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Numerical simulations of AGN evolution

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In this paper we present numerical N-body calculations of the evolution model of active galactic nuclei (AGN), consisting of compact stellar cluster, supermassive black hole and gaseous accretion disc. Besides gravitational interactions between stars and between stars and the black hole, the dissipative interaction of stars with the gaseous accretion disc and contact stellar collisions were taken into account. As numerical simulations of real stellar clusters containing more than 10^8 stars are not yet technically attainable in direct N-body simulations, we develop scaling methods that allow us to extrapolate the results of numerical simulations performed with smaller particle numbers to real stellar systems. Specifically, in this work we derive the scaling formula for stellar collisions. We then introduce a tentative scheme for stellar inelastic collisions in our numerical model to roughly estimate their effect. The preliminary result is that in simulating AGN dense stellar clusters contact collisions should be taken into account, as the mass lost during collisions, even if added to the gaseous accretion disc, has a different regime of accretion than that for intact stars. Therefore, stellar collisions might considerably reduce the rate of the black hole mass growth.

Keywords: Accretion, accretion discs – Galaxies, nuclei – Galaxies: active – Celestial mechanics – Methods: numerical

1 Introduction

The physical nature of AGN is far from being fully understood because of large distances and relatively small sizes of energy-producing regions. Therefore, the development of AGN dynamic theory still remains one of the main problems of AGN physics.

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At the end of the 20th century an important fact was established concerning the relation between the mass and velocity dispersion of stars in galactic bulges (spherical subsystems of galaxies) and the mass of the central black holes, which are present not only in AGN, but in practically all galaxies (Kormendy and Richstone 1995; Magorrian et al. 1998). Later on, many increasingly more accurate observations of the AGN central zones were made (Boker 2008; Van der Marel et al. 2007; Rossa et al. 2006) using the largest ground- and space-based telescopes. Those results confirm that the AGN phenomenon is closely related not only to the central supermassive black hole (SMBH) but also to the compact stellar cluster surrounding the SMBH, the existence of which was postulated a decade before (Vilkoviskij and Karpova 1996). The sizes of the central stellar clusters are of the same order of magnitude as those of galactic globular clusters (i.e., close to 1 pc), but the masses of CSC are much larger; usually they are about the masses of the central SMBH, $M_{\rm CSC} \sim (10^7$–$10^8)\, M_\odot$. This justifies the approach used in Vilkoviskij et al. (1996), considering AGN as a complex physical system consisting of three physically different but interacting subsystems: the central SMBH, the compact stellar cluster (CSC) and the gaseous subsystem including accretion disc (AD) and obscuring torus.

Previous AGN theories were based on various models of the gas accretion to the SMBH, taking into account different types of the accretion disc structure, magnetic field configuration, and accretion rate (Novikov and Thorne 1973; Shakura and Sunyaev 1973; Ostriker 1983). The presence of a compact stellar cluster harboring SMBH changes the problem qualitatively, since it requires analysis and calculation of dissipative interactions between stars and the accretion disc. An attempt to consider some aspects of stars interacting with the central active object and gas was made in Vilkoviskij (1983). Interaction between stars and AD were first investigated considering movements of individual stars in the gas disc, which results in a rapid decrease of the orbit's eccentricity and a diminution of the inclination angle between the orbits and the AD plane (Karas and Vokrohlicky 1993; Rauch 1999; Syer et al. 1991). Vilkoviskiy and Czerny (2002) have shown that stars in the AD will not decline their orbits to the AD plane completely, but will have some finite (fluctuating) velocity components normal to the disc due to pair interactions of the stars within the accretion disc. The semi-analytical model developed in Vilkoviskiy and Czerny (2002) shows that these interactions lead to the continuous inflow of stars to the centre of the system.

On the other hand, numerical studies of the relaxation-driven evolution of stellar systems demonstrate the enormous progress made in the past two decades. One of the important causes of the progress was a dramatic increase in the capabilities of fast special purpose (GRAPE) and then general purpose (GPU) computers, and direct N-body modeling software for them (Aarseth 1999a,b; Spurzem 1999; Harfst et al. 2007). In a large number of investigations, the evolution of globular stellar clusters has been studied and the validity of simplified theoretical models assessed, including many astrophysical processes such as stellar evolution, realistic tidal fields, primordial binaries, mergers, collisions and so on.

In Kazakhstan the first computer cluster specialized for N-body simulations was created in 2008 due to the support of the Volkswagen Foundation within the framework of the joint Kazakh–German project "STARDISK", which combined the endeavors of groups in Heidelberg University and Fessenkov Astrophysical Institute. Here we present some results of numerical investigations of AGN evolution, taking into

account the presence of the compact stellar cluster in AGN. The calculations were performed with phiGRAPE+GPU code (Harfst et al. 2007; Just et al. 2012). Its current version takes advantage of both MPI and CUDA parallelism.

2 The model of evolution of the AGN with central star cluster

While it is still impossible to calculate a one-to-one N-star model of the central stellar clusters in galactic nuclei with realistic particle numbers ($N_s \sim 10^8$), there is another possibility which permits modeling the evolution of large systems with a smaller number of model particles $N_p \ll N_s$. The main problem for the calculation of models with a large number of stars, interacting not only through gravitational forces, but also by means of dissipative star–gas friction and direct star–star inelastic collisions, is the dependence of the processes on the relation of model particles to real star numbers, N_p/N_s. But if gravitational star–star interactions still play an important role in the evolution of the whole system, one might introduce special scaling coefficients (depending on N_p/N_s) to preserve the ratio of characteristic time of a dissipative process (e.g., star–gas interaction or star–star direct collisions) to the relaxation time of the system. After that we have to study the convergence of the simulations with different N_p to choose the minimal possible N_p for a sufficiently accurate model simulation. If the characteristic time of a dissipative process is comparable to (or smaller than) the relaxation time of the stellar cluster, then that process has to be taken into account in the simulation of the CSC evolution.

To simulate the evolution of the system containing CSC, SMBH and AD, we need to have equations of motion for each element of the subsystems and the initial distribution of matter (for stars and gas). As opposed to the works which considered AD as an infinitesimally thin disc described by surface density (e.g., Rauch 1995), we employ a three-dimensional spatial distribution of the gas in AD and calculate the drag force (apart from the gravitational forces from other stars and SMBH) in each point of stellar trajectory.

Thus, our model integrates equations of motion for ith star that include interaction with all other stars, with the SMBH and with the accretion disc:

$$\vec{a}_i^* = (\vec{F}_i^{\rm CSC} + \vec{F}_i^{\rm SMBH} + \vec{F}_i^{\rm AD})/m_i, \tag{1}$$

where $\vec{a}_i^*$ is the acceleration of a star, and the forces are

$$\vec{F}_i^{\rm CSC} = -\sum_{i\neq j} \frac{Gm_i m_j \vec{r}_{ij}}{r_{ij}^3}, \vec{F}_i^{\rm SMBH} = -\frac{Gm_i M_{\rm BH}\vec{r}_i}{r_i^3}, \vec{r}_{ij} = \vec{r}_i - \vec{r}_j.$$

The drag force depends on the stellar radius r_s, gas density $\rho(\vec{r}_i)$ and the velocity of a star relative to the gaseous medium ($\vec{V}_{\rm rel}$):

$$\vec{F}_i^{\rm AD} = -Q_{\rm d}\pi r_s^2 \rho\, |V_{\rm rel}|\, \vec{V}_{\rm rel}, \tag{2}$$

where $Q_{\rm d}$ is the drag coefficient. The accretion disc model is the same as in Just et al. (2012), and it is introduced phenomenologically simply as density distribution of the following form:

$$\rho(R,z) = C\frac{M_{\rm d}}{hR_{\rm d}^3}\left(\frac{R}{R_{\rm d}}\right)^{-3/4}\exp\left[-0.7\left(\frac{R}{R_{\rm d}}\right)^4\right]\exp\left(-\frac{z^2}{2h^2R_{\rm d}^2}\right), \qquad (3)$$

where $R_{\rm d}$ is the cut-off radius of the disc, $h = h_z/R_{\rm d}$ (dimensionless thickness of the disc) and C is a constant.

The disc has Keplerian rotation curve needed to derive relative star–gas velocity for the calculation of the drag force. Since the effective mass of the disc is concentrated within the influence radius of the SMBH, we neglect the effect of inner stars on the rotation curve, which is thus defined solely by SMBH mass. While the angular velocity of a given disc circular element might change over time as SMBH mass grows due to accretion of stars, otherwise the disc is stationary. Physically such calculations might be justified for the real systems in which there is a continuous inflow of matter with a given specific momentum from the outer torus of AGN (stretched out of the AD), thus supporting mass and momentum inflow to the AD and keeping its parameters practically constant during the simulation. The mass of the disc $M_{\rm d}$ is taken to be 100 times smaller than the mass of the CSC, allowing us to neglect the gravitational effect of the disc.

The phiGRAPE+GPU code which we use allows simulating the black hole as a "live" massive particle (Merritt et al. 2007). However, we are not interested here in the "Brownian" motion of a "live" black hole, so we model it with a fixed potential well that comes from the central point with the mass 10 times smaller than the mass of the CSC. For detailed description of the algorithm of stellar accretion in our model see Just et al. (2012).

As initial conditions we take the distributions of positions and velocities according to the Plummer model (Plummer 1911) which describes equilibrium distribution of stars in the stellar system without a black hole. In the initial simulation during the violent relaxation phase lasting several crossing times (which is of the same order as $t_{\rm dyn}$ introduced below), we let the system with the central SMBH reach its quasi-equilibrium distribution, and only then we switch on the accretion disc. We consider the particles as equal-mass stars and do not take the stellar evolution into account at this stage.

The relaxation time for the cluster with N_s stars is defined as follows (Spitzer 1987):

$$t_{\rm rx} = 0.14\frac{N_s}{\ln(0.4N_s)}t_{\rm dyn}, \qquad (4)$$

where $t_{\rm dyn}$ is the dynamical time

$$t_{\rm dyn} = \left(\frac{r_{\rm half}^3}{GM_{\rm CSC}}\right)^{1/2}.$$

It should be noted that in dimensionless N-body units routinely used in N-body simulations $t_{\rm dyn}$ is invariant under the change of particle number and has the value of $\sim$0.66, as $G = 1$, $M_{\rm CSC} = 1$ and the half-mass radius for the Plummer model is $r_{\rm half} \sim 0.76$.

The scaling of N-body models involving only gravitational elastic interactions to real systems is performed on the basis of formula (4), which gives the characteristic

time scale of the cumulative effect of gravitational pair interactions between stars. If a non-gravitational process such as star–gas dissipative interaction is introduced into the model, it also has to be appropriately rescaled. For that we should keep the ratio $t_{\rm diss}/t_{\rm dyn}$ the same as in a real system with large particle number. This might be achieved through increasing either the effective cross-section of stars (or their radii) or the coefficient describing the star–gas interaction intensity with decreasing the total number of particles in the system. At this stage we treat model particles as dimensionless points, and therefore we employ the latter case, where the scaling is performed through the $Q_{\rm d}$ coefficient in formula (2), which is redefined so as to describe the dimensionless total dissipative cross section of all N stars normalized to the disc area:

$$Q_{\rm tot} = Q_d(N\pi r^2)/(\pi R_d^2). \tag{5}$$

As shown in Just et al. (2012), the scaling in this case amounts to changing this coefficient in the following manner:

$$Q_{\rm tot}(N_2) = \frac{t_{\rm rx}(N_1)}{t_{\rm rx}(N_2)} Q_{\rm tot}(N_1). \tag{6}$$

To investigate the convergence behavior of the simulations, we have calculated the evolution of the SMBH mass in model systems with different particle numbers ranging from 1000 to 64000. The results of the calculations are shown in Fig. 1, and we can see that for $N_p \geq 8000$ the model curves almost coincide, which means that model calculations with the above defined scaling show the "precision saturation" at $N_p \geq 8000$ and that the curve for the real system with, e.g., $N_s = 10^8$ is expected to lie very close to the curve for $N_p = 64000$.

Thus, one might conclude that the presence of the accretion disc substantially increases the mass growth rate of the SMBH, and this rate increases with the intensity of dissipation in the system. When the star–gas interaction is strong enough, the spatial distribution of stars becomes prominently axially symmetrical and flattened; a case of such spatial distribution is shown in Fig. 2 for $N_p = 8000$. Here one can see that stars form a "stellar disc" component that coincides spatially with the gaseous accretion disc. However, even at large values of $Q_{\rm tot}$, the flattening is finite, i.e., in our model the velocities of the stars retain component normal to disk plane due to pair interaction with other stars, the result which is in accordance with Vilkoviskij and Czerny (2002). A more detailed presentation of the AGN evolution models with star–gas interaction (without stellar collisions) is presented in Just et al. (2012).

3 Effects of stellar collisions in the AGN evolution model

In galactic nuclei the stellar density might, in principle, be high enough, so that stars may collide physically. For stars of radius r the average time between collisions is

$$t_{\rm coll} = \frac{1}{\nu} = \frac{1}{n\sigma V}, \tag{7}$$

where n is the stellar concentration, ν is the frequency of collisions for a typical star, $\sigma = \pi p(r_1 + r_2)^2$ – effective cross-section for the collision ($p \leq 1$ is the collision

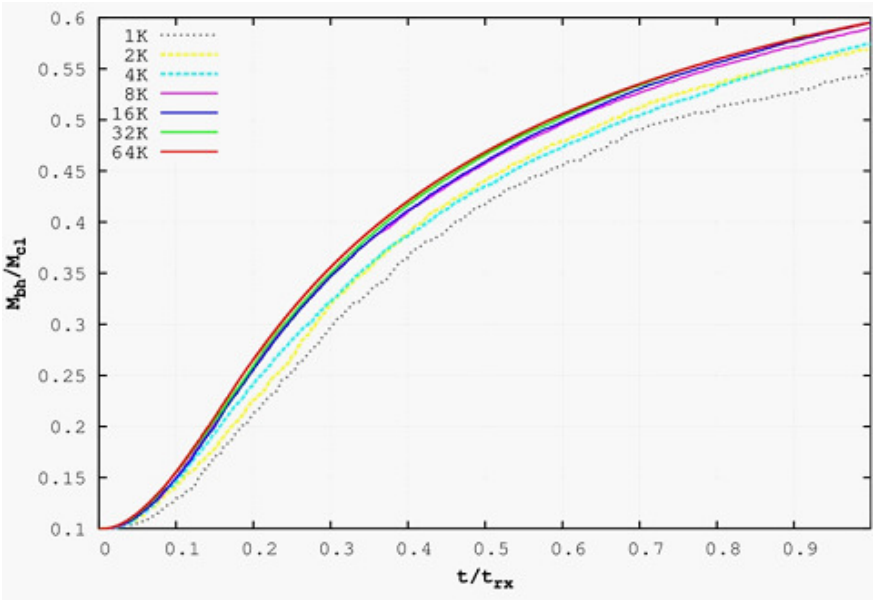


Figure 1 The model calculations of the growth of the SMBH mass (normalized to initial mass of the stellar cluster) shows the convergence of the simulations with increasing particle numbers. All runs are performed with the accretion disc. The only differing parameters in the runs are the number of particles and the $Q_{\rm tot}$ coefficient rescaled each time according to formula (6). The time is shown in the units of the relaxation time.

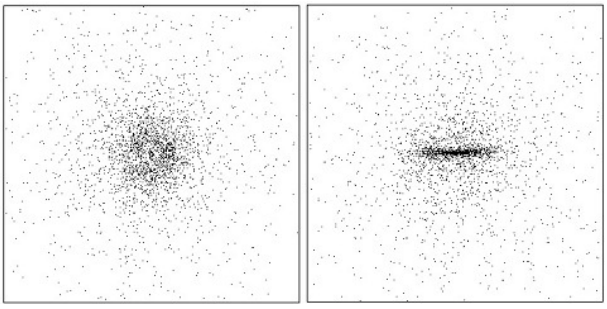


Figure 2 Spatial distribution of stars (projection on XZ-plane; the accretion disc is in XY-plane). Left – initial moment, right – at the time $t = t_{\rm rx}$.

parameter), and V is a characteristic velocity of a star. Assuming that model particles have equal masses and radii and noting that

$$n \approx \frac{N}{(4/3)\pi R^3}, V \approx R/t_{\rm cross} \approx R/t_{\rm dyn},$$

where R is a characteristic size of the system, one might compare the collision type (for a typical star) with the relaxation time of the system (r is the stellar radius):

$$\frac{t_{\rm coll}}{t_{\rm rx}} = \left(\frac{R}{r}\right)^2 \frac{\ln(0.4N)}{0.42pN^2}. \quad (8)$$

Stellar contact collisions in AGN were considered in Fokker–Planck simulations which also included realistic distribution of masses (Murphy et al. 1991). To understand if we need to introduce contact collisions at the resolution of our N-body AGN model, we must compare it with real systems. We performed a simulation of the model system with $N_p = 32000$ and with the smallest (within our models) accretion radius $R_{\rm accr} = 3{\times}10^{-4}$, to better resolve the interactions of stars in the innermost parts of the system. This simulation takes the star–gas interaction into account, but collisions are not yet introduced. To estimate the maximum stellar density value, we considered the evolution of 0.05% Lagrangian radius (i.e., radius of a sphere containing 0.05% of the system mass).

Suppose that with this model we simulate the object M87 which has the total stellar mass of $6.6{\times}10^{10}\ M_\odot$ and the black hole influence radius of 291 pc (see data in Just et al. (2012)). So for 0.05% Lagrange radius the mass is $M_{0.05\%} = (5 \times 10^{-4}) \times (6.6 \times 10^{10})\ M_\odot = 3.3 \times 10^7\ M_\odot$. The 0.05% Lagrange radius is around 0.001 N-body units (NBU) at its minimum To get it in units of pc, we normalize it to the influence radius (0.22 NBU in our model and in 291 pc M87). As a result, assuming that our model particles represent solar-mass stars, we get $3{\times}10^7$ particles in a sphere of radius ∼1.3 pc. As follows from (8), this corresponds to systems in which characteristic collision time for a typical star is 150 times larger than relaxation time of the system.

Alternatively, if we assume that with the same model we simulate the object M32 with stellar mass $3 \times 10^7\ M_\odot$ and influence radius of 2.3 pc one gets around 10^6 particles in a sphere of radius 0.01 pc. As follows from (8), in this case collision time for a typical star is only 5 times larger than the relaxation time, and so cumulatively stellar collisions might play a significant role.

It should be kept in mind that in reality the density values would be even higher, since $R_{\rm accr}$ in our model is unphysically large (only ∼3 times smaller than 0.05% Lagrange radius), so in the model, as opposed to real system, stars do not accumulate there as they are quickly accreted. Therefore, it is reasonable to consider introducing collisions at this stage before we improve the resolution of our numerical model that will allow us to use smaller values of $R_{\rm accr}$.

Thus, we need to introduce a scaling procedure for contact collisions in the same manner as we did for dissipative interaction between stars and gas. In this case our ansatz is that the mass loss rate should be the invariant parameter as we change the numbers of particles in the system. The mass loss (in the whole system) per unit time due to contact collisions is proportional to the frequency of the collisions in the whole system ($\sim N\nu$, where ν is defined for a typical star, see (9)) and mass of colliding stars (we assume that all stars have the same mass): $\Delta M = kN\nu m = kN^2\sigma V m/R^3$, m is the mass of a star, k = const, R is a characteristic size (e.g., 90% Lagrange radius) of the stellar system. Then the mass lost during relaxation time is $t_{\rm rx}\Delta M$. Because we need to keep the fractions of the mass lost due to collisions during relaxation time the same in model and real systems, we have

$$kN_1^2\sigma_1 V_1 m_1 t_{\rm rx1} R_2^3 = kN_2^2\sigma_2 V_2 m_2 t_{\rm rx2} R_1^3,$$

where subscript 2 stands for real system and 1 stands for model system. The values of velocities and characteristic sizes are invariant in N-body units, whereas $m = 1/N$. So we get the scaling rule for stellar radii as follows:

$$\frac{r_2}{r_1} = \sqrt{\frac{\sigma_2}{\sigma_1}} = \frac{N_1}{N_2}\sqrt{\frac{\ln(0.4N_2)}{\ln(0.4N_1)}}. \tag{9}$$

4 Tentative simulations of AGN evolution with contact stellar collisions

We have implemented a tentative scheme of stellar collisions in the phi-GRAPE+GPU code. The scheme is as follows: at every integration step, the distances between every pair of "active" particles are calculated (the active particles are those considered in the current integration step). If the distance is less than the linear collision parameter $p^{1/2}$ (see formula (8); $p^{1/2} = 0.6$ in our model to ensure that we consider strongly disruptive collisions), then a collision event is identified. Such algorithm does not slow down the computing considerably since at each step only a small fraction of stars are active. After collision, one particle is removed from the system, while another one continues its motion according to momentum conservation law (i.e. we suppose that two particles coalesce into one with the mass $(m_1+m_2)/2$, while another half of the total mass is lost). Such "toy model" does not preserve the total energy and does not depend on the angle between trajectories and impact parameter; we are interested here in tentative estimation of the effect of collisions in our model, not in details of that process. In other words, we would like to estimate how often stars approach each other in the inner part of the system to identify a collision event, given the scaling relation (9).

Stellar inflow toward the SMBH is considerably conditioned by the dissipation of the kinetic energy during the passage of the gas disc. Stellar collisions are expected to make this inflow less intense as some of the stars would be disrupted by collisions prior to reaching the accretion radius. Therefore, to estimate the effect of collisions, one might compare SMBH mass evolution due to stellar accretion with and without stellar contact collisions.

We have performed and compared a series of numerical simulations of the SMBH mass evolution with and without stellar collisions for different values of accretion radius, with all other model parameters being equal (except that in simulations without collisions particles are treated as dimensionless, and so the softening parameter is introduced into Newtonian potential). To perform scaling procedure (9) for collisions, we take the reference real dense stellar cluster to have 2×10^7 stars in a sphere of 1 pc radius, with solar-type stars with radii normalized to characteristic size of the system $r = 0.23\times10^{-7}$. We made simulation for three particle numbers – 2000, 4000 and 8000 – with and without collisions in each case. While simulations with $R_{\rm accr} > 5\times10^{-3}$ showed little difference between the curves representing models with and without collisions, the simulation with $R_{\rm accr} = 10^{-3}$ displayed considerable divergence of the curves (Fig. 3) already by one relaxation time. This result indicates that stellar disrupting collisions might indeed considerably reduce the amount of the stellar matter drifting beyond accretion radius. Moreover, this amount, in case of collisions taken into account, depends on the model parameter $R_{\rm accr}$. So, we conclude that

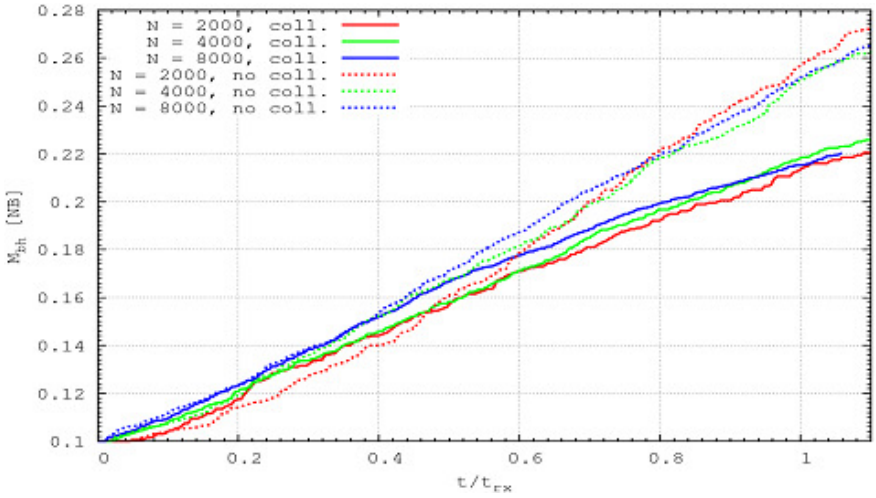


Figure 3 Black hole mass evolution with and without collisions. $R_{\rm accr} = 10^{-3}$.

with going to higher resolutions it is necessary to take the contact stellar collisions into account.

We again stress that these results are tentative, and further improvements are due to be introduced both into the technical scheme and into the physical model. As for the technical scheme, for example, the candidate stars for collision events should be searched not among active particles, but rather within neighbor spheres of each active particle. As for the physical model, we have to decide how to treat the fate of the gas produced during stellar collisions. In principle, it might be added to the mass of the accretion disc and, thus, eventually also accreted to the SMBH. Since we are ultimately interested in the matter accretion rate which is the dominant factor of the AGN phenomenon, it might seem that in this case it makes no difference if we consider stellar collisions or not, since the same mass will be accreted in both cases, either in the form of stars, or in the form of gas produced during disruptive collisions nearby the SMBH. However, the accretion of gas and accretion of stars are different processes, and the former one strongly depends on the ratio of the current luminosity to the Eddington luminosity, while accretion of stars is not strongly limited in that way. Furthermore, introducing stellar evolution might alter the situation, since, e.g., for some part of the stellar population (neutron stars and stellar-mass black holes) collisions might be neglected even in the close vicinity of the SMBH, and they are expected to be accreted onto the SMBH mostly as intact particles.

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