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Modern theory of star formation: 3D MHD simulations

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We review some results of three-dimensional magneto-hydrodynamic (MHD) simulations of star formation in galaxies. We consider 1) the formation of molecular clouds and their cores in converging flows of atomic gas; 2) possible links between magnetic and radiative effects in collapsing protostellar clouds; 3) fragmentation conditions of protostellar clouds; 4) hypothetic formation of giant planets together with stars.

Keywords: Molecular clouds; MHD turbulence; Collapse; Fragmentation; Star formation; Planet formation

1 Introduction

Modern theory of star formation in galaxies uses complicated three-dimensional (3D) numerical models of interstellar medium. The magnetic energy density of interstellar medium is comparable with the densities of gravitational, thermal and turbulent kinetic energies. Therefore advanced 3D models of interstellar medium take magnetic field into account – via magneto-hydrodynamic (MHD) approximation.

Effective computational technologies such as adaptive mesh refinement (AMR, see Banerjee and Pudritz, 2006), nested grid (see Machida *et al.*, 2011), and smoothed-particle hydrodynamics (SPH, see Price and Bate, 2009) are implemented in MHD models to increase the spatial resolution. For example, the large number of mesh levels (up to 27) allows us to consider in 3D the process of star formation from molecular cloud core to stellar interior (Banerjee and Pudritz, 2006).

In this brief review we point out a few achievements of 3D MHD star formation simulations, paying most attention to the investigations of protostellar clouds – gravitationally bound cores of molecular clouds.

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2 Formation of molecular clouds and their cores

Molecular clouds in the Galaxy have masses $\simeq 10^2$–$10^5\,M_\odot$ and sizes $\simeq$1–100 pc (Dudorov, 1991). They can form in warm neutral medium by thermal instability, nonlinearly amplified by transonic compressions (Ballesteros-Paredes *et al.*, 1999). The dynamic pressure of such compressions causes the affected regions to overshoot from cold neutral medium to molecular clouds. The transonic compressions most likely arise from turbulent fluctuations or gravitational instability occurring on large scales, e.g. during the creation of gas spirals by stellar spirals (Wada *et al.*, 2011).

Magnetic field can be important on the scales of molecular clouds and their substructures (Dudorov, 1991). Therefore, there is the tendency to include magnetic field to the models of molecular clouds formation. The highest spatial resolution in such models was obtained with the help of AMR codes RAMSES (Hennebelle *et al.*, 2008) and FLASH (Banerjee *et al.*, 2009) under the approximation of ideal MHD. For example, Hennebelle *et al.* (2008) investigate the head on collision of warm atomic flows with number density $n = 1$ cm^{-3} and temperature $T = 8000$ K. Initially, the gas is compressed to $n \simeq 100$ cm^{-3} due to thermal instability and dynamic pressure of colliding flows. Next, the ratio of gas mass to magnetic flux becomes supercritical ($\mu > 1$) because of permanent gas inflow from surrounding medium along magnetic lines. Therefore, the magnetic field cannot prevent gravitational instability but can delay it. The resulting cloud consists of cold filaments and cores ($T \simeq 10$ K) with density up to $n \approx 10^6$ cm^{-3}, mixed with warm gas. The typical value of column density is $\simeq 10^{21}$ cm^{-2} that agrees with observations of Taurus molecular cloud. The magnetic field correlates with density approximately as $B \propto n^{0.1}$ for $n < 10^4$ cm^{-3}, but $B \propto n^{0.5}$ for higher density. The direction of magnetic lines inside filaments and cores often differs very much from the initial direction.

Vazquez-Semadeni *et al.* (2011) improved the model of Banerjee *et al.* (2009) by taking magnetic ambipolar diffusion (MAD) into account. MAD is the motion of charged component of plasma relative to the neutral component under the action of magnetic force. The friction of charged and neutral component causes energy dissipation. Moreover, MAD causes the diffusion of magnetic field itself. Duffin and Pudritz (2008) incorporated the module for the computation of MAD into the code FLASH in order to improve the models of molecular and protostellar clouds. Vazquez-Semadeni *et al.* (2011), using the modified FLASH, consider the collision of warm atomic flows to study the formation of molecular clouds. Gravitationally unstable regions with $n > 2 \cdot 10^5$ cm^{-3} in the simulations are replaced by sink particles that allows the authors to investigate the long-time evolution of molecular clouds as well as to estimate the rate and efficiency of star formation. Computations show that these characteristics strongly depend on the initial value of μ but weakly depend on the efficiency of MAD (because MAD velocity on the size scales of molecular clouds is much less than mass velocity). Computations show also that a cloud becomes molecular and supercritical ($\mu > 1$) at roughly the same time. This suggests that converging flows on the scale $\approx$1 kpc induced by stellar spirals may form molecular clouds with $\mu > 1$. On the other hand, smaller-scale converging flows induced by supernova shocks or turbulence may create the partially atomic clouds with $\mu < 1$. The efficiency of star formation in the simulated clouds is not large (3–35%) because a significant fraction of molecular gas has $\mu < 1$, while star formation occurs in the regions with $\mu > 1$.

3 Radiation MHD models of protostellar clouds

The most significant increase of density (>15 orders of magnitude) at the process of star formation happens during the collapse of protostellar clouds. The collapse begins from the state with $n \simeq 10^4$ cm^{-3} and $T \simeq 10$ K. The gas temperature can drop to 5 K at $n \simeq 10^6$ cm^{-3} due to effective radiative cooling of dust during the collapse. The dust becomes optically thick to own radiation at $n \simeq 10^9$ cm^{-3}, when almost isothermal contraction is replaced by almost adiabatic one. The radiation transport equations must be solved to the correct modelling of thermal and ionization processes at non-isothermal collapse stages.

The equations of radiation transport are solved in a few 3D MHD simulations of protostellar clouds. We point out only two works where possible links between magnetic and radiative effects are shown.

1) Commercon *et al.* (2011) using RAMSES code investigate the relationship between magnetic braking and radiative heating of collapsing cloud. Due to AMR technology the effective number of cells is 262144^3, providing spatial resolution 2.16 AU in computational domain with size 2.76 pc. The following effect is discussed. Magnetic braking reduces centrifugal force and thus increases the infall velocity onto a protostar. This intensifies protostellar accretion and enhances radiative heating from accretion shock. The rise of temperature around the protostar leads to suppression of fragmentation. As a result, the protostar stays without companions.

2) Peters *et al.* (2011) using FLASH code study thermal and ionization feedback from protostars. The collapse and fragmentation of rotating cloud with radius $R =$ 0.5 pc and mass 1000 M$_\odot$ (15 Jeans mass) is considered. Young stars are modelled with the help of sink particles. Stellar luminosity is calculated analytically from the mass and accretion rate of the sink particles. A cluster of 14 young stars appears in the simulations. The most massive of stars generate expanding HII regions. These regions tend to disrupt helical geometry of magnetic field, responsible for collimated outflows. In turn, magnetic forces tend to confine HII regions. So, the interplay of magnetic field and HII regions around massive stars can be important.

4 Fragmentation conditions of protostellar clouds

The formulation of fragmentation criterion of protostellar clouds is a problem for star formation theory because there is no satisfactory analytical model of fragmentation. We hope that numerical models will help to solve this problem.

We investigate the combined influence of magnetic field, turbulence, and rotation on the collapse and fragmentation of protostellar clouds, using AMR code Megalion (Dudorov *et al.*, 2003). We consider uniform spherical cloud embedded into uniform medium as the initial state. Initial MHD turbulence is represented by an ensemble of Alfven waves with power-law spectrum. Initial angular velocity and large-scale magnetic field are uniform and collinear with each other.

We look for clumps in the density field and calculate their characteristics in order to select the fragments of the cloud. We define a *clump* as a set of contacting cells in which the density exceeds some limiting value $\rho_{\rm lim}$. In order to estimate the

gravitational boundness of a clump we consider gas motions relative to the mass center of the clump and take only the internal motions and interactions into account. The flows directed to the clump center promote the collapse (Hunter, 1979), therefore we postulate the clump boundness criterion in the form

$$\varepsilon_{\rm bnd} \equiv \frac{-E_{\rm in} + E_{\rm out} + E_{\rm tan} + E_{\rm t} + E_{\rm m}}{E_{\rm g}} - 1 < 0 \,, \qquad (1)$$

where $E_{\rm in}$, $E_{\rm out}$, $E_{\rm tan}$ are the energies of centripetal, centrifugal and tangential motions of the gas, $E_{\rm t}$ and $E_{\rm m}$ are thermal and magnetic energies of the clump, $E_{\rm g}$ is the absolute value of its gravitational energy. The quantity $\varepsilon_{\rm bnd}$ is called *the boundness parameter*.

We define the *fragmentation* as the appearance of two or more *fragments* – self-gravitating clumps fulfilling the criterion (1).

Figure 1 shows an example of results when the initial relations of thermal, rotational, large-scale magnetic and turbulent kinetic energies of the cloud to the absolute value of its gravitational energy are 0.15, 0.05, 0.2, 0.1, respectively. The influence of mesh refinement on the density field can be seen at the top of Fig. 1: well resolved fragments are surrounded by poorly resolved envelope. The example of function $\varepsilon_{\rm bnd}(\rho_{\rm lim})$ is shown at the bottom of Fig. 1: this function bifurcates at the point $\rho_{\rm lim} = \rho_{\rm fb} \approx 2240$ when massive central clump is separated on two fragments.

Our computations show that slightly supersonic initial turbulence in the combination with moderate initial rotation are sufficient to stimulate the fragmentation of protostellar clouds at the isothermal collapse stages. We analyse the results of about 200 computations with different parameters and suppose that cloud fragmentation happens if the two conditions are fulfilled:

rotating gas configuration is unstable to non-axisymmetric perturbations, (2)

two (or more) of the perturbations are gravitationally unstable waves. (3)

Differentially rotating polytropes are unstable to non-axisymmetric perturbations if the ratio of rotational energy to the absolute value of the gravitational energy, β, exceeds a critical value $\beta_{\rm cr}$ (see Tassoul, 1978). We consider $E_{\rm tan}/E_{\rm g}$ as an analogue of β and find that $E_{\rm tan}/E_{\rm g} \in (0.06, 0.38)$, with arithmetic mean 0.2, at the left neighbourhood of the point $\rho_{\rm fb}$ (see example in Fig. 1). We suggest that the variation of $E_{\rm tan}/E_{\rm g}$ in this neighbourhood is large because various computations differ in amplitude and geometry of perturbations. For comparison, secular and dynamical instabilities of Maclaurin spheroids are characterised by $\beta_{\rm cr} = 0.138$ and 0.274, respectively.

The critical wavelength of gravitationally unstable fast magnetosonic waves propagating across a uniform magnetic field in a homogeneous medium is $\lambda_{\rm Jm} = \sqrt{\pi\left(c_{\rm s}^2 + v_{\rm a}^2\right)/(G\rho)}$, where $c_{\rm s}$ is the sound speed, $v_{\rm a}$ is the Alfvén velocity, G is the gravitational constant, and ρ is the density (Severny, 1953). By analogy with $\lambda_{\rm Jm}$, we introduce the critical effective radius of gravitationally unstable non-homogeneous clump created by the interference of magnetosonic waves:

$$R_{\rm cr} \equiv \frac{1}{2}\sqrt{\frac{\pi\langle c_{\rm s}^2 + v_{\rm a}^2\rangle}{G\langle\rho\rangle}} \,, \qquad (4)$$

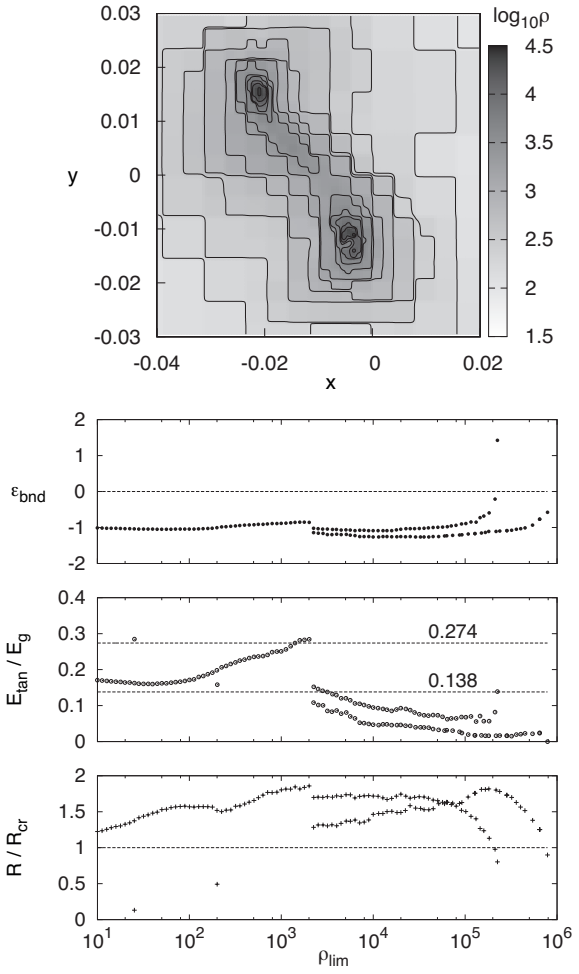


Figure 1 The final state of dense central region in 3D MHD model of protostellar cloud, computed with the help of AMR code Megalion. Top – the decimal logarithm of density in the plane perpendicular to initial angular velocity (grey scale and contours from 2 to 4.4 via 0.2). Bottom – the clump characteristics versus limiting density $\rho_{\rm lim}$: the boundness parameter (1), $\varepsilon_{\rm bnd}$, the ratio of "rotational" energy to the absolute value of gravitational energy, $E_{\rm tan}/E_{\rm g}$, and the ratio of effective radius to the critical value (4), $R/R_{\rm cr}$.

where angle brackets denote averaging over clump volume. Introducing the effective radius $R = (3V/(4\pi))^{1/3}$ of a clump with volume V we consider the dependencies of $R/R_{\rm cr}$ on $\rho_{\rm lim}$ (see example in Fig. 1). At the right neighbourhood of the point $\rho_{\rm fb}$ the major fragments in pairs have $R/R_{\rm cr} \in (0.9, 2.1)$ with arithmetic mean 1.3, while the minor fragments have $R/R_{\rm cr} \in (0.7, 1.4)$ with arithmetic mean 1.0. So, the condition $R/R_{\rm cr} > 1$ can serve as criterion of the gravitational instability of magnetized clumps.

How are conditions (2) and (3) linked to the initial conditions? First, initial turbulence in a protostellar cloud causes the fluctuations of angular momentum. Some

of them can lead to sufficiently rapid rotation of central region of cloud during the isothermal stages of collapse. However such coincidence is infrequent: condition (2) was fulfilled only in 10 % of computations without initial rotation. Moderate initial rotation increases significantly the chance to satisfy the condition (2): e.g. up to 50 % at initial $\beta = 0.05$. Second, the condition (3) can be fulfilled during the isothermal stages of collapse if the initial size of turbulent perturbations is comparable with cloud radius.

Note that simultaneous fulfilling of condition (3) for three and more perturbations is less probable than for two perturbations. For this reason usually two fragments of the first generation appear in a collapsing cloud. Because of individual collapse, some of the fragments can undergo subsequent fragmentation and so on (conception of hierarchical fragmentation of Hoyle, 1953). We point out that every next act of fragmentation is less probable than the previous one for two reasons. (i) The spin rotation of parental fragment transforms mainly into orbital rotation of filial fragments, so that condition (2) in the last may be unfulfilled. (ii) Due to inhomogeneity of collapse, only a small region of parental fragment reaches high density, that may not satisfy the condition (3). So, in contrast to Hoyle, we conclude that hierarchical fragmentation must quickly stop and cannot lead to a large number of fragments.

5 Formation of giant planets together with stars

Let us consider the work of Machida *et al.* (2011) who first carried out 3D MHD simulations from the stage of protostellar cloud to the stage of a protostar with circumstellar disk about 10^4 yr old. Spatial resolution 0.58 AU in computational domain with size 0.73 pc is obtained due to technology of nested grid. Ohmic resistivity is taken into account. The initial axis of rotation is parallel to the magnetic field. The computations show that a protostar (the region with $n > 10^{13}$ cm^{-3} represented by sink particle) is born in the center of optically thick disk. Initially, the disk is more massive than the protostar, and can be recurrently fragmented due to self-gravitation. Most fragments appear at a distance of 20–40 AU from the protostar but then fall onto it. Only fragments of the last generation survived until the end of the computation. Ohmic resistivity restricts magnetic field in the disk to the level which allows the fragments with substellar masses to appear. For instance, some of the fragments have masses 6–12 Jupiter mass and can be classified as giant protoplanets.

The result mentioned above means that giant planets can form simultaneously with stars. This conclusion agrees with the recent discovery of a planet orbiting around a young star (Lagrange *et al.*, 2010).

6 Conclusions

We tried to review some new results of the computations of star formation process. We focused on the conclusions derived from three-dimensional magneto-hydrodynamic simulations. The following findings are interesting.

- Molecular clouds and their cores can form in the warm neutral medium via combined action of dynamic pressure, thermal instability and self-gravitation.

- Expanding HII zones tend to disrupt helical geometry of magnetic field, responsible for collimated outflows from newborn stars. In turn, magnetic field helps thermal pressure in confining HII zones.
- Magnetic braking reduces centrifugal force and thus increases the infall velocity on to a protostar. This may enhance accretion luminosity and prevent the formation of companions.
- A protostellar cloud can be fragmented during isothermal stages of collapse if 1) rotating gas configuration is unstable to non-axisymmetric perturbations, and 2) two ore more of these perturbations are gravitationally unstable waves.
- Magnetic field in circumstellar disks can be limited by Ohmic resistivity to the level allowing the formation of brown dwarfs and giant planets via gravitational instability.

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