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Some principle problems of stellar dynamics

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The following problems of Galactic Dynamics are very briefly discussed: a) do chaotic orbits exist for real galaxies? b) do steady distribution functions exist for steady non-integrable potentials? c) can the phase-space fluid be turbulent? d) why the maximum entropy gravitating sphere is of infinite mass and energy?

Keywords: Stellar dynamics – Methods: Analytical – Galaxies: Modeling

1 Introduction

Many principle problems of Stellar Dynamics were discussed by Kurth [1, 2], Ogorodnikov [3, 4, 5], Kuzmin [6], Idlis [7], Tsitsin [8, 9, 10], Binney [11, 12], Kandrup [13, 14], Pfenniger [15], Antonov [16] and others. We must agree that the foundations of its basic statements are doubtful and need justification. In this paper we shall consider some of them.

2 Chaotic orbits?

It is proved in Non-linear Dynamics (e.g., [17]) that Hamiltonian systems with two or three degrees of freedom (that are of interest for Galactic Dynamics) are not integrable, in general. Non-integrable systems are dense [18]. Chaotic stellar orbits are possible, and it is confirmed by orbit computations (e.g., [19]). Toda's lattice is a well-known system that is integrable. It is proved that any finite term truncation of Toda's potential is not integrable [21]. Chaos is seen in computer calculations [20]. Is it not a paradox?

Idlis [22] asserted that three isolating integrals exist for any steady self-gravitating system. No one could understand his proof (e.g., [23]). But even it is true, one can ask: what will be with non-self-gravitating systems (and real galaxies are not self-gravitating)? Besides, potentials of real galaxies can be approximated with both

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integrable (e.g., Stäckel's) and non-integrable forms. What are invariant manifolds in the phase space of real galaxies?

Discussing the problem we must remember that galaxies were idealized as isolated collisionless steady systems. We must watch orbits for infinite time (that is not possible even in simulations) to conclude that an integral of motion is not isolating. So, maybe, chaotic orbits in common sense are not possible for real galaxies.

3 Jeans' theorem

According to Jeans' theorem, a distribution function (DF) of collisionless star systems is constant along any phase space trajectory. For steady systems this means that only isolating integrals of motion can be arguments of the DF, for the DF cannot depend on infinitely multi-valued non-isolating integrals (as it was established by Kuzmin [24] and Lynden-Bell [25]). There is no problem for integrable potentials (e.g., for spherical galaxies).

But what happens in a case when some orbits are conditionally periodic and others are chaotic [26]? Maybe, the smoothed steady DF admitting triaxial velocity distribution does not exist for arbitrary (non-integrable) axisymmetric potentials. We can suggest that time-dependent integrals of motion must be arguments of the DF (that will be time-dependent) in general. This model was developed in a paper published in 1934 by Heckmann & Straßl [27].

4 Coarse-grained DF

Stellar Dynamics deals mainly with the fine-grained DF. But the coarse-grained DF (as it was introduced by Poincaré and Ehrenfests, see [28]) is of real significance for Galactic Astronomy, as it was stressed by Binney [26]. His considerations were based on the probable existence of chaotic orbits. But similar ones are valid also for chaotic time dependence of the fine-grained DF. Let us start with a system of N point masses. Define the DF. Compare evolutions of our N-body system and a phase-space fluid (obtained as an exact solution of Boltzmann's equation). If there was a phase mixing, some time later layers of various values of the DF will be infinitely close to each other. And the new DF resulting from averaging a new state of N points will not resemble the exact DF at that moment. So, a usual model of phase-space fluid will not work. A phase stream will be turbulent, and some kind of space-time averaging of the DF is neccessary.

Note that the coarse-grained DF does not satisfy the collisionless Boltzmann equation [29, 30].

5 The maximum entropy state

It is well known that the maximum entropy state of graviplasma is an isothermic sphere. It is obtained under conditions so that values of mass and energy are fixed.

But not only size but mass and energy of the isothermic sphere are infinite. What is the reason for the the paradox?

The velocity space of finite mass systems is restricted with a velocity of escape, and it must be taken into account during integrating DF over velocities, though a simple truncation of DF is not enough. Also, a system must be spatially restricted, and its size must be of the order of a virial radius $GM^2/(-E)$, but it is not clear how to realize this idea. Of course, a system can be embedded into a container, but what can this container be for real galaxies?

In principle, it is not correct to use an entropy as a functional of DF for unbounded discrete systems. We must use the method of phase space cells and seek for the most probable distribution of stars over cells (without using the Stirling formula). But how to do it?

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