Astronomical and Astrophysical Transactions (*AApTr*), 2014, Vol. 28, Issue 3, pp. 265–274,
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The probability of discovering close binary stars as eclipsing variables in works by M.A. Svechnikov

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Received 15 May, 2013

A method of estimating the probabilities of discovering close binary stars as eclipsing variables in Svechnikov's works is described. The dependences of the probabilities on the mass of the primary component, M_1, mass ratio q, semimajor axis A of the orbit and angle i of orbit inclination are presented.

Keywords: Stars: binaries: close binaries: probabilities of discovery

1 Introduction

Close binary stars (CBS) can be observed as double-lined spectroscopic binaries and eclipsing variables. The fundamental stellar characteristics (masses, radii, luminosity) are precisely measured for these stars. However observational data are too distorted by observational selection effects and do not show the real number of these systems in space. Estimating the probability of discovering CBS as eclipsing variables and taking the effects of observational selection into account are important problems for statistical studies of binary stars.

2 Geometric and photometric probabilities

There are geometric and photometric probabilities. The geometric probability is related to the fact that only at a certain angle of the orbit of a binary star in relation to the observer, it might be assumed to be an eclipsing variable. The geometric probability can be estimated as follows. Let an observer be on the surface of a sphere of a very large radius and its center be at point O. Eclipses will be visible to the observer on the surface of a spherical belt inscribed on the sphere during the rotation of family cones with an aperture angle φ (Fig. 1). Then the geometric probability

$$W_{\rm g} = \frac{S_{\rm ball\,belt}}{S_{\rm sphere}} = \sin\varphi \quad \text{or} \quad W_{\rm g} = \frac{\delta}{A} = \frac{R_1 + R_2}{A}, \tag{1}$$

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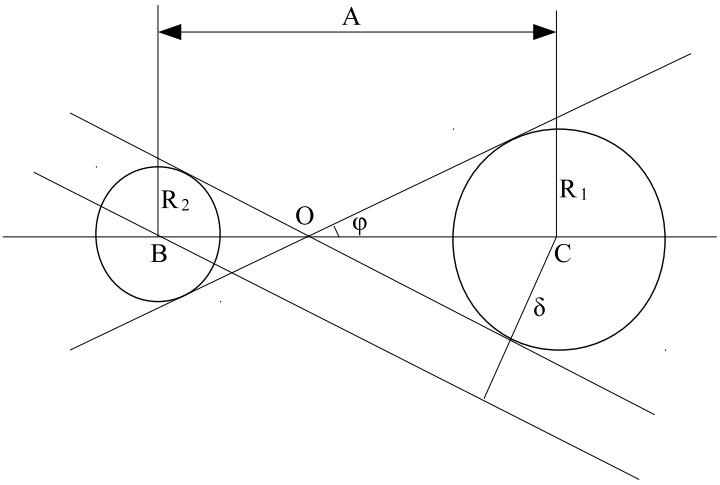


Figure 1 Geometric probability.

where δ is the relative distance between the projections of the centers of the stars on the chart plane. In this way, the geometric probability was defined by Popov (1970), Kraicheva et al. (1981), Farinella and Paolicchi (1978), Giuricin et al. (1983).

Photographic observations promoted mass discoveries and studies of variable stars. The photometric probability is the probability of discovering a variability star by comparing a pair of photographic plates, using one of the methods of detection. According to Kurochkin (1971), it can be represented as:

$$W_{\rm ph}\,(m, \Delta m) = \int\limits_0^L F\,(\Delta m)\, G\,(\Delta m)\; d\,(\Delta m), \tag{2}$$

where L is the amplitude of brightness variations; $G\,(\Delta m)$ is the theoretical probability (calculated for a given light curve); $F\,(\Delta m)$ is the empirical probability (calculated directly from the search results of eclipsing variable stars and depending on the detection method of a variable star, observer's attentiveness and other subjective factors).

According to Plaut (1966), for relatively bright stars ($m \leq 15^m$) the empirical probability can be expressed as:

$$F\,(\Delta m) = \begin{cases} 0, & \Delta m < 0^m.25; \\ -0.04 + 0.5\Delta m, & 0^m.25 \leq \Delta m \leq 1^m.7; \\ \approx 0.83, & \Delta m > 1^m.7. \end{cases} \tag{3}$$

The theoretical probability $G\,(\Delta m)$ is the probability of the fact that the difference in star brightness $\Delta m = m\,(\theta_2) - m\,(\theta_1)$ at time moments t_1 and t_2 will be greater than the detection threshold h ($h = 0^m.25$ was adopted for photographic observations). It can be calculated if the light curve of the star is given. Kviz suggested a smart geometric method of calculating the probability according to a given light curve (Kviz 1956). We will mark the first phase of observation θ_1 at the horizontal axis, the second phase of observation θ_2 at the vertical axis (Fig. 2). Also mark points m–h

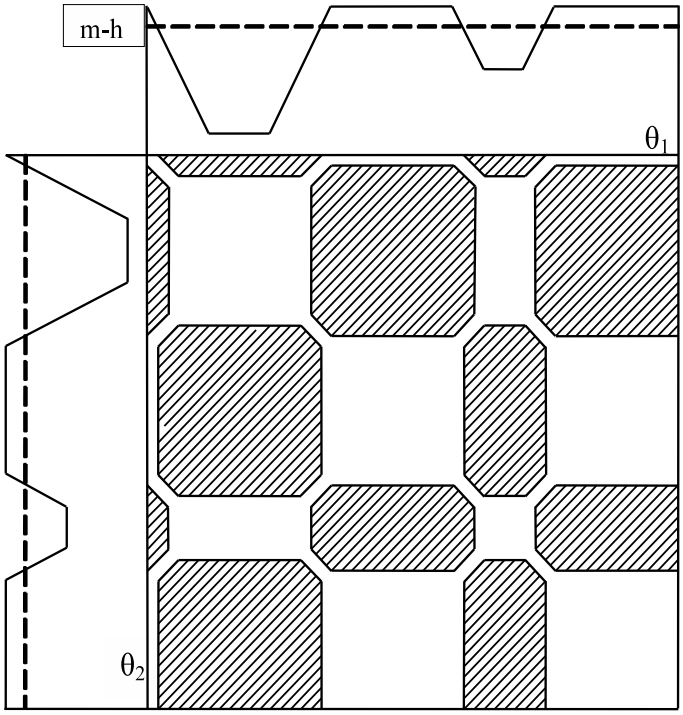


Figure 2 Kviz square.

at the light curves, draw straight lines through them, cutting off the area where the detection of variability will be impossible (in this case the second observation localizes the interval m–h). Therefore, it is easy to highlight areas where the difference of the light $m_2 - m_1 \leq h$ (not shaded) by the geometric construction. The ratio of the shaded area to the area of the square determines the theoretical probability of detection.

In this way, the photometric probability was calculated in the paper by Popov (1970) for detached and semidetached binaries. But in the article it was assumed that if $\Delta m > 0^m.3$ then the probability of discovery was equal to 1, and if $\Delta m < 0^m.3$ then it was equal to 0, i.e., subjective factors were not considered.

3 Probabilities of discovering close binary systems in Svechnikov's works

For the first time the overall probability of discovering close binaries of various types as eclipsing variables as the result of massive photographic observations was estimated in works by Svechnikov et al. (1989), Svechnikov and Eretnova (1991), Eretnova and Svechnikov (1991, 1995) as a function of the primary mass M_1, mass ratio q, semimajor axis A of the orbit and angle i of orbit inclination. The case of uniformly bright stellar disks was examined (hypothesis "U"). The overall probability W was presented as the product of geometric and photometric probabilities: $W = W_{\rm g} \cdot W_{\rm ph}$.

We assumed that if the depth of the eclipse would be less than the detection threshold h, we would not be able to detect the variability of binary stars, even when the system is observed on one plate away from the eclipse and on the other in the middle of the primary minimum. The detection threshold was taken as $\Delta m_{\min} = 0^m.25$. Then the geometric probability can be written as

$$W_{\rm g} = \frac{\delta_{\rm lim}}{A} = \frac{R_1 + p_{\rm lim} R_2}{A}, \tag{4}$$

where $\delta_{\rm lim}$ is the limiting value for the distance between the projections of the centers of the components on the chart plane for which the depth of the primary minimum will be $\Delta m_{\min}$; $p_{\rm lim}$ is the corresponding geometric depth of the eclipse.

Then we determined $W_{\rm g}$ as a function of M_1, q, A and i. Let us consider an example of detached close binaries both components of which are main sequence stars (DMS type according to Svechnikov's classification (Svechnikov 1969)). Since the empirical relation between mass and radius for main sequence stars has the form (Svechnikov and Taidakova 1985):

$$\lg M = \begin{cases} 1.56 \lg R - 0.153, & M > 1.4\, M_\odot; \\ 0.98 \lg R - 0.038, & M \le 1.4\, M_\odot, \end{cases} \tag{5}$$

we have for the geometric probability:

$$W_{\rm g} = \begin{cases} 1.253\, M_1^{0.641} \left(1 + q^{0.641} p_{\rm lim}\right)/A, & M_1 \ge M_2 \ge 1.4\, M_\odot; \\ \left(1.253\, M_1^{0.641} + 1.093 p_{\rm lim}\, M_2^{1.0205}\right)/A, & M_1 \ge 1.4\, M_\odot > M_2; \\ 1.093\, M_1^{1.0205} \left(1 + q^{1.0205} p_{\rm lim}\right)/A, & 1.4\, M_\odot > M_1 \ge M_2. \end{cases} \tag{6}$$

The geometric depth of the eclipse, $p_{\rm lim}$, is a function of the ratio of the radii of the components $k = \dfrac{R_2}{R_1}$ and the photometric phase α of the eclipse: $p_{\rm lim} = p_{\rm lim}\,(k, \alpha_{\rm lim})$. In DMS systems, at the primary minimum the eclipse is of $S \to L$ type (the smaller in size star eclipses the larger one star). Then the brightness L of the system is given by

$$L = 1 - \alpha k^2 L_1, \tag{7}$$

where L_1 is primary component luminosity.

Using Pogson formula $m = -2.5 \lg L + m_0$, we obtain

$$\alpha = \frac{0.2057}{k^2 L_1}. \tag{8}$$

The surface brightness ratio of components J_1 and J_2 (in assumption of the black-body radiation model) is:

$$\gamma = \frac{J_1}{J_2} = \frac{L_1}{L_2} \cdot \frac{R_2^2}{R_1^2} = \frac{\exp(c_2/\lambda T_2) - 1}{\exp(c_2/\lambda T_1) - 1}, \tag{9}$$

where T_1 and T_2 are the effective temperatures of components, λ is the observational wavelength for the star and $c_2 = \dfrac{hc}{k} = 1.43833 \dfrac{\rm cm}{\rm deg}$.

Using the empirical relation between mass and effective temperature for main sequence stars (Svechnikov and Taidakova 1985):

$$\lg M = 1.575 \lg T_{\rm eff} - 5.893, \tag{10}$$

we obtain L_1:

$$L_1 = \begin{cases} \dfrac{\gamma}{\gamma + q^{1.252}}, & M_1 \geq M_2 \geq 1.4\, M_\odot; \\ \dfrac{1.316\gamma M_1^{1.282}}{1.316\gamma M_1^{1.282} + M_2^{2.041}}, & M_1 \geq 1.4\, M_\odot > M_2; \\ \dfrac{\gamma}{\gamma + q^{2.041}}, & 1.4\, M_\odot > M_1 \geq M_2. \end{cases} \tag{11}$$

Knowing L_1 we find $\alpha_{\rm lim}$ and then (from the tables byTsesevich (Eclipsing Variable Stars ed. by Tsesevich 1971)) determine $p_{\rm lim} = p_{\rm lim}(k, \alpha_{\rm lim})$. After this, it is easy to find the geometric probability $W_{\rm g} = W_{\rm g}(M_1, q, A)$.

Our work takes into account both the theoretical and empirical probability of discovering when calculating the photometric probability. The empirical probability was determined according to Plaut [3]. The theoretical probability is calculated by the Kviz method. Thus, the photometric probability can be written as:

$$W_{\rm ph} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} F\left(\theta_1, \theta_2\right)\, d\theta_1\, d\theta_2, \tag{12}$$

where θ_1, θ_2 are the phase angles at time moments t_1 and t_2.

$W_{\rm ph}$ was estimated by the Monte Carlo method. To do this, a sample of N independent pairs of random numbers ξ_i, η_i uniformly distributed over the interval $[0, 2\pi]$ was selected. The sample size determines the accuracy of this estimation and should be fairly large. We used $N = 5000$, which resulted in a standard relative deviation of about 5%. Thus,

$$W_{\rm ph} = \frac{1}{N} \sum_{i=1}^{N} F\left(\xi_i, \eta_i\right). \tag{13}$$

Most of the eclipsing variables were discovered by comparing a small number (3–4) of pairs of photographic plates. The transition from $W_{\rm ph}(1)$(comparing a single pair of plates) to $W_{\rm ph}(4)$ (comparing four independent pairs of plates) was carried out using the formula:

$$W(n) = 1 - [1 - W(1)]^n. \tag{14}$$

Modeling of the light curve was realized according to formulas from Eclipsing Variable Stars ed. by Tsesevich (1971). The brightness loss of the system for an $S \to L$ eclipse is given by

$$1 - L = \alpha k^2 L_1 \tag{15}$$

and in the case of an $L \to S$ eclipse by

$$1 - L = \alpha L_2 = \alpha(1 - L_1). \tag{16}$$

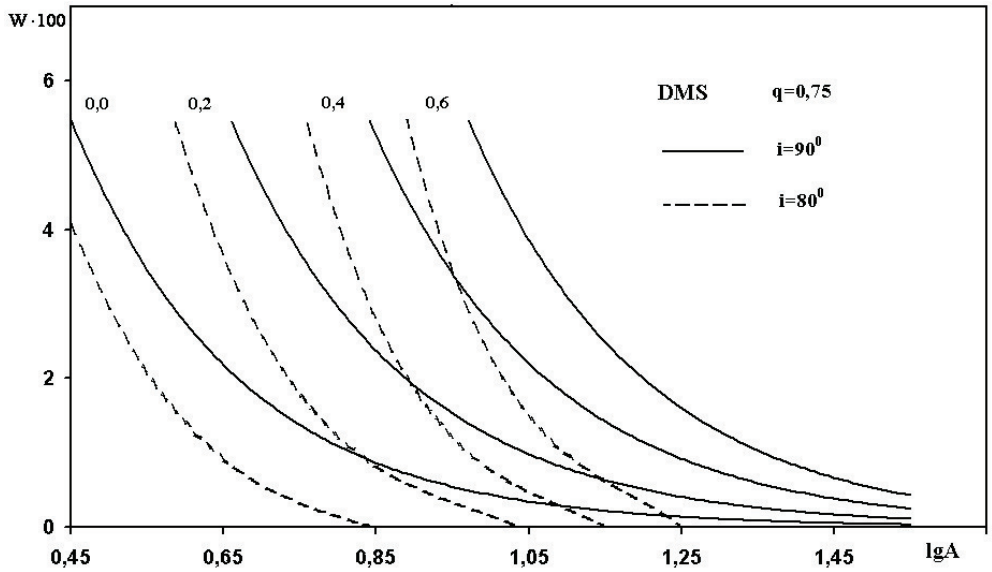


Figure 3 The probabilities W of discovering DMS systems as eclipsing variables are presented as a function of semimajor axis A. The numbers next to the curves indicate the corresponding $\lg M_1$.

The photometric phase is

$$\alpha = \frac{1}{180}\left(\frac{\varphi_1}{k^2} + \varphi_2\right) - \left(p + \frac{1}{k}\right)\frac{1}{k}\sin\frac{\varphi_1}{\pi}, \tag{17}$$

where

$$\varphi_1 = \arccos\frac{2 + 2kp + k^2p^2 - k^2}{2(1+kp)}, \quad \varphi_2 = \arccos\frac{k + 2p + kp^2}{2(1+kp)}. \tag{18}$$

The geometric depth of the eclipse is

$$p = \frac{A}{R_2}\sqrt{1 - \sin^2 i\cos^2\theta} - \frac{1}{k}. \tag{19}$$

After that, we determined $W_{\rm ph} = W_{\rm ph}(M_1, q, A, i)$ and overall probability $W = W_{\rm g} \cdot W_{\rm ph} = W(M_1, q, A, i)$.

The dependences of probabilities on these parameters for different types of close binary systems are shown in Figs. 3–6. The mass of the primary component and the semimajor axis are measured in solar units. The overall probabilities of discovering DMS systems depend only weakly on the mass ratio q (Svechnikov et al. 1989; Svechnikov and Eretnova 1991). It can be seen from Fig. 3 that as orbit inclination, i, is reduced and the semimajor axis, A, is increased, the probabilities W of DMS systems fall off rapidly. Here the dependence of W on the semimajor axis is stronger. The probabilities of discovering are equal to zero at small angles of orbit inclination ($i = 70$–$80°$) even for small A.

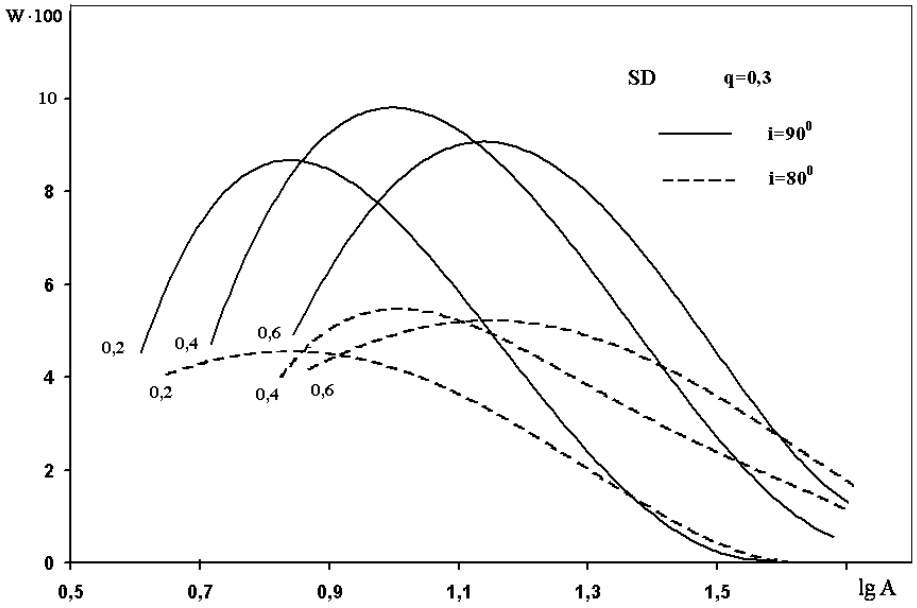


Figure 4 The probabilities W of discovering SD systems as eclipsing variables are presented as a function of semimajor axis A. The numbers next to the curves indicate the corresponding $\lg M_1$.

The probability of discovering semidetached systems (SD) first increases and then decreases while A increases (Eretnova and Svechnikov 1991) (Fig. 4). Moreover, this tendency becomes stronger by increasing i. An increase of the probability of discovering with increasing A is typical of short-periodic SD systems where the satellite subgiant is smaller than the main-sequence star. In SD systems, at the primary minimum subgiant eclipses the main sequence star. Therefore, if we reduce A in such systems, we will get a reduction of the size of inner critical surface and, consequently, a reduction of the size of subgiant at a constant size of main sequence star. Herewith the depth of the primary minimum will decrease, which will decrease the probability of discovering.

The probabilities of discovering contact close binary stars in early spectral classes (CE systems according to Svechnikov (1969)) do not depend on the semimajor axis of the orbit, because both components of these systems fill their inner critical surfaces. The probabilities depend only weakly on M_1 and q (Eretnova and Svechnikov 1995) and increase as i rises, but are essentially constant for $i \geq 80°$ (Fig. 6). It can be explained by the fact that for $i \geq 80°$ the eclipses become complete at the primary minimum and a further increase in i has no significant effect on the depth of the principal maximum and, therefore, on the probability of discovery.

The case of total limb darkening (hypothesis "D") has been examined in works by Eretnova et al. (2006, 2008) (Figs. 5, 6). The difference between hypotheses "U" (the case of uniformly bright stellar disks) and "D" is not too big. This is explained by the following: 1) to observe the eclipse the disks of stars have to overlap sufficiently so

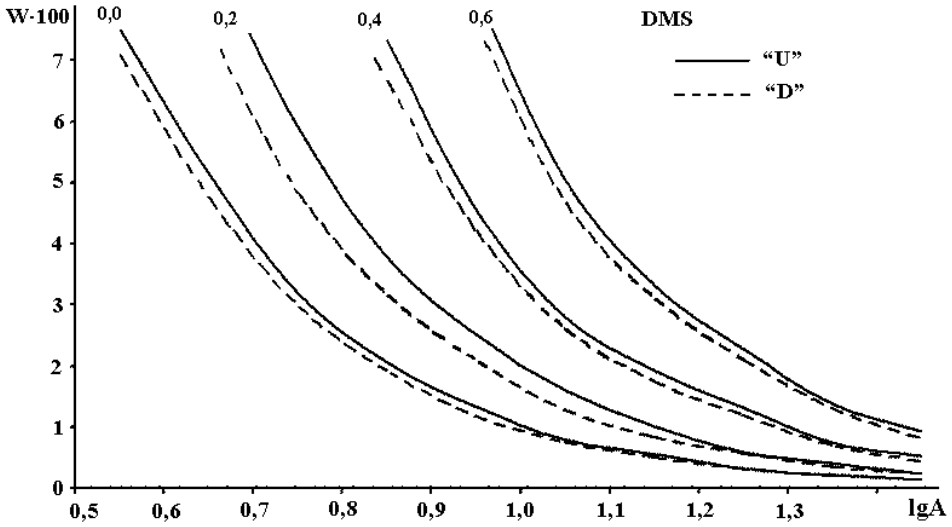


Figure 5 Comparison of probabilities W of discovering DMS systems as eclipsing variables is shown for $q = 0.95$ and $i = 90^\circ$ under hypothesis "D" (dashed curve) and hypothesis "U" (smooth curve). The numbers next to the curves indicate the corresponding $\lg M_1$.

that the difference in the brightness of components is equal to the detection threshold. Since the brightness of a star at the limb under hypothesis "D" is lower, the fraction of occulted surface required to reach the detection threshold should be greater here than under hypothesis "U". But this leads to a reduction in the geometric probability of discovery. In CE systems gravitational darkening of the disk occurs because of the effect of ellipsoidality. On the surface of a deformed star, the force of gravity is

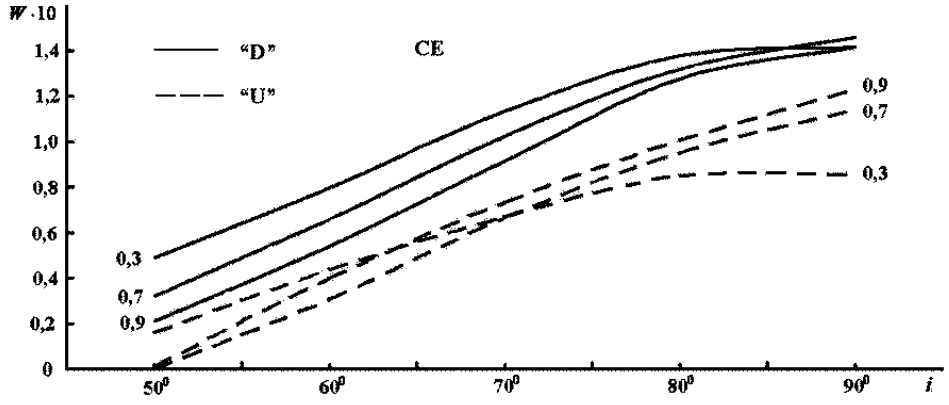


Figure 6 The comparison of probabilities W of discovering CE systems as eclipsing variables is shown for $\lg M_1 = 0.8$ under hypothesis "D" (smooth curve) and hypothesis "U" (dashed curve). The numbers next to the curves indicate the corresponding q.

greater at the poles than at the equator. Thus, at the poles the brightness may be enhanced, which leads to an increase in the geometric probability; 2) the photometric probability is determined by the shape of the light curve and the difference in the depths of the minima, which are different under hypotheses "U" and "D". In DMS and CE systems at the primary minimum the eclipse is of $S \to L$ type, therefore, after the second contact the brightness of the system continues to decrease in the case of "D" and the depth of the minimum will be greater than for the case of "U". It leads to an increase in the photometric probability of discovery.

It should also be noted that the method for estimating the probabilities under hypothesis "U" is much simpler and less time consuming.

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