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Some self-consistent models for disc galaxies

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Some special solutions of the integral equations for the distribution function of gravitating discs are discussed.

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1 Introduction

It is a fact that the main mass of a typical spiral galaxy is concentrated in a thin disc. A lot of observational data on structure and kinematics of nearby galaxies is available now. It will be of great interest to link observations with theories of Galactic Dynamics. The first task is to construct available models of mass distribution. The problem was studied by many authors (e.g. [1, 2]). The power-law discs by Bisnovatyi-Kogan [3] and Evans [4] are examples of such models.

The final purpose of modelling stellar systems is to find a steady distribution function (DF). According to Jeans' theorem as applied to steady stellar discs DF has the form $f(r, v_r, v_\theta) = \Psi(E, L)$. Here r is the distance to the centre of the system, (v_r, v_θ) is the planar velocity vector, E is the energy, L is the modulus of the angular momentum, $E = (1/2)(v_r^2 + v_\theta^2) - \Phi(r)$, $L = rv_\theta$ ($\Phi(r)$ is the positive potential). Some authors studied disc models starting with the velocity distribution (modified Maxwellian or Schwarzschildian one) [5–7],but most authors preferred the way "from the potential to DF" [2] (e.g. [8–14]). A general theory of equilibrium collisionless discs was developed in [15–18]. Pichon & Lynden-Bell [19] discussed the kinematics of steady models and constructed DFs for which Toomre's stability parameter Q was given.

2 General formulae

We use the following notations: $\mu(r)$ is the surface density, r_* is the boundary radius ($r_* = \infty$ for infinite systems), $V(r)$ is the streaming velocity. We shall suppose that

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the potential $\Phi(r)$ is a monotonously decreasing function. Denote $y = 2[\Phi(r) - \Phi(r_*)]$, $\xi = rv_\theta$, and $x = y - v_r^2 - (r/a)v_\theta^2$ with $(r/a)^2 = 1 + \lambda r^2$ and $\lambda = \text{ const} \geq -r_*^2$ (λ is the velocity anisotropy parameter).

Then $\Psi = \Psi(x, \xi)$ and [17]

$$\mu < v_r^m v_\theta^n >= (2r^{n+1})^{-1} \iint\limits_{D(r)} \xi^n [x_*(\xi) - x]^{(m-1)/2} \Psi(x, \xi)\, \mathrm{d}x\mathrm{d}\xi$$

where $D(r) = \{(x, \xi) : x \geq 0 \wedge x \leq x_*(\xi)\}$ and $x_*(\xi) = y - (\xi/a)^2$ corresponds to Lindblad's parabola ($v_r = 0$) at the (x, ξ) plane. The cases $(m, n) = (0, 0)$, $(m, n) = (0, 1)$ and $(m, n) = (2, 0)$, $(m, n) = (0, 2)$ are of the most interest. So,

$$\mu(r) = (2r)^{-1} \iint\limits_{D(r)} \Upsilon(x, \xi)[x_*(\xi) - x]^{-1/2}\, \mathrm{d}x\mathrm{d}\xi$$

and

$$V(r) = (2\mu r^2)^{-1} \iint\limits_{D(r)} \xi \Xi(x, \xi)[x_*(\xi) - x]^{-1/2}\, \mathrm{d}x\mathrm{d}\xi$$

where $\Upsilon = \dfrac{1}{2}[\Psi(x, \xi) + \Psi(x, -\xi)]$, $\Xi = \dfrac{1}{2}[\Psi(x, \xi) - \Psi(x, -\xi)]$. These formulae will be considered as integral equations for finding the even Υ and the odd Ξ parts of DF. Some considerations how to find Ξ were suggested in [17]. Examples of solutions were given by Kutuzov [20]. Below we shall discuss the first equation.

3 Some special cases

The case of the ellipsoidal velocity distribution, $\Upsilon = \Upsilon(x)$ is the simplest one and was considered in [16, 17]. Now we shall briefly discuss a more general model with $\Upsilon(x, \xi) = \xi^{2\alpha}\phi(x)$ [17, 18]. After some algebra it can be found that

$$\frac{\mu}{r} a^{-2\alpha} = \pi^{1/2} \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)} \int\limits_0^y (y - x)^\alpha \phi(x)\, \mathrm{d}x$$

and the problem of finding $\phi(x)$ is reduced to the generalized Abel equation. Its detailed analysis was given in [18].

Suppose that $\Upsilon(x, \xi) = \delta(x)\psi(\xi)$ with $\lambda = -r_*^2$ [21]. In this model all stars reach the boundary circle. The similar spherical model was qualitively discussed by Poincaré [22]. Then the problem is reduced to the Abel equation and

$$f(\eta) = \frac{1}{\pi} \left[\frac{h(0)}{\eta^{1/2}} + \int\limits_0^\eta \frac{h'(t)\mathrm{d}t}{(\eta - t)^{1/2}} \right]$$

where $t = a^2 y$, $\eta = \xi^2$, $h(t) = (r/a)\mu$, $f(\eta) = \psi(\xi)/2\xi$.

Also the opposite model when DF does not depend on our Jacobi integral x (and on energy in the case $\lambda = 0$) (i.e. $\Upsilon(x, \xi) = \psi(\xi)$) is of some interest. It is easy to find after some algebra that

$$\psi(\xi) = \frac{1}{\pi}\left[\frac{F(0)}{\xi^{1/2}} + \int\limits_0^{\xi} \frac{F'(z)\mathrm{d}z}{(\xi - z)^{1/2}}\right]$$

with $F(z) = \dfrac{1}{4}\mu(r)\dfrac{a}{r}$, $z = ay^{1/2}$.

It is evident that disc models analogous to Gerhard's spheres [23] can be constructed. We consider a slightly simplified version of them. Denote $\xi_c^2(x) = a^2(y - x)$ (i.e. ξ_c is the angular momentum for circular motions with a given x). Then

$$\mu = \frac{1}{r}\int_0^y \int_0^{\xi_c(x)} \frac{a\Upsilon(x, \xi)\,\mathrm{d}\xi\mathrm{d}x}{(\xi_c^2 - \xi^2)^{1/2}}.$$

Now we put $t = \xi^2/\xi_c^2$ and suppose that $\Upsilon(x, \xi) = \varphi(x)h(t)$, $\varphi(x)$ being an unknown function and $h(t)$ (Gerhard's circulation function) being a given one. Then we can write down the integral equation for $\varphi(x)$. If $h(t) = t^k$ it looks as follows:

$$2r\mu = \frac{\pi^{1/2}\Gamma(k + 1/2)}{\Gamma(k)}\int_0^y \frac{\varphi(x)\mathrm{d}x}{(y - x)^{1/2}}.$$

The problem is reduced to the Abel equation.

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References

[1] A. M. Fridman and V. L. Polyachenko, *Physics of Gravitating Systems*. Vol. 1. (New York, Springer, 1984).
[2] J. Binney and S. Tremaine, *Galactic Dynamics* (Princeton, Princeton University Press, 1987).
[3] G. S. Bisnovatyi-Kogan, Sov. Astron. Lett. **1** 177 (1975).
[4] N. W. Evans, Mon. Not. R. Astron. Soc. **267** 333 (1994).
[5] E. W. Ng, Astrophys. J. **150** 387 (1969).
[6] J. R. J. Lafon, Astron. Astrophys. **46** 461 (1976).
[7] E. Zweibel, Astrophys. J. **222** 103 (1978).
[8] K. C. Freeman, Mon. Not. R. Astron. Soc. **134** 15 (1966).
[9] G. S. Bisnovatyi-Kogan and Ya. B. Zel'dovich, Astrofizics **6** 207 (1970).
[10] M. Miyamoto, Publ. Astron. Soc. Japan **23** 21 (1971).

[11] M. Miyamoto, Astron. Astrophys. **30** 441 (1974).
[12] M. Miyamoto, Publ. Astron. Soc. Japan **27** 431 (1975).
[13] G. S. Bisnovatyi-Kogan and Ya. B. Zel'dovich, *Dynamics and Evolution of Stellar Systems*, ed. K. F. Ogorodnikov (Moscow, Leningrad, VAGO, GAO AN SSSR, 1975), p. 138.
[14] N. W. Evans and J. C. Colett, Mon. Not. R. Astron. Soc. **264** 35 (1993).
[15] S. Aoki, Publ. Astron. Soc. Japan **25** 75 (1973).
[16] S. Kalnajs, Astrophys. J. **205** 751 (1975).
[17] L. P. Ossipkov, Sov. Astron. Lett. **4** 36 (1978).
[18] N. Hiotelis and P. Patsis, Astrophys. Space Sci. **206** 21 (1993).
[19] C. Pichon and D. Lynden-Bell, Mon. Not. R. Astron. Soc. **282** 1143 (1996).
[20] S. A. Kutuzov, Astron. Astrophys. Transact. **7** 11 (1995).
[21] L. P. Ossipkov, *Structure and Evolution of Stellar Systems*, ed. T. A. Agekian, A. A. Mülläri, V. V. Orlov (Saint Petersburg, Publ. Saint Petersburg University, 1997), p. 266.
[22] H. Poincaré, Bull. Soc. astron. France **20** 153 (1906).
[23] O. E. Gerhard, Mon. Not. R. Astron. Soc. **250** 812 (1991).