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On the systematics of the Galactic centre distance determination from orbits of central stars

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A simple analysis of the problem of solving stellar orbit around Sgr A* to derive absolutely (geometrically) the distance to the Galactic centre, R_0, was used to demonstrate that neglect of plausible radial velocity of Sgr A* relative to the LSR (20–60 km s^{-1}) leads to a substantial systematic error in R_0 – of at least 1.3–5.6%, i.e., ∼0.1–0.5 kpc, for star S2/S0-2. Sgr A*'s tangential motion can measurably increase this error. Relativistic and non-Keplerian $\mathcal{O}\left(V_r^2/c^2\right)$ effects should also be allowed for in orbit modelling to avoid an additional bias of the R_0 estimate.

Keywords: Galaxy; Galactic centre distance; Orbits of central stars; Motion of Sgr A*; Relativity

1 Introduction

Modelling stellar orbits at the Galactic centre (GC) allows us to measure the distance to it from the Sun, R_0, with formal uncertainties which are unprecedentedly low for absolute (i.e., not using luminosity calibrations) methods of determination of R_0 [1]. The pioneering estimates of R_0 from solving the star S2/S0-2's orbit around the compact concentration of dark mass, associated with the radio source Sgr A*, were derived (i) using a *Keplerian* model (ii) on the *assumption that the intrinsic velocity of Sgr A* is negligible* [2, 3].

In this study, a schematic description for the problem of solving stellar orbit is proposed. This description, which seemed to be useful in gaining better insight into observational backgrounds for different required parameters, is used to evaluate the impact both of *an unaccounted motion of Sgr A** (i.e., the focus of S2 orbit) and of the most material *relativistic* and *non-Keplerian effects* on an R_0 value found from a Keplerian fitting.

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2 Problem on determination of orbital parameters, distance to and mass at orbital focus (Sgr A*)

The completeness of solution of the problem in question is determined by the type of available data on motion of an individual star (e.g., S2).

If *star's proper motions alone are available*, all six orbital parameters are solved, except that only the absolute value of the inclination angle, i, is determined, i.e., the direction of revolution remains unresolved (e.g., [4]). Besides, the semimajor axis is derived in angular units (in arcsec), hereafter a''. The distance to the focus, R_0, and the central mass, M, can not be solved.

If *proper motions and at least a single measurement of radial velocity of star are available*, the problem is completely solved (for a value of star's radial velocity, V_r, being significantly different from focus' value of V_r):

A. *The sign of V_r determines the sign of i*, and, consequently, also the direction of rotation (e.g., [4]).

B. *The absolute value of V_r determines values R_0 and M.* To gain greater insight into the fact of the matter, the problem can be symbolically divided into two subproblems: (1) the determination of orbital parameters from the proper motions alone and (2) the determination, knowing the orbit, of the distance to focus (R_0) and of the central mass from the measurement(s) of V_r. These subproblems are almost independent for modelling the motion of stars around Sgr A* from numerous proper motion measurements and a single (or few) V_r one(s). In this case, V_r measurement(s) contribute(s) almost nothing to the knowledge of orbit, and vice versa proper motion measurements do not directly determine neither R_0 nor M. Such breaking the problem down seems to be realistic at least for early R_0 estimates [2, 3].

If so, the value of $|V_r|$ may be considered as determining R_0 and M from known orbital parameters as follows.

(**i**) The knowledge of $|V_r|$ determines the total space velocity, V, since the ratio between them can be found from the orbit elements for any moment t:

$$\frac{V_r^2}{V^2} = \frac{[e \sin v \sin u + (1 + e \cos v) \cos u]^2 \sin^2 i}{1 + 2e \cos v + e^2}, \tag{1}$$

where e is the eccentricity; v is the true anomaly, i.e., a known function of orbit elements and t (e.g., [5]); $u = v + \omega$ is the argument of latitude; ω is the argument of pericenter.

(**ii**) The knowledge of V determines the value of a *in linear units*, and, consequently, the ratio between the latter and a in angular units gives R_0:

$$V = na \left(\frac{1 + 2e \cos v + e^2}{1 - e^2} \right)^{1/2}, \qquad R_0 = \frac{a\,[\mathrm{kpc}]}{a''}. \tag{2}$$

Here, $n = 2\pi/P$, P is the orbital period.

(**iii**) The value of a in linear units determines the central mass $M = n^2 a^3/G$, where G is the gravitational constant.

Table 1 Lower limit for systematic error in R_0 because of neglect of a possible radial motion of Sgr A*.

Observational	$\langle V_r \rangle$	V_r(Sgr A*)	$\delta_{\rm sys}$	$\sigma_{\rm sys}(R_0)$ (kpc)	
period	(km s^{-1})	(km s^{-1})		$R_0 = 7.5$ kpc	$R_0 = 8$ kpc
2003 April–June	−1500	20	0.013	0.10	0.11
		60	0.040	0.30	0.32
2004 July–August	−1075	20	0.019	0.14	0.15
		60	0.056	0.42	0.45

3 Systematic uncertainty in $\mathbf{R_0}$ owing to a nonzero motion of orbital focus (Sgr A*)

Neglect of a possible *radial* motion of Sgr A* relative to the Local Standard of Rest (LSR), $V_r^* \equiv V_r$(Sgr A*), is equivalent to the introducing a corresponding systematic error in all V_r values. This error is equal to a value of V_r^* and is the same in all measurements of V_r. From Eqs. (1) and (2) follows that the relative systematic error in V_r velocity fully converts to the relative systematic error in R_0, i.e.,

$$\delta_{\rm sys} \equiv \sigma_{\rm sys}(V_r)\,/|V_r| = V_r^*\,/|V_r| = \sigma_{\rm sys}(R_0)\,/R_0\ . \tag{3}$$

These simple considerations make it possible readily to evaluate the systematic error in R_0. The higher is $|V_r|$, the lower is $\delta_{\rm sys}$ at the same V_r^*. Thus highest absolute values of V_r measured shortly (but not immediately) after the pericentre passage of S2 give a *lower* limit for $\sigma_{\rm sys}(R_0)$. Table 1 presents values of $\sigma_{\rm sys}(R_0)$ calculated for these measurements of V_r [3] with Sgr A*'s velocities of $V_r^* = 20$ and 60 km s^{-1}, plausible for massive objects in the GC [6, 3], and with $R_0 = 7.5$ and 8.0 kpc [7, 8, 3]. In Table 1, $\langle V_r \rangle$ is the average of velocities V_r [3] over the observational period.

Table 1 demonstrates that neglect of possible radial velocity of the orbital focus (Sgr A*) can lead to a substantial systematic error in R_0: for star S2, $\sigma_{\rm sys}(R_0) \geq 1.3$–$5.6\% = (0.1$–$0.45$ kpc$) \times (R_0/8$ kpc) with $V_r^* = 20$–60 km s^{-1}. Notice that the value of $\sigma_{\rm sys}(R_0)$ can not be reduced statistically.

The effect of nonzero *proper* motion of Sgr A*, V_μ^*, on R_0 can be shown to increase the effect of nonzero radial velocity V_r^* of Sgr A* by factor up to ≈1.9 for equal components of motion ($V_\mu^{*2} = 2V_r^{*2}$); see details in [9].

These estimates are recently corroborated by results of direct introducing components of Sgr A* motion as free parameters [10, 11].

4 Relativistic and non-Keplerian effects for $\mathbf{R_0}$

The same method as in Sections 2 and 3 can be applied to estimate the systematic error in R_0 due to leading-order (β^2, $\beta \equiv V_r/c$) Newtonian and relativistic effects: the Rømer delay, the transverse Doppler shift, and the gravitational redshift [12].

Over the period of observations [4, 3], the positive contribution of β^2 effects, ΔV_r, to the measured (negative) velocities V_r of S2 (see [12]) give decreasing R_0, since $\Delta_{\rm sys} \equiv \Delta V_r/V_r = \Delta R_0/R_0$ (see similar Eq. 3). Use of calculations [12] for the effects result in: for $\langle V_r\rangle(2003.35) = -1500$ km s^{-1}, $\Delta V_r = +26$ km s^{-1} and $\Delta R_0 = -0.14$ kpc $\times$ $(R_0/8$ kpc); for $V_r(2002.42) = -510$ km s^{-1} just after the pericentre passage [4], $\Delta V_r = +134$ km s^{-1} and $\Delta R_0 = -1.7$ kpc $\times$ $(R_0/8$ kpc) (!). However, with regard to more recent V_r values the *total* β^2 bias is $\Delta R_0 \sim -0.1$ kpc, which is consistent with results [11].

The gravitational redshift contributes also a constant (positive) zeroth-order term to the observed V_r: $\Delta V_r/c = B_{0,\rm G} \equiv GM/\left(2c^2a\right)$ [12]. For S2, $B_{0,\rm G}$ is not completely negligible ($\sim 6 \pm 1$ km s^{-1}). Notice that while $B_{0,\rm G}$ is degenerate with V_r^* in the case of *single* star's orbital solution, the degeneracy can be broken by a solution for *a number* orbits with different values of a.

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