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On the theory of the two-component gravitational lens

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The propagation of light rays in the spherical-symmetric gravitational field of the Galaxy described both by dark matter as well as baryonic matter is searched. The focus of the article is on the case when contributions to the gravitational lensing of these components are comparable to each other. It is shown that positions of images of the lensing object depend on the profiles of dark matter and baryonic matter. Besides, it was noted that positions of images from the gravitational lens are drastically (by four orders of magnitude) smaller than the distance between the gravitational lens and the observer.

Keywords: Dark matter; Halo mass profile; Scalar field; Two-component gravitational lens

1 Introduction

The basic components of a galaxy are its central part (the nucleus), the bulge, the gaseous-dust disc, the cluster of stars and the halo of dark matter (Chernin 2008). It should be emphasized that dark matter is about 90% of the total mass of a galaxy. That is why the influence of dark matter on the gravitational lensing effect was the topic and focus of most researchers (Bliokh and Minakov 1989; Zakharov and Sazhin 1998).

But if light rays propagate in the vicinity of the central part of a galaxy it is necessary to take into account the influence of baryonic matter on the gravitational lensing effect. We will identify this type of lens as a two-component gravitational lens.

To examine such a lens it is necessary to know the distribution of baryonic matter in the galaxy. Though this problem was investigated by a number of authors, we limit ourselves to distributions by Idliss (1961) and King (1966). As for the profile for dark matter, we will take the well-known Navarro–Frenk–White (NFW) profile, merely (Navarro et al. 1995).

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2 Two-component gravitational lens

The focus of this article is on investigating the propagation of light rays in the spherical-symmetric gravitational field of the Galaxy described both by dark matter as well as baryonic matter. We examine the case when contributions by the gravitational lensing of these components are comparable to each other. Later on, for simplicity, the movement of light rays will be considered in the coordinate plane $\theta = \frac{\pi}{2}$.

The basic equation that describes the propagation of light rays throughout the gravitational lens is the aberrational equation. It relates the impact parameter p with the deflecting angle θ and distance from the observer and the trajectory of light rays P. All subsequent denotations correspond to the denotations in the monograph by Bliokh and Minakov (1989).

The general form of the aberrational equation is the algebraic equation of an nth order. To compose the aberrational equation, we note that, according to Bliokh and Minakov (1989), the following expression takes place

$$P_i = p + \Delta P_i, \quad (i = 1, 2) \tag{1}$$

where ΔP_i is the displacement of light rays from the rectilinear trajectory in the gravitational lens. Due to the minuteness of the deflecting angle we have the approximate relation

$$\Delta P = x \cdot \tan\theta \approx x \cdot \theta, \tag{2}$$

where x is the position of the observer behind the gravitational lens on its principal axis.

For the following analyses, note that the deflecting angle θ must depend on impact parameter P, i.e., $\theta = \theta(P)$. In fact, according to general relativity the deflecting angle in the gravitational field of an opaque body (Sun) is inversely proportional to the impact parameter, i.e., $\theta(P) = \frac{\alpha}{P}$, where $\alpha = \frac{4\pi GM}{c^2}$. In Chechin et al. (2012), we show that the deflecting angle may depend on the impact parameter in the first order as well as in the second order. That is why for a transparent cosmic object, i.e., a galaxy, we set the parametric representation of the deflecting angle

$$\theta(P) = \alpha_0 P^0 + \alpha_1 P^1 + \alpha_2 P^2 + ... + \alpha_n P^n = \alpha_0 + \alpha_1 P + \alpha_2 P^2 + ... + \alpha_n P^n. \tag{3}$$

Substituting eq. (3) into eqs. (1) and (2), we get the general expression for the aberrational equation

$$P = p + (\alpha_0 + \alpha_1 P^1 + \alpha_2 P^2 + ... + \alpha_n P^n)x. \tag{4}$$

The choice of the order of the aberrational equation depends on the numerical values of coefficients $\alpha_0, \alpha_1, \alpha_2, ..., \alpha_n$. In this paper, we will consider the aberrational equation of an order not higher than two.

So, the aberrational equation of the second order

$$\alpha_2 x P^2 + (\alpha_1 x - 1)P + \alpha_0 x = 0 \tag{5}$$

let symbolize in the standard reduced form

$$P^2 + \left(\frac{\alpha_1 - \frac{1}{x}}{\alpha_2} \right) P + \frac{\alpha_0}{\alpha_2} = 0. \tag{6}$$

Its roots are ($x = R_0$)

$$P_{1,2} = \frac{1}{2} \left(\frac{\alpha - \frac{1}{R_0}}{\alpha_2} \right) \pm \sqrt{\frac{1}{4} \left(\frac{\alpha - \frac{1}{R_0}}{\alpha_2} \right)^2 - \frac{\alpha_0}{\alpha_2}}. \tag{7}$$

And, finally, for the linear aberrational equation

$$P = \alpha_0 x + \alpha_1 P x \tag{8}$$

the impact parameter is determined by the expression ($x = R$)

$$P = -\frac{\alpha_0 R_0}{\alpha_1 R_0 - 1} = -\frac{\alpha_0}{\alpha_1 - \frac{1}{R}} = \frac{\alpha_0}{\left(\frac{1}{R_0} - \alpha_1 \right)}. \tag{9}$$

Based on these results it is possible to calculate the actual parameters $\alpha_0, \alpha_1, \alpha_2$ for the profiles mentioned before.

Using the multiplicative representation of the refractive index discussed in Bliokh and Minakov (1989), for the two-component substance (dark matter and baryonic matter) we write down an approximate expression for the general refractive index as

$$n = n_{\rm DM} n_{\rm BM} \approx 1 + \eta_{\rm DM} + \eta_{\rm BM}, \tag{10}$$

where

$$\left. \begin{array}{l} n_{\rm DM} = 1 + \eta_{\rm DM} \\ n_{\rm BM} = 1 + \eta_{\rm BM} \end{array} \right\}. \tag{11}$$

We combine the mentioned profiles into two groups – the NFW profile and Idliss distribution, the NFW profile and King distribution.

1. From Chechin et al. (2012) it follows that the refractive index of a galaxy with the NFW profile for dark matter and baryonic matter distribution according to Idliss has the form

$$n_{\rm NFW-Id} = 1 + \frac{8\pi G}{c^2} \rho_0 r_0^2 - \frac{8\pi G}{5c^2} \rho_0 r_0^2 \left(\frac{r}{r_0} \right) - \frac{4\pi G}{3c^2} \rho_0' r_0^2 \left(\frac{r^2}{r_0^2} \right). \tag{12}$$

To estimate the baryonic matter density in the center of the galaxy we take advantage of the results by Kardashev (1983). As the mass of the central part of our Galaxy is about $3 \cdot 10^5 M_{\rm sun}$ in the cubic parsec, then the central baryonic mass density will be

$$\rho_0 = 3 \cdot 10^5 M_{\rm sun} / 4 \cdot 10^{66}\,{\rm cm}^3 \approx 10^{-28}\,{\rm g/cm}^3.$$

Using the well-known expression from optics (Drude 1935) for the deflection angle

$$\Delta\theta = \int_{-\infty}^{\infty} \frac{d}{dr}(\ln n)dr = 2\ln n\,\Big|_{-\infty}^{r},$$

we find the total angle

$$\theta_{\mathrm{NFW-Id}} = 2\left(\frac{8\pi G}{c^2}\rho_0 r_0^2 - \frac{8\pi G}{5c^2}\rho_0 r_0^2\left(\frac{r}{r_0}\right) - \frac{4\pi G}{3c^2}\rho_0' r_0^2\left(\frac{r^2}{r_0^2}\right)\right). \tag{13}$$

In this expression the numerical magnitudes of the coefficients are as follows:

$$\left.\begin{aligned} \alpha_0 &= \frac{8\pi G}{c^2}\rho_0 r_0^2 = 16\pi \\ \alpha_1 &= -\frac{8\pi G}{5c^2}\rho_0' r_0^2 = -3\pi\cdot 10^{-23}\left[\mathrm{cm}^{-1}\right] \\ \alpha_2 &= -\frac{4\pi G}{3c^2}\rho' = -3\pi\cdot 10^{-46}\left[\mathrm{cm}^{-2}\right] \end{aligned}\right\}. \tag{14}$$

Putting them into eq. (13) we get the magnitude of the angle:

$$\theta_{\mathrm{NFW-Id}} \cong 4'', 3. \tag{15}$$

Setting that the test position of the observer is described by magnitude $x = 3.7\cdot 10^{27}\,\mathrm{cm}$ (position of quasar QSO 0957+561). As parameters included into eq. (7) have the following numerical values

$$\frac{1}{2}\left(\frac{\alpha_1 - \dfrac{1}{x}}{\alpha_2}\right) = 0.5\times 10^{23}\,\mathrm{cm}, \tag{16}$$

$$\frac{\alpha_0}{\alpha_2} = -5.3\times 10^{46}\,\mathrm{cm}^2, \tag{17}$$

the roots of the aberrational equation (or impact parameters) are, thus,

$$\left.\begin{aligned} P_1 &\cong 2.8\times 10^{23}\,\mathrm{cm} \\ P_2 &\cong -1.8\times 10^{23}\,\mathrm{cm} \end{aligned}\right\}. \tag{18}$$

From these calculations, it can be seen that the impact parameter (position of the lensing star image) has a positive value and a negative value. In doing this their modules are not equal to each other. Therefore the observer will see two images of the same object that are located on the opposite sides of the gravitational lens' principal optical axis. Note that they are located at distances drastically smaller than the test position of the observer

$$P_1 \cong 2.8\cdot 10^{23}\,\mathrm{cm} << \mathrm{x} = 3.7\cdot 10^{27}\,\mathrm{cm}; \mathrm{P}_2 \cong -1.8\cdot 10^{23}\,\mathrm{cm} << \mathrm{x} = 3.7\cdot 10^{27}\,\mathrm{cm}.$$

Further, the refractive indices of a galaxy with the NFW profile of dark matter and King distribution of baryonic matter have the form

$$n_{\mathrm{NFW-K}} = 1 + \frac{8\pi G}{c^2}\rho_0 r_0^2 - \frac{8\pi G}{3c^2}\rho_0'' r_0^2 - \frac{8\pi G}{5c^2}\rho r_0^2\left(\frac{r}{r_0}\right) + \frac{8\pi G}{c^2}\rho_0'' r_0^2\left(\frac{r}{r_0}\right). \quad (19)$$

The corresponding deflective angle will, hence, be

$$\theta_{NFW-K} = \left(\frac{8\pi G}{c^2}\rho_0 r_0^2 - \frac{8\pi G}{3c^2}\rho_0'' r_0^2 - \frac{8\pi G}{5c^2}\rho r_0^2\left(\frac{r}{r_0}\right) + \frac{8\pi G}{c^2}\rho_0'' r_0^2\left(\frac{r}{r_0}\right)\right). \quad (20)$$

Using the previous results we find the required coefficients in expression (20)

$$\left.\begin{aligned} \alpha_0 &= \frac{8\pi G}{c^2}\rho_0 r_0^2 \approx 16\pi \\ \alpha_0 &= \frac{8\pi G}{3c^2}\rho_0'' r_0^2 \approx -5\pi \\ \alpha_1 &= \frac{8\pi G}{5c^2}\rho_0 r_0^2 \approx -3.2\pi\cdot 10^{-23}\left[\mathrm{cm}^{-1}\right] \\ \alpha_1 &= \frac{8\pi G}{c^2}\rho_0'' r_0^2 \approx 16\pi\cdot 10^{-23}\left[\mathrm{cm}^{-1}\right] \end{aligned}\right\}, \quad (21)$$

and the numerical value of the deflection angle

$$\theta_{\mathrm{NFW-K}} \cong 2'', 6. \quad (22)$$

That is why the roots of the aberrational equation will be

$$P_1 = -\frac{\alpha_0}{\alpha_1 - \dfrac{1}{R_0}} \cong 5.3\times 10^{23}\,\mathrm{cm}, \quad (23)$$

$$P_2 = -\frac{\alpha_0}{\alpha_1 - \dfrac{1}{R_0}} \cong 0.3\times 10^{23}\,\mathrm{cm}. \quad (24)$$

From eqs. (23) and (24) we can see that the lensing star has two images that are close to each other and lie on the same side of the principal optical axis. In doing this they are placed at significantly smaller distances than the test position of the observer

$$P_1 \cong 5.3\cdot 10^{23}\,\mathrm{cm} << x = 3.7\cdot 10^{27}\,\mathrm{cm}; P_2 \cong 0.3\cdot 10^{23}\,\mathrm{cm} << x = 3.7\cdot 10^{27}\,\mathrm{cm}.$$

In conclusion we summarize the main results of our article. First, the positions of images of a lensing object depend on the profiles of dark matter as well as baryonic matter. As a result, they may lie on one side of the principal optical axis, in particular. Second, the positions of images of a lensing object are drastically (by four orders of magnitude) smaller than the distance from the observer to the gravitational lens.

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