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Interpolation and forecast of irregularity of the deformable Earth rotation

V.V. Bondarenko*, V.V. Perepelkin

Moscow Aviation Institute (State Technical University)

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The mathematical model of irregularity of the Earth rotation is constructed based on gravitational-tidal mechanism of the Sun and the Moon influences by the methods of celestial mechanics. The irregularity of the Earth rotation is related as fluctuations of the length of the day and correction *UT1–UTC*. The comparison of the forecast based on the model and IERS measurements proves that the model fits thin dynamic peculiarities of the Earth rotation adequately.

Keywords: Earth rotation irregularities; Day length fluctuations; UT1–UTC correction

It was accepted in the second half of XX century that the Earth rotates around its axes irregularly. The observed irregularities of the Earth rotation are separated in the science literature as short period ones (within one year), inter-year variations (6–100 years), secular variations (during the period of the Earth axes precession) and on cosmogony time intervals. The irregularity of the Earth axes rotation and variations of the length of the day on short time intervals generally depend on perturbing moments of forces from the Earth deformations which correspond to perturbations of components of the Earth inertia tensor under the action of lunar and solar tides that have celestial-mechanics origin.

The possibility to study the irregularities of the Earth axes rotation more precisely appeared with the creation of the unit high-precision uniform physical scale of atomic time *TAI* independent of daily and orbital motion of the Earth. The refinement of the fluctuations of the Earth rotation velocity depends on the precision of determination of the international time *UT1* fixed with the Earth rotation naturally. The scale *UT1* is used to construct the international coordinated time *UTC* which is atomic time *TAI* corrected on integer number of seconds to correspond approximately with the international time *UT1* fixed with the Earth rotation [1, 2]. Therefore the creation of a mathematical model of variations of the length of the day $l.o.d.(t)$ on the short time intervals and obtaining the value of correction *UT1–UTC* are one of the ways of mathematical representation of the irregularity of the Earth rotation. The Euler–Lowville dynamic equations with variable inertia tensor are used to study the deformable Earth axis rotation [3–5]:

$$J\omega^{\bullet} + [\omega \times J\omega] = M, \quad M = M_{\mathrm{K}} + M^{\mathrm{S}} + M^{\mathrm{L}}, \quad J = J^{*} = \delta J. \tag{1}$$

*E-mail: v.v.bon@yandex.ru

Here $\omega = (p, q, r)$ is vector of the Earth rotation angular velocity in the fixed coordinate system, r is axial component being the dominant value by 6 orders of magnitude greater than components p and q, J^* is inertia tensor of "frozen" the Earth, variations of inertia tensor δJ contain the harmonic components of gravitational daily tides of the Moon and the Sun and maybe others (annual, half-annual, month's, two-weeks, etc.), $M^{\mathrm{S,L}}$ are gravitational-tidal perturbed moments of the Sun and the Moon correspondently, M_{K} are additional perturbed terms obtained by differentiation of angular momentum vector of the deformable Earth. Let us write the equation of the system (1) for component of the Earth axes rotation:

$$C^* r^{\bullet} + (B^* - A^*)\, pq + (J_{qr} p - J_{pr} q)\, r = M_r^{\mathrm{S}} + M_r^{\mathrm{L}}. \tag{2}$$

Here J_{pr}, J_{qr} are small non-diagonal elements of the inertia tensor. The expression for M_r^{S} (for example) takes the form after averaging for quick variable φ (angle of self rotation) [5]:

$$M_r^{\mathrm{S}} = 3\omega_0^2 \left(\chi_{1r}^{\mathrm{S}} \sin^2\theta + \chi_{2r}^{\mathrm{S}} \sin\theta\cos\theta\right). \tag{3}$$

The values χ_{1r}^{S} and χ_{2r}^{S} in equation (3) are conditioned by half-daily and daily tides correspondently, they are obtained as a result of φ averaging of coefficients at expressions $\sin^2\theta$ and $\sin\theta\cos\theta$ and must be determined based on observations data. The structure of fluctuations of the length of the day is obtained by integration of equation (2):

$$l.o.d.\,(\tau) = c + a_{\mathrm{c}}^{\mathrm{S}} \cos(2\pi\tau) + a_{\mathrm{s}}^{\mathrm{S}} \sin(2\pi\tau) + b_{\mathrm{c}}^{\mathrm{S}} \cos(4\pi\tau) + b_{\mathrm{s}}^{\mathrm{S}} \sin(4\pi\tau) +$$

$$a_{\mathrm{c}}^{\mathrm{L}} \cos(2\pi\nu_{\mathrm{m}}\tau) + a_{\mathrm{s}}^{\mathrm{L}} \sin(2\pi\nu_{\mathrm{m}}\tau) + b_{\mathrm{c}}^{\mathrm{L}} \cos(2\pi\nu_{\mathrm{f}}\tau) + b_{\mathrm{s}}^{\mathrm{L}} \sin(2\pi\nu_{\mathrm{f}}\tau). \tag{4}$$

Here $\nu_{\mathrm{m}} = 13.28$, $\nu_{\mathrm{f}} = 26.68$ are the frequencies of month's and two-weeks oscillations of lunar perturbation, unknown values c and all a and b are values that must be calculated from IERS measurements.

Let us write the expression for correction *UT1–UTC* for within one year time intervals from equation (4):

$$(UT1 - UTC)\,(\tau)\,/365.25 = -\int \mathrm{l.o.d.}\,(\tau)\, d\tau = \mathrm{const} - c\tau -$$

$$(2\pi)^{-1}\left(a_{\mathrm{c}}^{\mathrm{S}} \sin(2\pi\tau) - a_{\mathrm{c}}^{\mathrm{S}} \cos(2\pi\tau)\right) - (4\pi)^{-1}\left(b_{\mathrm{c}}^{\mathrm{S}} \sin(4\pi\tau) - b_{\mathrm{s}}^{\mathrm{S}} \cos(4\pi\tau)\right) -$$

$$(2\pi\nu_{\mathrm{m}})^{-1}\left(a_{\mathrm{c}}^{\mathrm{L}} \sin(2\pi\nu_{\mathrm{m}}\tau) - a_{\mathrm{s}}^{\mathrm{L}} \cos(2\pi\nu_{\mathrm{m}}\tau)\right) -$$

$$(2\pi\nu_{\mathrm{f}})^{-1}\left(b_{\mathrm{c}}^{\mathrm{L}} \sin(2\pi\nu_{\mathrm{f}}\tau) - b_{\mathrm{s}}^{\mathrm{L}} \cos(2\pi\nu_{\mathrm{f}}\tau)\right). \tag{5}$$

The numerical simulation of within one year tidal irregularity of the Earth axis rotation was carried out according to constructed model (4) and (5) by the least squares method based on IERS observations and measurements data [1]. The forecast of the length of the day (*l.o.d.*) fluctuations for 2006 is presented on Fig. 1: 1 is curve of IERS data; 2 is theoretical curve obtained with the help of nine parameters model by interpolation of *l.o.d.* measurements data in 2005. The estimation of mean-square error σ_9 for this model was carried out:

$$\sigma_9 = 0.201 \cdot 10^{-3}.$$

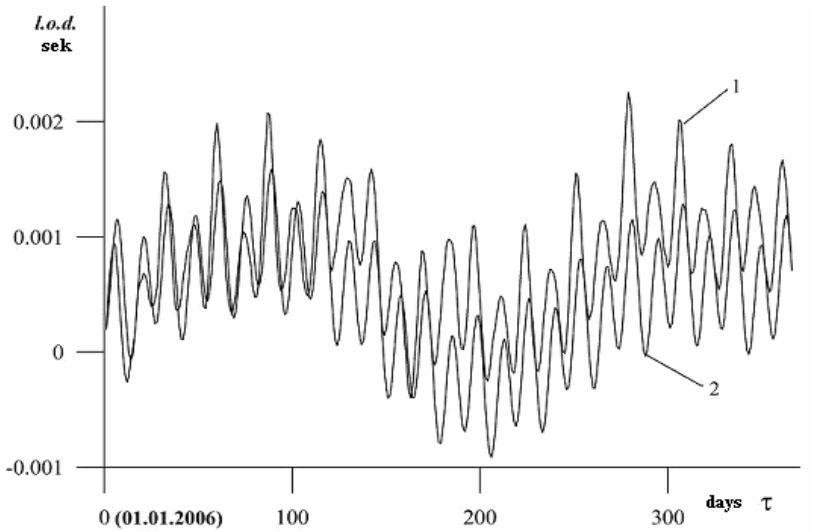


Figure 1 The forecast of the length of the day (*l.o.d.*) fluctuations for 2006.

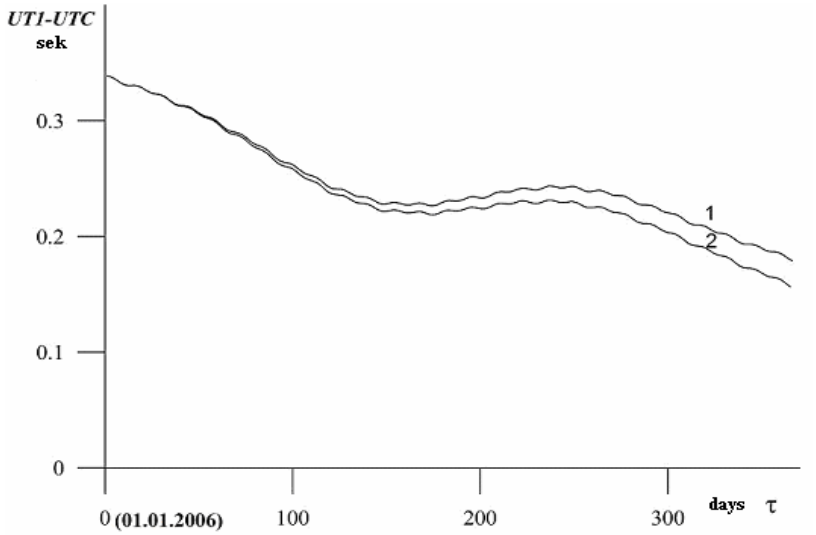


Figure 2 The forecast of correction *UT1–UTC* for 2006.

The forecast of correction *UT1–UTC* for 2006 based on expression (5) and obtained forecast for *l.o.d.* is given on Fig. 2. The comparison of nature and theoretical trajectories of irregularity of tidal oscillations of rotation angular velocity certifies that constructed model fits IERS measurements data adequately. This work was supported by the Russian Foundation for Basic Research (project 10-02-00595).

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