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Scooping helium out in the early B-type main sequence stars

E.I. Staritsin*

K.A. Barchatova Kourovka Astronomical Observatory, B.N. Yeltsin Ural Federal University, 51 Lenin Avenue, Ekaterinburg, 620083, Russia

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The observed process of surface helium enrichment through the main sequence phase of B-stars evolution is analysed on the basis of current hydrodynamical model of matter mixing in rotating star interior. It is shown that shear turbulence and semiconvection are the main processes of partial matter mixing in the interiors of a rotating star with a mass of 8 $M_\odot$. The calculated surface helium enhancement during the second half of main sequence stage of evolution looks like an observable one.

Keywords: Stars: structure and evolution – Stars: early-type

1 Introduction

The early B–type main sequence (MS) stars show the process of surface helium abundance enrichment through the MS phase of evolution. The average value of the ratio $N_{\rm He}/N_{\rm H}$ enhancement towards the end of MS phase runs to 26±10% for stars with masses $4 < M/M_\odot < 12$ according to Lyubimkov et al. (2004) and it runs to 23±13% for stars with masses $8 < M/M_\odot < 16$ according to Huang and Gies (2006). The helium formation takes place in the star core due to hydrogen nuclear burning. The enhancement of surface helium abundance is a source of information that some mixing processes act in star interior. The purpose of this article is to study the ability of the current hydrodynamical model of matter mixing in star interior for the interpretation of the surface helium enrichment during star evolution at MS phase.

2 Hydrodynamical model of matter mixing in star interior

The current hydrodynamical model of matter mixing in rotating star interior includes the following processes: turbulent entrainment, meridional circulation, shear turbulence and semiconvection. The model of matter mixing describes the transport of chemicals and angular momentum in the vertical direction in the rotating star interior. An angular velocity $\Omega(r)$ has an approximately constant value on a surface of

*Email: Eugenij.Staritsin@usu.ru

constant pressure due to high anisotropy of shear turbulence in star interior. The angular velocity $\Omega(r)$ depends on a distance to a constant-pressure surface. This is a case of shellular stellar rotation (Zahn 1975; Zahn 1992).

2.1 Turbulent entrainment

The MS stars with a mass of $M > 1.5\ M_\odot$ consist of a convective core and a radiative envelope. Thermal convection in the central part of a star is a reason for turbulence. The convective turbulence region is surrounded by a stable layer. Over time, the upper turbulent layer "diffuses" into the stable layer, thus increasing the size of the mixed region. An interface between turbulent region and the stable one migrates at speed V_e determined by the turbulent entrainment law:

$$V_\mathrm{e}/V_\mathrm{t} = A \cdot Ri_\mathrm{B}^{-n}, \tag{1}$$

V_t is the turbulence velocity adjacent to the interface. V_t is a measure of strength of the turbulence. n and A are parameters of the entrainment law. Ri_B is a bulk Richardson number:

$$Ri_\mathrm{B} = \frac{\triangle b \cdot \ell}{V_\mathrm{t}^2},$$

ℓ is a length scale for the turbulent motions. $\triangle b$ is the buoyancy jump across the interface:

$$\triangle b = \int_h N^2 dr,$$

$h \sim V_\mathrm{c}/N$, V_c is a convective velocity, N is a local Brunt-Vaisala frequency. $\triangle b$ is a measure of "stiffness" of the boundary.

The following values of parameter $n : 1.05 \pm 0.21$ are determined due to three-dimensional numerical simulation of hydrogen core burning in a 23 $M_\odot$ stellar model (Meakin and Arnett 2007). The model for V_t determination using only a one-dimensional model of a star was recently developed by Meakin and Arnett (2010). Using that

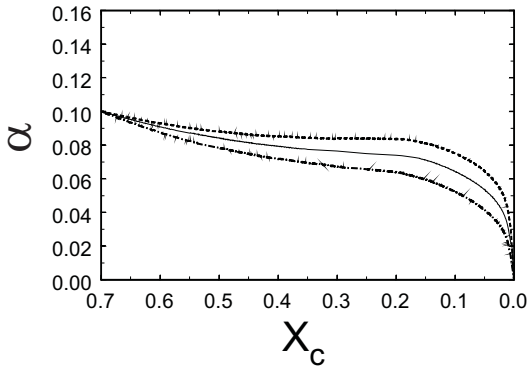


Figure 1 Distance α between the boundaries of a convective unstable region and of a convective turbulent one in a star with mass 24 $M_\odot$ versus hydrogen abundance in the convective turbulence region, X_c, for various values of parameter n: 1.0 (solid line), 0.85 (dashed line), and 1.2 (dash-dotted line).

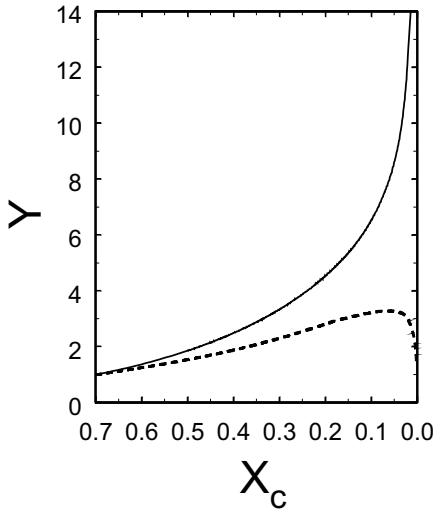


Figure 2 Physical parameters in the convective turbulence region of a star with mass 24 $M_\odot$ as a function of hydrogen abundance in the convective turbulence region X_c: the rate of buoyancy jump creation (solid line) and the cubed turbulence velocity (dashed line). All functions are normalized on zero-age values.

model and values of parameter n, distance d between the boundaries of a convective unstable region and of a convective turbulent one can be determined:

$$d = V_e \cdot \triangle t, \tag{2}$$

$\triangle t$ is time step in numerical simulation of a stellar evolution.

The distance d expressed in the pressure scale height H_P, $\alpha = d/H_P$, diminishes through the MS phase of evolution (Fig. 1). The reason is simple. The rate of nuclear burning increases due to increase of a star luminosity. As a result the turbulent velocity and the rate of buoyancy jump creation increase. But increase of the first one is less than that of the second one (Fig. 2). So V_e becomes slower and the distance diminishes (Staritsin 2013).

2.2 Meridional circulation and shear turbulence

These two processes act in the layers of star situated above the convective turbulence region. The transport of angular momentum by meridional circulation and shear turbulence in the vertical direction is described by the equation (Tassoul 1979)

$$\frac{\partial(\rho\varpi^2\Omega)}{\partial t} + \mathrm{div}(\rho\varpi^2\Omega\mathbf{u}) = \mathrm{div}(\rho\nu_V\varpi^2\mathrm{grad}\Omega), \tag{3}$$

where ρ is the density, ϖ is the distance to the rotation axis, t is the time, $\mathbf{u}$ is the meridional circulation velocity, and ν_V is the turbulent viscosity in the vertical direction. Solving equation (3), we can determine the angular velocity distribution.

The vertical component of the meridional circulation velocity in the case of shellular stellar rotation can be represented as (Zhan 1992)

$$U_r(r,\theta) = U(r)\mathrm{P}_2(\theta),$$

where $U(r)$ is the amplitude of the vertical component of the meridional circulation velocity; and $\mathrm{P}_2(\theta)$ is the second-degree Legendre polynomial. The amplitude $U(r)$ can be determined from the steady-state energy equation

$$\rho T \mathbf{u}\,\mathrm{grad}\, s = \rho\varepsilon_n + \mathrm{div}(\chi\,\mathrm{grad}\, T) - \mathrm{div}\mathbf{F}_h, \tag{4}$$

where T is temperature, s is specific entropy, ε_n is the rate of nuclear energy release, χ is thermal conductivity, $\mathbf{F}_\mathrm{h}$ is the flux of thermal energy due to horizontal turbulence: $\mathbf{F}_\mathrm{h} = -\nu_\mathrm{h}\rho T\partial s/\partial \mathbf{i}_\theta$, and ν_h is the turbulent viscosity in the horizontal direction (i.e., along the surface of constant pressure).

2.3 Semiconvection

One mechanism for the transport of chemical elements through a layer with a variable chemical composition is called semiconvection (Schwarzschild and Harm 1958). A linear analysis of the stability of a gaseous medium with radiation (Kato 1966) showed that when

$$\nabla_a \le \nabla \le \nabla_a + \frac{\beta}{4-3\beta}\nabla_\mu,$$

an oscillatory instability develops in a layer with a variable chemical composition. Here ∇ is the temperature gradient, ∇_a is the adiabatic temperature gradient, ∇_μ is the gradient in mean molecular weight, and β is the ratio of the gas pressure to the total pressure. The oscillatory instability is a reason of partial mixing of matter in that layer. The quantitative picture of such mixing has not been studied in detail so far. So a state of neutrality relative to semiconvection is believed to be established as a result of such mixing. Two criteria for neutrality in the semiconvective zone were put forward: the S criterion (Schwarzschild and Harm 1958)

$$\nabla = \nabla_a, \tag{5}$$

and, as was suggested by Sakashita and Hayashi (1961), the L criterion (Ledoux 1958)

$$\nabla = \nabla_a + \frac{\beta}{4-3\beta}\nabla_\mu.$$

Semiconvection is not efficient in momenta transport in comparison with meridional circulation (Staritsin 2009).

2.4 Chemicals transport

The hydrogen mass transport is described by the diffusion equation (Chaboyer and Zahn 1992)

$$\frac{\partial(\rho X)}{\partial t} = \mathrm{div}(\rho\, D\,\mathrm{grad}\, X), \tag{6}$$

where X is the hydrogen mass fraction; the diffusivity D is determined by the sum: $D = D_{\rm V} + D_{\rm sc}$, $D_{\rm V}$ is the turbulent diffusivity in the vertical direction and $D_{\rm sc}$ is the diffusivity due to semiconvection. The turbulent viscosity $\nu_{\rm t}$ and diffusivity $D_{\rm t}$ differ in magnitude in different flows and can change within the flow by several orders of magnitude, but their ratio – the turbulent Schmidt number $Sc_{\rm t}$,

$$Sc_{\rm t} = \nu_{\rm t}/D_{\rm t} \tag{7}$$

changes only slightly both within the flow under consideration and from flow to flow. Meridional circulation is not efficient in the transport of chemicals in the vertical direction in comparison with shear turbulence (Staritsin 2009).

2.5 Turbulent viscosity

The turbulent-viscosity coefficient in the vertical direction is determined by equating the work done by the gravitational force when two arbitrary fluid elements exchange places and the kinetic-energy excesses in the flow. The destabilizing role of the exchange of energy between the element and surrounding matter due to the reradiation of photons and the horizontal transport of chemical elements due to the anisotropy of the turbulence is taken into account. The vertical turbulent-viscosity coefficient is (Talon and Zahn 1997)

$$\nu_{\rm V} = \frac{2Ri_{\rm c}}{N_T^2/(K+D_{\rm h}) + N_\mu^2/D_{\rm h}} \left(\frac{d(\varpi\Omega)}{dr} \right)^2, \tag{8}$$

where $K = (4acT^3)/(3c_P\rho^2\kappa)$, $N_T^2 = (g\delta/H_p)(\nabla_a - \nabla)$, $N_\mu^2 = (g/H_p)\nabla_\mu$, $N^2 = N_T^2 + N_\mu^2$ is the square of the local Brunt-Vaisala frequency, $Ri_{\rm c}$ is the critical value of a local Richardson number, $D_{\rm h}$ is the turbulent diffusivity in the horizontal direction, a is the Stefan constant, c is the speed of light, c_P is the specific heat at constant pressure, κ is the opacity, g is the mean gravity at the surface of constant pressure, $\delta = (4-3\beta)/\beta$.

There are some expressions for the turbulent viscosity in the horizontal direction (Maeder 2003; Mathis, Palacios and Zahn 2004). All of them include some free parameters. In the current work we use the simplest expression (Zahn 1992):

$$\nu_{\rm h} = C_z r \left| U(r) \right|, \tag{9}$$

where C_z is a parameter. The numerical value of this parameter is chosen so that the criterion for applicability of equation (9)

$$\nu_{\rm V} \ll \nu_{\rm h} \ll K$$

is satisfied.

3 Example of 8 $M_\odot$ star evolution

The MS evolution of a rotating star with a mass of 8 $M_\odot$ is calculated using Paczynski's program (Paczynski 1970) modified in order to take into account the effect of

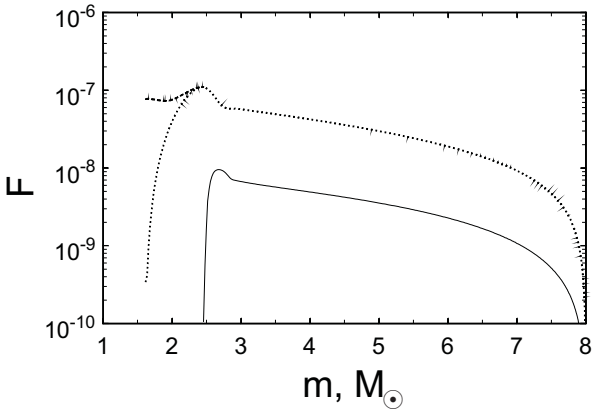


Figure 3 Turbulent flux of hydrogen mass F (in $M_\odot$/year) in the interiors of a star with mass 8 $M_\odot$ when the hydrogen abundance in the convective turbulence region is $X_c = 0.5$ (solid line) and $X_c = 0.1$ (dotted line) and total semiconvective and turbulent flux of hydrogen mass when the hydrogen abundance in the convective turbulence region is $X_c = 0.1$ (dashed line).

hydrodynamic processes and rotation on the stellar structure (Staritsin 1999, 2009, 2013). The initial value of the equatorial rotation velocity is set at 250 km/s. That value of rotation velocity is typical of young stars of similar masses. The stellar structure equations are solved simultaneously with equations (1–9) for the case of the following free parameters choice: $A = 1.11 \cdot 10^{-3}$, $C_z = 200$, $Sc_t = 0.1$.

The ratio of the angular velocities for the convective core and the stellar surface in young massive stars is limited both below and above (Staritsin 2005, 2007). Inside these limits, the time scale of the angular momentum transport by meridional circulation and shear turbulence is comparable to the nuclear one. Outside these limits, the absolute value of the meridional circulation velocity is high and the time scale of the angular momentum transport is short as compared to the nuclear one. The initial model of the evolutionary sequence was obtained by simultaneously solving the stellar structure equations and equations (3, 4, 8, 9), starting from rigid rotation and keeping hydrogen distribution unchanged. A chemically homogeneous model with the ratio of the angular velocities for the convective core and the stellar surface $\Omega_c/\Omega_s = 1.23$ is taken as the initial model of the evolutionary sequence. The time scale of the angular momentum transport in the initial model is the same as the nuclear one.

During the first half of MS evolution the shear turbulence is the single process of chemicals transport in vertical direction in the layers above convective turbulence region. The flux of hydrogen mass due to shear turbulence decreases drastically in the layers with variable chemical composition situated between the convective turbulence region and the initial chemically homogeneous envelope of star (Fig. 3). Shear turbulence transports the hydrogen mass from the outer parts of the star into the layer with variable chemical composition. As a result the condition for semiconvection appears in that layer. The flux of hydrogen mass due to semiconvection is much larger than that of shear turbulence in the layer situated near the convective turbulent region (Fig. 3). Semiconvection transports hydrogen from the layer with variable chemical

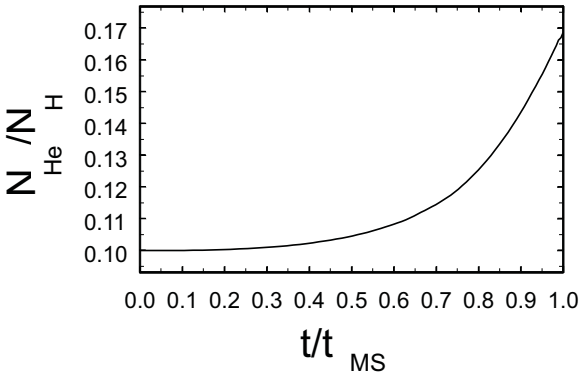


Figure 4 Variations in calculated helium enhancement with relative time $t/t_{\rm MS}$ for a star with mass 8 $M_{\odot}$.

composition into the convective turbulent region during the second half of MS evolution. That is why hydrogen is transported from the turbulent envelope through the layer with variable chemical composition to the convective turbulent region. Helium is transported in the opposite direction from the convective turbulent region to the turbulent envelope.

The calculated $N_{\rm He}/N_{\rm H}$ enhancement through the MS phase is shown in Fig. 4. The average of the calculated $N_{\rm He}/N_{\rm H}$ enhancement for the second half of MS evolution looks like the observable one.

Conclusion

The current hydrodynamical model of matter mixing in star interior is able to explain the surface helium enrichment observed in the early B-type main sequence stars.

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