shadow is usually the first observed parameter. Therefore, we use it to evaluate the accuracy of the predictions of the theory and the Schwarzschild, Reissner–Nordstrom, Kerr metrics are used as the first approximation.

The paper is organised as follows: Section 2 is devoted to black hole metrics as GR solutions and real astronomical objects, Section 3 gives an introduction to BH shadows, Section 4 extends the shadow description by the new parameters, Section 5 is devoted to the shadow from rotating BHs and Section 6 presents conclusions.

## 2 Black hole: GR solution and a real astronomical object

We start from the metric of a non-rotating non-charged (Schwarzschild) BH (in the Planck system of units $G = c = \hbar = 1$):

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where

$$A(r) = B(r)^{-1} = 1 - \frac{2M}{r}, \tag{2}$$

$M$ is the mass of the BH. If the electromagnetic field is taken into account, one has to apply the Reissner–Nordstrom metric 1:

$$A(r) = B(r)^{-1} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{3}$$

where $Q$ is the electric charge. The rotation can be taken into account with the help of the Newman–Janis algorithm [15]. Applying it to 3 results in the Kerr–Newman metric in the form:

$$\begin{aligned} ds^2 = & - \left(1 - \frac{2(M - \frac{Q^2}{2r})r}{\rho^2}\right) dt^2 - \frac{4(M - \frac{Q^2}{2r})ar\sin^2\theta}{\rho^2} d\phi dt \\ & + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \end{aligned}$$

$$+ \quad \left( r^2 + a^2 + \frac{2(M - \frac{Q^2}{2r})a^2 r \sin^2\theta}{\rho^2} \right) \sin^2\theta d\phi^2, \tag{4}$$

where

$$\rho^2 = r^2 + a^2 \cos^2\theta,$$
$$\Delta(r) = r^2 - 2(M - \frac{Q^2}{2r})r + a^2, \tag{5}$$

where $a = J/M$ is the BH rotation spin and $J$ is the angular momentum. If $Q = 0$ one obtains an uncharged rotating BH described by the Kerr metric.

In astronomy BHs of various masses were detected by different methods. BHs of stellar masses ($4 \div 15\, M_{\rm Sun}$) are observed in close binary systems via the light curves and radial velocities [16]. Super-massive BHs are discovered in the centres of galaxies with masses estimated in various ways. For example, the masses could be constrained from the gas dynamics or by the behaviour of the central stars. It was a great success when the Event Horizon Telescope was lucky to obtain a direct image of the shadows (in the range of radio waves) for the BH in the centre of the M87 galaxy [2]. Its mass appears to be equal to $6.5 \times 10^9\, M_{\rm Sun}$. The mass of a supermassive BH at the centre of our Galaxy (Sag $A^*$) was measured by the analysis of nearest stars motions [17]: it is equal to $4.15 \times 10^6\, M_{\rm Sun}$. Unfortunately a lot of dust and gas at the line of sight makes it difficult to get an image of its shadow.

Dozens of fusion events involving BHs were detected at the LIGO and VIRGO gravitational-wave detectors. The first one happened in 2015 [18]. It is important that the maximum detected BH mass formed by the merging of two smaller ones is about $150\, M_{\rm Sun}$ [19]. Mergers of stellar mass BHs with neutron stars [20] were also detected.

## 3 Shadows from black holes

A characteristic feature of BH is the presence of an event horizon. The latter represents an imaginary surface which the light cannot leave to reach an external observer. In the simplest case of a Schwarzschild BH the horizon radius is equal to $R_{\rm sh} = 2\, M$. However, the dark spot from the BH will be larger due to gravitational effects in the background of the surrounding stars, plasma, gas and the accretion disk. Such a spot is called the shadow of a BH. The shadow radius for the Schwarzschild BH is equal to $D = 3\sqrt{3}\, M$ [21]. The edge of the shadow corresponds to the place where a photon transits to an unstable orbit around the BH. All photons flying to the BH with an impact parameter less than the radius of the shadow reach the horizon and then are captured by it. The remaining photons fly to infinity and, so, can reach a remote observer. Note that the shadow has a round shape for a non-rotating BH. Rotation distorts the shape of the shadow. An increase of the rotation speed causes a stronger difference from the round shape. Slowly rotating objects are approximated by non-rotating metrics.

The first captured shadow belongs to a supermassive BH in the centre of the galaxy M87 [3]. This BH represents one of the largest BH like objects at the night sky (in terms of the angular size). Its characteristic size is of the order of $\mu$as. The supermassive BH in the centre of our galaxy Sag $A^*$ has a similar size. But M87 is not situated at the plane of our galaxy and represents an elliptical one. So the intergalactic matter in both galaxies has interference with the image acquisition. That is why the first image was taken at the galaxy M87. In addition jets from the central object are visible in the centre of M87 that gives the possibility to assume the direction of the BH rotation. The discussed images were obtained by a network of radio telescopes located around the globe. Due to the low resolution it appeared to be possible to estimate only the mass of the BH and to constrain a set of BH models [22]. For example, for the Reissner–Nordstrom metrics the limit on the normalised physical charge has the form $\overline{q} < 0.9$ (the maximum charge of the Reissner–Nordstrom BH is equal to $\overline{q} = 1$). Due to the heavy smudging the edges of the shadow are not visible. Strong gravitational lensing of synchrotronic radiation from the hot plasma around the BH forms a light ring. The project team was lucky to determine the direction of the BH rotation using the distribution of the brightness from the light ring and obtained that the rotation vector is directed in the opposite direction for the Earth observer. The consideration of the polarisation allowed the project team to improve the image of the luminous ring [23].

## 4 New parameters for shadows

The success in the shadow imaging of a supermassive BH at the centre of M87 [3] opens up a new opportunity to test extended theories of gravity at event horizon scales. At such scales it becomes possible to detect the corrections to GR rapidly diminishing during a distance increase [24, 25]. When the amount of observational data is limited extended theories of gravity can give similar explanations as GR. For example, when the contribution of additional dimensions in the Randal–Sandrum model is considered one obtains, in fact, the Reissner–Nordstrom metric ($q \to Q^2$) with a positive or negative sign in the charge term [26, 27]. $q$ is called the tidal charge. Unlike an electric charge, a tidal charge can have any sign at $1/r^2$. For this metric, the shadow radius was obtained analytically in [24] and is equal to:

$$D = \sqrt{\frac{(8q^2 - 36q + 27) + \sqrt{(8q^2 - 36q + 27)^2 + 64q^3(1-q)}}{2(1-q)}}. \tag{6}$$

When $q > 1$ the object has no horizon. Hence the minimum size of the BH shadow is equal to 4 $M$. As one can conclude from 6 for $q > 9/8$ a photon sphere is absent.

One of the possible ways to describe a spherically symmetric, static, asymptotically flat space-time in extended gravity is to use a metric generalising Schwarzschild space-time (1).

In the symmetric case $A(r) = -B(r)^{-1}$. Of course, in general $A(r) \neq -B(r)^{-1}$. In the symmetric case corrections originating from extended theories of gravity can be represented as additional terms of the Taylor series expansion continuing Schwarz-

schild metrics. Therefore the modelling of the shadow in such model includes the equations of photons motion around the BH. So,

$$u(r) = \left(\frac{d\hat{r}}{d\phi}\right)^2 = \frac{\hat{r}^4}{D^2 A(\hat{r}) B(\hat{r})} - \frac{\hat{r}^2}{B(\hat{r})}, \tag{7}$$

where $D = L/E$ is the aiming parameter of the photon beam. The edge of the shadow corresponds to the transition of light particles to an unstable orbit. The description of this transition is as follows:

$$u(r) = 0, \quad \frac{du(r)}{dr} = 0, \quad \frac{d^2u(r)}{d^2r} > 0. \tag{8}$$

The size of the shadow corresponds to the maximal solution of Eqs (8). We proceed this solution numerically. In [28, 29] we considered such a problem both for symmetric and asymmetric metrics. The inclusion of $1/r^{-3}$ correction allows us to describe a wider range of shadows' sizes including the ones less than 4 $M$. Configurations with a horizon are still possible, but without a photonic sphere. It appears that there is a BH without a shadow.

A model for a non-rotating BH shadow is not enough to test the metrics that are more complicated than the Reissner–Nordstrom one. Many theories are able to describe the shadow with the same size. To determine the complete set of theory parameters additional tests of the BH potential are required, for example, strong gravitational lensing of bright objects around the BH, the last stable orbit of the accretion disk, the distribution of background intensity from ionised plasma around the BH, ... Earlier [28, 29] we estimated the accuracy required to constrain extended theories in a BH non-rotating case. For strong gravitational lensing this accuracy can be about $10^3$ times less than the angular size of the BH is required. The background intensity also requires an accuracy of $\approx 10^3$ times less than its maximum value.

## 5 Shadows from rotating black holes

One of the possible ways to include rotation in a maximally simple manner is to apply the Newman–Janis algorithm 1. Its key idea is the substitution $m(r) \to M$ [30]. The term $m(r)$ allows us to incorporate the various theories of gravity. Generically for corrections represented as terms of the Taylor series the procedure is the following:

$$m(r) = M - \frac{q}{2r} - \frac{C_3}{2r^2} - ... - \frac{C_n}{2r^{n-1}} - .... \tag{9}$$

The coordinates of the shadow edge $[\alpha, \beta]$ on the image plane are:

$$\begin{gathered} \alpha = \frac{\xi_-}{\sin\theta_i}, \\ \beta = \pm\sqrt{\eta_- + (a - \xi_-)^2 - \left(a\sin\theta_i - \frac{\xi_-}{\sin\theta_i}\right)^2}, \end{gathered} \tag{10}$$

where $\theta_i$ is the angle of inclination of the BH rotation axis,

$$\begin{aligned} \xi_- &= \frac{4r_0^2\xi_A - (r_0^2+a^2)\xi_B}{a\xi_C}, \\ \eta_- &= \frac{r_0^3[\eta_A a^2 - r_0\eta_B^2]}{a^2\eta_C^2}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} \xi_A &= M - \frac{q}{2r_0} - \frac{C_3}{2r_0^2} - \ldots - \frac{C_n}{2r_0^{n-1}} - \ldots, \\ \xi_B &= r_0 + M + \frac{C_3}{2r_0^2} + \ldots + \frac{(n-2)C_n}{2r_0^{n-1}} + \ldots, \\ \xi_C &= r_0 - M - \frac{C_3}{2r_0^2} - \ldots - \frac{(n-2)C_n}{2r_0^{n-1}} - \ldots, \\ \eta_A &= 4M - \frac{4q}{r_0} - \frac{6C_3}{r_0^2} - \ldots - \frac{2nC_n}{r_0^{n-1}} - \ldots, \\ \eta_B &= r_0 - 3M + \frac{2q}{r_0} + \frac{5C_3}{2r_0^2} + \ldots \\ &\quad + \frac{(n+2)C_n}{2r_0^{n-1}} + \ldots, \\ \eta_C &= r_0 - M - \frac{C_3}{2r_0^2} - \ldots - \frac{(n-2)C_n}{2r_0^{n-1}} - \ldots. \end{aligned}$$

In [31] the accuracy was estimated for the corrections in the form of the Taylor expansion. To proceed with this we compared the shapes of BH shadows with the same size. For the series up to $r^{-3}$, strong rotation with $a = 0.9$ and the values of $q$ and $C_3$ comparable with $M$ (for example, $q = 0.17$, $C_3 = -0.5$), the deviation of the shape from the Kerr BH reaches 2% of the shadow size. To constrain a rapidly rotating BH, an order of magnitude lower accuracy is required, about one hundredth of the size of the shadow.

## 6 Conclusions

The resolution of the first BH shadow images was quite low: about half of the object size [3]. A further improvement of observation techniques from the Earth's surface could increase the resolution only severalfold [32]. Thus, the limit for the ground-based radio telescopes has been almost reached. In order to be able to test the extended theories of gravity a resolution of several orders of magnitude higher is required [33, 31]. Hence it is necessary to think about using spacecraft observations.

Further measuring of the size of the shadow is enough only for the theories with metrics like the Reissner–Nordstrom one without additional Taylor series. To be able to cross this boundary the number of observational types must be increased. It is

important that using the last stable orbit, strong gravitational lensing of bright stars [33], the distribution of the shadow background intensity the required resolution can be about 0.001 of the shadow size for corrections comparable with the BH mass value. On the contrary, the study of rapidly rotating BH shadow looks more promising. For the same values of the amendments the required accuracy could be 0.01 of the BH shadow size [31].

Models without a rotation are applicable only in the case of small spin value. In this case, it becomes possible to neglect the rotation. Usually this is not the case and rotation must be included. It is very important that inclusion of a rotation (especially a strong one) increases by itself the number of different BH potential probes. Therefore the required resolution can be less by an order. To reduce the resolution requirements for telescopes for testing of extended theories of gravity we need the development of new methods including the rotation in a more simple way. Asymmetric corrections in the form of a series could make it possible to model new configurations of a BH shadow image.

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## References

[1] Jerome A. Orosz, Jeffrey E. McClintock, Jason P. Aufdenberg, Ronald A. Remillard, Mark J. Reid, Ramesh Narayan, and Lijun Gou. The Mass of the Black Hole in Cygnus X-1. *Astrophys. J.*, 742:84, 2011.

[2] B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. *Phys. Rev. Lett.*, 116(6):061102, 2016.

[3] Kazunori Akiyama et al. First M87 Event Horizon Telescope Results. V. Physical Origin of the Asymmetric Ring. *Astrophys. J.*, 875(1):L5, 2019.

[4] Salvatore Capozziello and Mariafelicia De Laurentis. Extended Theories of Gravity. *Phys. Rept.*, 509:167–321, 2011.

[5] Thomas P. Sotiriou and Valerio Faraoni. f(R) Theories of Gravity. *Rev. Mod. Phys.*, 82:451–497, 2010.

[6] Antonio De Felice and Shinji Tsujikawa. f(R) theories. *Living Rev. Rel.*, 13:3, 2010.

[7] Christos Charmousis, Edmund J. Copeland, Antonio Padilla, and Paul M. Saffin. General second order scalar-tensor theory, self tuning, and the Fab Four. *Phys. Rev. Lett.*, 108:051101, 2012.

[8] Eugeny Babichev, Christos Charmousis, and Antoine Lehébel. Asymptotically flat black holes in Horndeski theory and beyond. *JCAP*, 04:027, 2017.

[9] Christian Pfeifer and Sebastian Schuster. Static spherically symmetric black holes in weak f(T)-gravity. *Universe*, 7(5):153, 2021.

[10] Yun Soo Myung and De-Cheng Zou. Black holes in new massive conformal gravity. *Physical Review D*, 100(6), Sep 2019.

[11] Tommaso De Lorenzo, Costantino Pacilio, Carlo Rovelli, and Simone Speziale. On the effective metric of a planck star. *General Relativity and Gravitation*, 47(4), Mar 2015.

[12] Jian-Ping Hu, Li-Li Shi, Yu Zhang, and Peng-Fei Duan. Analytical time-like geodesics in modified hayward black hole space-time. *Astrophysics and Space Science*, 363(10), Sep 2018.

[13] Haroldo C. D. Lima Junior, Luís C. B. Crispino, Pedro V. P. Cunha, and Carlos A. R. Herdeiro. Mistaken identity: can different black holes cast the same shadow?, 2021.

[14] A. Barrau, Julien Grain, and S. O. Alexeyev. Gauss-Bonnet black holes at the LHC: Beyond the dimensionality of space. *Phys. Lett. B*, 584:114, 2004.

[15] E. T. Newman and A. I. Janis. Note on the Kerr Spinning-Particle Metric. *Journal of Mathematical Physics*, 6:915–917, June 1965.

[16] K. A. Postnov and A. M. Cherepashchuk. Masses of stellar black holes and testing theories of gravitation. *Astron. Rep.*, 80:1075–1085, 2003.

[17] R. Abuter, A. Amorim, M. Bauböck, J. P. Berger, H. Bonnet, W. Brandner, Y. Clénet, V. Coudé du Foresto, P. T. de Zeeuw, et al. A geometric distance measurement to the galactic center black hole with 0.3% uncertainty. *Astronomy & Astrophysics*, 625:L10, May 2019.

[18] B.P. Abbott, R. Abbott, T.D. Abbott, M.R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R.X. Adhikari, et al. Observation of gravitational waves from a binary black hole merger. *Physical Review Letters*, 116(6), Feb 2016.

[19] R. Abbott, T.D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, R.X. Adhikari, V.B. Adya, C. Affeldt, M. Agathos, et al. Gw190521: A binary black hole merger with a total mass of 150 $M_{\rm sun}$. *Physical Review Letters*, 125(10), Sep 2020.

[20] R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, A. Adams, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, et al. Observation of gravitational waves from two neutron star–black hole coalescences. *The Astrophysical Journal Letters*, 915(1):L5, Jun 2021.

[21] S. Chandrasekhar. Mathematical theory of black holes.

[22] Prashant Kocherlakota, Luciano Rezzolla, Heino Falcke, Christian M. Fromm, Michael Kramer, Yosuke Mizuno, Antonios Nathanail, Héctor Olivares, Ziri Younsi, Kazunori Akiyama, et al. Constraints on black-hole charges with the 2017 eht observations of M87*. *Physical Review D*, 103(10), May 2021.

[23] Kazunori Akiyama et al. First M87 Event Horizon Telescope Results. VII. Polarization of the Ring. *Astrophys. J. Lett.*, 910(1):L12, 2021.

[24] Alexander F. Zakharov. Constraints on a charge in the Reissner–Nordstrom metric for the black hole at the Galactic Center. *Phys. Rev.*, D90(6):062007, 2014.

[25] Alexander F. Zakharov. Constraints on (tidal) charge of the supermassive black hole at the Galactic Center with trajectories of bright stars. 2018.

[26] Naresh Dadhich, Roy Maartens, Philippos Papadopoulos, and Vahid Rezania. Black holes on the brane. *Phys. Lett.*, B487:1–6, 2000.

[27] S. O. Alexeyev and D. A. Starodubtseva. Black holes in models with noncompact extra dimensions. *J. Exp. Theor. Phys.*, 111:576–581, 2010.

[28] S. O. Alexeyev, A. N. Petrov, and B. N. Latosh. Maeda–Dadhich Solutions as Real Black Holes. *Phys. Rev.*, D92(10):104046, 2015.

[29] V. Prokopov, S. Alexeyev, and O. Zenin. Black hole shadows: How to fix the extended gravity theory, 2021.

[30] Naoki Tsukamoto. Black hole shadow in an asymptotically flat, stationary, and axisymmetric spacetime: The Kerr–Newman and rotating regular black holes. *Physical Review D*, 97(6), Mar 2018.

[31] S.O. Alexeyev and V.A. Prokopov. Shadows from spinning black holes in extended gravity. *J. Exp. Theor. Phys.*, 130(5):666–670, 2020.

[32] Prof. Dr. Heino Falcke private talk.

[33] S.O. Alexeyev, B.N. Latosh, V.A. Prokopov, and E.D. Emtsova. Phenomenological Extension for Tidal Charge Black Hole. *J. Exp. Theor. Phys.*, 128(5):720–726, 2019.