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Estimation of errors at the calibration of spectral data at the Irbene RT-32 radio telescope

O. Antyufeyev[1,2,*], M. Bleiders[1], O. Patoka[1,2], V. Bezrukovs[1], A. Aberfelds[1], I. Shmeld[1], A. Orbidans[1], J. Steinbergs[1], V. Shulga[2,3]

[1]*Engineering Research Institute "Ventspils International Radio Astronomy Centre" of Ventspils University of Applied Sciences, Inzenieru Str. 101, Ventspils, LV-3601, Latvia*

[2]*Institute of Radio Astronomy of the National Academy of Sciences of Ukraine, 4, Mystetstv St., Kharkiv, 61002, Ukraine*

[3]*International Center of Future Science of the Jilin University, 2699 Qianjin St., 130012 Changchun, PR China*

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We analyze calibration errors that occur during the single-dish observations at the Irbene RT-32 radio telescope when calibration is done using noise diode. Only frequency-switched mode of spectral line observations is considered. The purpose of the article is to find possible values of noise diode temperature T_{cal} and period of switching oscillator frequency $2t_q$. For this, we have analyzed the formula for calculating the antenna temperature of the source. It is shown that noise diode temperature affects the standard deviation of source antenna temperature σ_{T_s} and the error of source antenna temperature δ_{T_s} caused by instability of system temperature. t_q affects the stability of the characteristics of the receiving system. In this article, the dependencies between T_{cal}, t_q, σ_{T_s} and δ_{T_s} were found and analyzed. Based on these dependencies, possible values of T_{cal} and t_q are found.

Keywords: Radio telescope, calibration, single-dish, spectral observation

1 Introduction

During radio astronomical observation in single-dish mode two calibration methods are used. High level of the calibration signal is required for the first method. Signal accumulation from astronomical source stops during the calibration. The first method is usually used at radio telescopes of millimeter waves. In this case, the signal from the black body is usually used as a calibration signal. The second method involves mixing the calibration and astronomical source signal. Thus, calibration and observation happen simultaneously. At that calibration signal cannot be strong in order

*Email: antyuf@rian.kharkov.ua

to prevent substantial increase in system temperature. This method is often used at radio telescopes of cm waves. In this case calibration signal is generated by noise diode.

Our article is devoted to the second method. Some articles have been dedicated to this topic and these articles usually describe the calculation of the antenna temperature of the source (see Winkel et al.(2012), Maddalena (2002), Braatz (2009)). The paper of Winkel et al.(2012) presents numerical experiments to determine the errors of various calibration methods. We focus on theoretical analysis of errors of this source antenna temperature resulting from fitting, instability of system parameters and search for the suitable temperature of noise diode and period of switching oscillator frequency. We consider the calibration of single-dish data of the RT-32 radio telescope at Irbene in C-band.

2 Basic definitions and formulation of the problem

We used the method proposed by Winkel et al.(2012) to calculate the antenna temperature of the source. In this article we consider only the case of frequency modulation. Calibration and observation processes happen simultaneously. Observation time (Fig. 1) is divided into small parts (HS periods). During the HS period, elevation angle, atmospheric opacity, radio telescope gain DPFU (degree per flux unit), receiver system gain g and system temperature T_{sys} are considered constant.

During the HS period there is accumulation of signal P at the different modes of noise diode and shift of oscillator frequency (Fig. 1). Definitions c_on and c_off in Fig. 1 correspond to the on and off modes of the noise diode. Definitions "sig" and "ref" correspond to the shift of the oscillator frequency to $\pm\Delta f_m$. In all cases, the

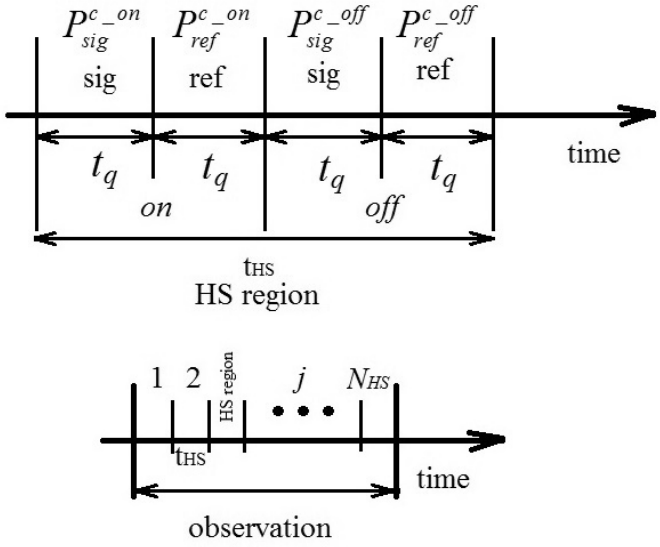


Figure 1 Time structure of the observation process.

signal P accumulates in the interval t_q. For each HS region, data are accumulated over time $t_{HS} = 4t_q$ and then T_s is determined.

Equations for calculation of antenna temperature of the source T_s can be written as (see Winkel et al.(2012)):

$$\begin{aligned} T_s(f) = \tfrac{1}{4} \Bigg(& T_{cal}(f)\, k_{ref}(f-\Delta f_m)\, \frac{P_{sig}^{c\text{-}off}(f-\Delta f_m) - P_{ref}^{c\text{-}off}(f-\Delta f_m)}{P_{ref}^{c\text{-}off}(f-\Delta f_m)} + \\ & + T_{cal}(f) \left[k_{ref}(f-\Delta f_m) + 1 \right] \frac{P_{sig}^{c\text{-}on}(f-\Delta f_m) - P_{ref}^{c\text{-}on}(f-\Delta f_m)}{P_{ref}^{c\text{-}on}(f-\Delta f_m)} \Bigg) \\ & + \tfrac{1}{4} \Bigg(T_{cal}(f)\, k_{sig}(f+\Delta f_m)\, \frac{P_{ref}^{c\text{-}off}(f+\Delta f_m) - P_{sig}^{c\text{-}off}(f+\Delta f_m)}{P_{sig}^{c\text{-}off}(f+\Delta f_m)} \\ & T_{cal}(f) \left[k_{sig}(f+\Delta f_m) + 1 \right] \frac{P_{ref}^{c\text{-}on}(f+\Delta f_m) - P_{sig}^{c\text{-}on}(f+\Delta f_m)}{P_{sig}^{c\text{-}on}(f+\Delta f_m)} \Bigg) \end{aligned} \tag{1}$$

$$k_{ref}(f-\Delta f_m) = \left(fitting \left(\frac{P_{ref}^{c\text{-}on}(f-\Delta f_m)}{P_{ref}^{c\text{-}off}(f-\Delta f_m)} \right) - 1 \right)^{-1}$$

$$k_{sig}(f+\Delta f_m) = \left(fitting \left(\frac{P_{sig}^{c\text{-}on}(f+\Delta f_m)}{P_{sig}^{c\text{-}off}(f+\Delta f_m)} \right) - 1 \right)^{-1}$$

Here T_{cal} is the temperature of the noise diode. We used the method of least squares to fit $\frac{P_{ref}^{c\text{-}on}(f-\Delta f_m)}{P_{ref}^{c\text{-}off}(f-\Delta f_m)}$ and $\frac{P_{sig}^{c\text{-}on}(f+\Delta f_m)}{P_{sig}^{c\text{-}off}(f+\Delta f_m)}$ by low degree polynomial. It should be adjusted for absorption in the atmosphere and converted to units of flux density

$$S_s = \frac{e^{\tau_0 \cdot AM}}{DPFU} T_s \tag{2}$$

Here $DPFU$ is radio telescope gain, τ_0 is the zenith opacity, AM $= 1/\sin(\mathrm{El})$ is air mass. Then we can increase the number of HS regions to get the desired standard deviation of the source flux density. The resulting source flux density will be equal to the average $\langle S_s \rangle$ over all HS regions.

In this article we consider some types of errors arising at calculation of T_s and its relation with T_{cal} and t_q for determination of suitable values T_{cal} and t_q. The most studied of these errors is standard deviation of antenna temperature of the source σ_{T_s} caused by noises of the system. Also we have considered errors arising at the fitting of k_{ref} , k_{sig} and error δ_{T_s} arising from the instability of T_{cal}, receiver system gain g and system temperature T_{sys}. These errors are errors in determining the amplitude of the spectral line. Errors of δ_{T_s}, errors in the fitting of the k_{ref} and k_{sig} weakly affect the σ_{T_s} and this effect is not taken into account in this article.

Basic equations used for calculations are shown below. Described above signals P are noise signals characterized by a mean value and standard deviation. Receiver system gain g is supposed to be linear. On these assumptions mean values and standard deviations of signals P in every spectrometer channel may be written as:

$$
\begin{aligned}
&\overline{P_{sig}^{c_on}(f-\Delta f_m)} = g\,(T_{sys}+T_{cal}+T_s)\\
&\overline{P_{sig}^{c_off}(f-\Delta f_m)} = g\,(T_{sys}+T_s)\\
&\overline{P_{ref}^{c_on}(f-\Delta f_m)} = g\,(T_{sys}+T_{cal})\\
&\overline{P_{ref}^{c_off}(f-\Delta f_m)} = gT_{sys}\\
&\overline{P_{sig}^{c_on}(f+\Delta f_m)} = g\,(T_{sys}+T_{cal})\\
&\overline{P_{sig}^{c_off}(f+\Delta f_m)} = gT_{sys}\\
&\overline{P_{ref}^{c_on}(f+\Delta f_m)} = g\,(T_{sys}+T_{cal}+T_s)\\
&\overline{P_{ref}^{c_off}(f+\Delta f_m)} = g\,(T_{sys}+T_s)\\
&\sigma_{P_{sig}^{c_on}(f-\Delta f_m)} = \sigma_{P_{ref}^{c_on}(f+\Delta f_m)} = g\frac{T_{sys}+T_{cal}+T_s}{\sqrt{\Delta f\cdot t_q}}\\
&\sigma_{P_{sig}^{c_off}(f-\Delta f_m)} = \sigma_{P_{ref}^{c_off}(f+\Delta f_m)} = g\frac{T_{sys}+T_s}{\sqrt{\Delta f\cdot t_q}}\\
&\sigma_{P_{ref}^{c_on}(f-\Delta f_m)} = \sigma_{P_{sig}^{c_on}(f+\Delta f_m)} = g\frac{T_{sys}+T_{cal}}{\sqrt{\Delta f\cdot t_q}}\\
&\sigma_{P_{ref}^{c_off}(f-\Delta f_m)} = \sigma_{P_{sig}^{c_off}(f+\Delta f_m)} = g\frac{T_{sys}}{\sqrt{\Delta f\cdot t_q}}
\end{aligned}
\tag{3}
$$

Here Δf is spectral resolution of spectrometer.

Formula for the standard deviation of a random variable, which is the function of some other random independent variables, is determined from error analysis. If $z, A, B, ...$ are random variables and $z = f(A, B, ...)$ then

$$
\sigma_z = \sqrt{\left(\frac{\partial f}{\partial A}\right)^2\Bigg|_{\substack{A=\bar{A}\\B=\bar{B}}}\cdot\sigma_A^2 + \left(\frac{\partial f}{\partial B}\right)^2\Bigg|_{\substack{A=\bar{A}\\B=\bar{B}}}\cdot\sigma_B^2 + ...} \tag{4}
$$

In this article relative errors are defined as ratio of triple standard deviation to mean value, which corresponds to the confidence level 99.7%. That is, all measurements with a probability of 99.7% have smaller errors than those that we found. In order to determine error with confidence level 95% and 68% it should be lowered by 1.5 and 3 times correspondingly.

3 Increase of standard deviation of source antenna temperature caused by the presence of calibration signal

The use of the calibration signal during observation leads to an increase of the standard deviation of the source antenna temperature. In this section we calculate its value.

First we calculated standard deviation of T_s. To achieve this we assume that $T_s << T_{cal}$ and $T_s << T_{sys}$. Since k_{ref} and k_{sig} are fitted by polynomial, their standard deviations are 0. At that $\overline{k_{ref}} = \overline{k_{sig}} = \frac{T_{sys}}{T_{cal}}$. Equation (1) can be rewritten as

$$T_s \approx \tfrac{1}{4}\left[T_{sys}\left(\frac{P_{sig}^{c_off}(f-\Delta f_m)}{P_{ref}^{c_off}(f-\Delta f_m)}-1\right)+\left(T_{sys}+T_{cal}\right)\left(\frac{P_{sig}^{c_on}(f-\Delta f_m)}{P_{ref}^{c_on}(f-\Delta f_m)}-1\right)\right]+$$
$$+\tfrac{1}{4}\cdot\left[T_{sys}\left(\frac{P_{ref}^{c_off}(f+\Delta f_m)}{P_{sig}^{c_off}(f+\Delta f_m)}-1\right)+\left(T_{sys}+T_{cal}\right)\left(\frac{P_{ref}^{c_on}(f+\Delta f_m)}{P_{sig}^{c_on}(f+\Delta f_m)}-1\right)\right]$$

T_s is the function of random variables $P_{sig}^{c_on}$, $P_{sig}^{c_off}$, $P_{ref}^{c_on}$ and $P_{ref}^{c_off}$. Using (3) we can write

$$\sigma_{T_s}^2=\left(\tfrac{1}{4}\right)^2\left[T_{sys}^2\left(\left(\frac{\sigma_{P_{sig}^{c_off}(f-\Delta f_m)}}{\overline{P_{ref}^{c_off}(f-\Delta f_m)}}\right)^2+\left(\frac{\overline{P_{sig}^{c_off}(f-\Delta f_m)}}{\overline{P_{ref}^{c_off}(f-\Delta f_m)}}\frac{\sigma_{P_{ref}^{c_off}(f-\Delta f_m)}}{\overline{P_{ref}^{c_off}(f-\Delta f_m)}}\right)^2\right)+\right.$$
$$\left.+\left(T_{sys}+T_{cal}\right)^2\left(\left(\frac{\sigma_{P_{sig}^{c_on}(f-\Delta f_m)}}{\overline{P_{ref}^{c_on}(f-\Delta f_m)}}\right)^2+\left(\frac{\overline{P_{sig}^{c_on}(f-\Delta f_m)}}{\overline{P_{ref}^{c_on}(f-\Delta f_m)}}\frac{\sigma_{P_{ref}^{c_on}(f-\Delta f_m)}}{\overline{P_{ref}^{c_on}(f-\Delta f_m)}}\right)^2\right)\right]+$$
$$+\left(\tfrac{1}{4}\right)^2\left[T_{sys}^2\left(\left(\frac{\sigma_{P_{ref}^{c_off}(f+\Delta f_m)}}{\overline{P_{sig}^{c_off}(f+\Delta f_m)}}\right)^2+\left(\frac{\overline{P_{ref}^{c_off}(f+\Delta f_m)}}{\overline{P_{sig}^{c_off}(f+\Delta f_m)}}\frac{\sigma_{P_{sig}^{c_off}(f+\Delta f_m)}}{\overline{P_{sig}^{c_off}(f+\Delta f_m)}}\right)^2\right)+\right.$$
$$\left.+\left(T_{sys}+T_{cal}\right)^2\left(\left(\frac{\sigma_{P_{ref}^{c_on}(f+\Delta f_m)}}{\overline{P_{sig}^{c_on}(f+\Delta f_m)}}\right)^2+\left(\frac{\overline{P_{ref}^{c_on}(f+\Delta f_m)}}{\overline{P_{sig}^{c_on}(f+\Delta f_m)}}\frac{\sigma_{P_{sig}^{c_on}(f+\Delta f_m)}}{\overline{P_{sig}^{c_on}(f+\Delta f_m)}}\right)^2\right)\right]$$

Using (4) and taking into account that $T_s << T_{cal}$ and $T_s << T_{sys}$, the standard deviation of T_s is written as

$$\sigma_{T_s}=\sqrt{\left(\tfrac{1}{4}\right)^2\cdot 2\left[T_{sys}^2\tfrac{2}{\Delta f\cdot t_q}+\left(T_{sys}+T_{cal}\right)^2\tfrac{2}{\Delta f\cdot t_q}\right]},\text{ or}$$

$$\sigma_{T_s}=\sqrt{\frac{T_{sys}^2+\left(T_{sys}+T_{cal}\right)^2}{4\cdot\Delta f\cdot t_q}} \tag{5}$$

The smallest standard deviation $\sigma_{T_s}^{\min}$ is achieved with $T_{cal}=0$. It is equal to $\sigma_{T_s}^{\min}=\sqrt{\frac{T_{sys}^2}{2\cdot\Delta f\cdot t_q}}$. Thus, we obtain the additional standard deviation percentage, which is due to the influence of the calibration signal

$$\delta_{\sigma_{T_s}}=\frac{\sigma_{T_s}-\sigma_{T_s}^{\min}}{\sigma_{T_s}^{\min}}=\left(\sqrt{\frac{1+\left(1+\frac{T_{cal}}{T_{sys}}\right)^2}{2}}-1\right)\cdot 100\% \tag{6}$$

Figure 2 shows the dependence of additional standard deviation percentage on T_{sys}/T_{cal}.

T_{cal} influence can be presented as Δt, the additional time required for compensating increase σ_{T_s} due to the presence of calibration signal. At $T_{cal}\neq 0$ we can achieve $\sigma_{T_s}^{\min}$ by increasing observation time by Δt. In this case

$$\sigma_{T_s}=\sqrt{\frac{T_{sys}^2+\left(T_{sys}+T_{cal}\right)^2}{4\Delta f(t_q+\Delta t)}}=\sigma_{T_s}^{\min}=\sqrt{\frac{T_{sys}^2}{2\Delta f t_q}}.$$

Hence we can calculate the additional time percentage δ_{time} necessary for compensating calibration signal influence

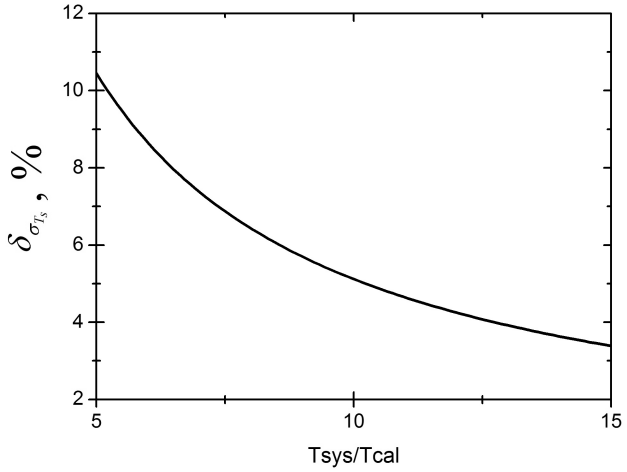


Figure 2 Dependence of additional standard deviation percentage on T_{sys}/T_{cal}.

$$\delta_{time} = \frac{\Delta t}{t_q} = \left(\frac{1}{2}\left(1 + \frac{T_{cal}}{T_{sys}}\right)^2 - \frac{1}{2}\right) 100\%$$

4 Errors of fitting k_{ref} and k_{sig}

There are errors arising from fitting during calculation of k_{ref} and k_{sig}. In this section we obtain standard deviation of k_{ref} and its dependence on t_q, T_{sys} and T_{cal}, number of spectrometer channels and frequency resolution. k_{ref} and k_{sig} are random variables. But their mean values and standard deviations coincide. So we are going to analyze k_{ref} only. For k_{sig} the results will be the same.

k_{ref} can be written as

$$k_{ref} = \frac{1}{F_{fit} - 1}, \tag{7}$$

where $F_{fit} \equiv fitting\left(\frac{P_{ref}^{c_on}(f-\Delta f_m)}{P_{ref}^{c_off}(f-\Delta f_m)}\right)$

Fitting does not influence mean values of the resulting function. So, mean values $\overline{k_{ref}}$ and $\overline{F_{fit}}$ may be written using (3)

$$\overline{F_{fit}} = \left(\frac{\overline{P_{ref}^{c_on}(f-\Delta f_m)}}{\overline{P_{ref}^{c_off}(f-\Delta f_m)}}\right) = \frac{T_{cal}}{T_{sys}} + 1 \tag{8}$$

$$\overline{k_{ref}} = \frac{1}{\overline{F_{fit}} - 1} = \frac{T_{sys}}{T_{cal}} \tag{9}$$

Standard deviation of k_{ref} is calculated using equations (3), (4), (8)

$$\sigma_{k_{ref}} = \sqrt{\left(\frac{\partial k_{ref}}{\partial F_{fit}}\right)^2\Bigg|_{F_{fit}=\overline{F_{fit}}} \cdot \sigma^2_{F_{fit}}} = \\ = \left|\frac{-1}{(\overline{F_{fit}}-1)^2}\right| \sigma_{F_{fit}} = \left(\frac{T_{sys}}{T_{cal}}\right)^2 \sigma_{F_{fit}} \tag{10}$$

$\sigma_{F_{fit}}$ depends on the specific type of function F_{fit}, which will be used during the fitting. In this article we consider two cases when F_{fit} doesn't depend on frequency and when it depends on frequency linearly.

Relative error of k_{ref} is calculated using the following equation

$$\delta_{k_{ref}} = 3\frac{\sigma_{k_{ref}}}{\overline{k_{ref}}} = 3\frac{T_{sys}}{T_{cal}}\sigma_{F_{fit}} \cdot 100\% \tag{11}$$

4.1 Standard deviation of the variable which describes signals on one of spectrometer channels

To determine $\sigma_{F_{fit}}$ first we calculated standard deviation for variable $F_{one} = \frac{P^{c_on}_{ref}}{P^{c_off}_{ref}}$ which describes signals on one of spectrometer channels. Using equations (3), (4) we obtain

$$\sigma_{F_{one}} = \sqrt{\left(\frac{\partial F_{one}}{\partial P^{c_on}_{ref}}\right)^2\Bigg|_{P^{c_on}_{ref}=\overline{P^{c_on}_{ref}}} \cdot \sigma^2_{P^{c_on}_{ref}} + \left(\frac{\partial F_{one}}{\partial P^{c_off}_{ref}}\right)^2\Bigg|_{P^{c_off}_{ref}=\overline{P^{c_off}_{ref}}} \cdot \sigma^2_{P^{c_off}_{ref}}}$$

Hence standard deviation for variable F_{one} is

$$\sigma_{F_{one}} = \left(\frac{T_{cal}}{T_{sys}} + 1\right)\sqrt{\frac{2}{\Delta f \cdot t_q}} \tag{12}$$

4.2 Standard deviation $\sigma_{k_{ref}}$ at F_{fit} independent from frequency

Let us assume that function $F_{fit} = C$, where C is same for all spectrometer channels. To determine $\sigma_{F_{fit}}$ and $\sigma_{k_{ref}}$ it is necessary to find standard deviation of C. Using the least squares method we find

$$C = \frac{1}{N}\sum_{i=0}^{N-1} F_{one}(i)$$

N is the number of spectral channels of spectrometer. We assume that there is no correlation between adjacent channels. Since all $F_{one}(i)$ are independent from one another and have same $\sigma_{F_{one}}$, using formula (4) we obtain

$$\sigma_{F_{fit}} = \sigma_C = \frac{1}{\sqrt{N}}\sigma_{F_{one}} \tag{13}$$

Using (9), (10), (11) we find relative error $\delta_{k_{ref}}^{const}$ of function k_{ref} at F_{fit} independent from frequency

$$\delta_{k_{ref}}^{const} = 3\left(\frac{T_{sys}}{T_{cal}} + 1\right)\sqrt{\frac{2}{N \cdot \Delta f \cdot t_q}} \cdot 100\% \tag{14}$$

4.3 Standard deviation $\sigma_{k_{ref}}$ at F_{fit} which linearly depends on frequency

Let us assume that function $F_{fit}(k) = B \cdot k + A$, where k is the number of spectrometer channel. Variables A and B are calculated using the least squares method previously mentioned by Taylor (1982)

$$A = \frac{\left(\sum_{i=0}^{N-1} i^2\right)\left(\sum_{i=0}^{N-1} F_{one}(i)\right) - \left(\sum_{i=0}^{N-1} i\right)\left(\sum_{i=0}^{N-1} i \cdot F_{one}(i)\right)}{N\left(\sum_{i=0}^{N-1} i^2\right) - \left(\sum_{i=0}^{N-1} i\right)^2} \tag{15}$$

$$B = \frac{N\left(\sum_{i=0}^{N-1} i \cdot F_{one}(i)\right) - \left(\sum_{i=0}^{N-1} i\right)\left(\sum_{i=0}^{N-1} F_{one}(i)\right)}{N\left(\sum_{i=0}^{N-1} i^2\right) - \left(\sum_{i=0}^{N-1} i\right)^2}$$

Thus, $F_{fit}(k)$ linearly depends on $F_{one}(i)$ through A and B. Using formula (4) we can obtain

$$\sigma_{F_{fit}}^2(k) = \sum_{i=0}^{N-1}\left(\frac{\partial F_{fit}(k)}{\partial F_{one}(i)}\right)^2 \sigma_{F_{one}}^2 =$$
$$= \sum_{i=0}^{N-1}\left(\frac{kNi - k\left(\sum_{i=0}^{N-1} i\right) + \left(\sum_{i=0}^{N-1} i^2\right) - i\left(\sum_{i=0}^{N-1} i\right)}{N\left(\sum_{i=0}^{N-1} i^2\right) - \left(\sum_{i=0}^{N-1} i\right)^2}\right)^2 \sigma_{F_{one}}^2$$

As a rule T_{sys} and T_{cal} depend on frequency. However, this dependence is usually weak within spectrometer bandwidth. So, for error calculation we consider T_{sys} and T_{cal} independent from frequency. It means that all $\sigma_{F_{one}}$ do not depend on frequency and are calculated using (12). Taking into account that $\sum_{i=0}^{N-1} i = \frac{N(N-1)}{2}$ and $\sum_{i=0}^{N-1} i^2 = \frac{N(N-1)(2N-1)}{6}$, we obtain

$$\sigma_{F_{fit}}^2(k) = \frac{144}{72}\frac{6k^2 + 6k - 6Nk + 2N^2 - 3N + 1}{N(N-1)(N+1)}\sigma_{F_{one}}^2 \tag{16}$$

$\sigma_{F_{fit}}(k)$ has maximum values $\sigma_{F_{fit}}^{\max}$ on the edges of spectrometer frequency band at $k = 0$ and $k = N - 1$. Further analysis will be carried out for $\sigma_{F_{fit}}^{\max}$

$$\sigma_{F_{fit}}^{\max} = \frac{2}{\sqrt{N}}\sigma_{F_{one}} \tag{17}$$

Using (9), (10), (11) we obtain relative error $\delta_{k_{ref}}^{linear}$ of k_{ref} function at F_{fit} which linearly depends on frequency

$$\delta_{k_{ref}}^{linear} = 2 \cdot 3\left(\frac{T_{sys}}{T_{cal}} + 1\right)\sqrt{\frac{2}{N \cdot \Delta f \cdot t_q}} \cdot 100\% \tag{18}$$

4.4 Summary for k_{ref} and k_{sig}

Figures 3 and 4 show dependences of $\delta^{const}_{k_{ref}}$ and $\delta^{linear}_{k_{ref}}$ on T_{sys}/T_{cal} at $t_q = 10$ sec, $t_q = 20$ sec and $t_q = 30$ sec. We assume that spectral resolution is $\Delta f = 1$ kHz, number of spectral channels is $N = 7000$. As we can see in Fig. 3 and Fig. 4 relative errors $\delta^{const}_{k_{ref}}$ and $\delta^{linear}_{k_{ref}}$ are $\sim$1% at $5 < T_{sys}/T_{cal} < 15$ for each of the considered t_q. The calculation was done for k_{ref}. Since k_{ref} and k_{sig} are independent, their joined influence on T_s is $\sqrt{2}$ less than k_{ref}. Averaging S_s by N_{HS} of HS regions lowers this error by $\sqrt{N_{HS}}$ times. Thus, errors arising at the fitting are negligible and will not be considered further in this article.

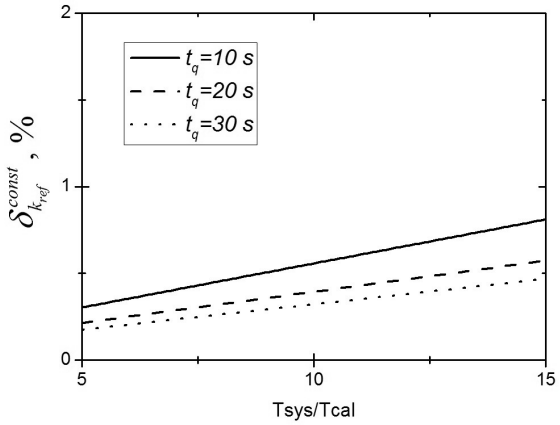


Figure 3 Dependences of $\delta^{const}_{k_{ref}}$ on T_{sys}/T_{cal} at $t_q = 10$ sec, $t_q = 20$ sec and $t_q = 30$ sec.

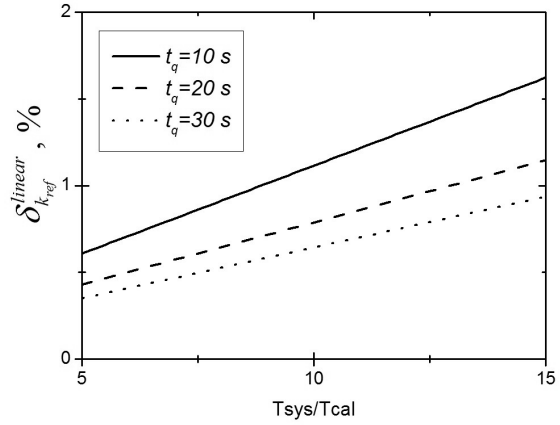


Figure 4 Dependences of $\delta^{linear}_{k_{ref}}$ on T_{sys}/T_{cal} at $t_q = 10$ sec, $t_q = 20$ sec and $t_q = 30$ sec

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5 Error of δ_{T_s} at instability of different characteristics of the receiver system

Analysis of the errors arising from instability of some system characteristics is more complicated. These errors can be substantial if T_{cal} is low compared to T_{sys}. Equations (1) and (3) correspond to each other if the temperature of the noise diode T_{cal}, receiver system gain g and system temperature T_{sys} are considered constant during the HS period. If this assumption is invalid there are errors at the calculation of T_s. We analyze these errors assuming that T_{cal}, T_{sys} and g have different values for different one-quarters of HS period. Analysis of these errors shows that

1. If during 1/4 HS period, corresponding to definition "sig" in Fig. 1 system temperature is $T_{sys}+\Delta T_{sys}$, and during 1/4 HS period, corresponding to definition "ref" in Fig. 1 system temperature is T_{sys}, then there is no error in T_s. The whole spectrum shifts to a constant value. It is easy to eliminate.
2. Similar situation arises if there is an error in T_{cal} instead of T_{sys} in the previous case.
3. If during 1/4 HS period, corresponding to definition "sig" in Fig. 1 receiver system gain is $g+\Delta g$, and during 1/4 HS period, corresponding to definition "ref" in Fig. 1 receiver system gain is g then relative error of antenna temperature of the source is $\delta_{T_s}=\frac{\Delta g}{g}=\delta_g$.
4. If during 1/4 HS period, corresponding to the on mode of the noise diode (Fig. 1) temperature of the noise diode is T_{cal} and during 1/4 HS period, corresponding to the off mode of the noise diode (Fig. 1) temperature of the noise diode is $T_{cal}+\Delta T_{cal}$ then relative error of antenna temperature of the source is $\delta_{T_s}=\frac{1}{\frac{T_{cal}}{\Delta T_{cal}}+1}=\frac{1}{\frac{1}{\delta_{T_{cal}}}+1}\approx\delta_{T_{cal}}$.
5. If during 1/4 HS period, corresponding to the on mode of the noise diode (Fig. 1) receiver system gain is g and during 1/4 HS period, corresponding to the off mode of the noise diode (Fig. 1) receiver system gain is $g+\Delta g$ then relative error of antenna temperature of the source is $\delta_{T_s}=\frac{\frac{1}{2}+\frac{T_{sys}}{T_{cal}}}{\frac{g}{\Delta g}-\frac{T_{sys}}{T_{cal}}}\approx\frac{T_{sys}}{T_{cal}}\delta_g$ at $T_{sys}>>T_{cal}$ and $\Delta g<<g$.

The performed analysis shows that the biggest dependence of error δ_{T_s} on δ_g and $\delta_{T_{cal}}$ arises in the last case. But it is possible to estimate δ_g only if we set T_{cal} much higher than T_{sys}. Since in our case $T_{sys}>>T_{cal}$, P signals at the on and off modes of the noise diode correlate with each other. Therefore we assume that instability of the receiver system gain is one of the components of instability T_{sys} and we consider only relative error δ_{T_s} arising from the instability of T_{sys}. This analysis is described in the next section.

5.1 Error of δ_{T_s} at instability of T_{sys}

We assume that during observation of one source $DPFU$ and zenith opacity remain stable. Then relative error of average source flux density $\langle S_s\rangle$ is the same as relative error of average antenna temperature of the source T_s. To find this error, first we

calculate the error of T_s which arises from instability of T_{sys} for one HS period with index j.

Let us assume that during 1/4 HS period corresponding to the on mode of the noise diode (Fig. 1) system temperature is T^j_{sys} and during 1/4 HS period corresponding to the off mode of the noise diode (Fig. 1) system temperature is $T^j_{sys}+\Delta T^j_{sys}$. Thus, equations (3) can be written as

$$\begin{array}{l}
\overline{P^{c_on}_{sig}\left(f-\Delta f_m,j\right)} = g\left(T^j_{sys}+T_{cal}+T_s\right)\\
\overline{P^{c_off}_{sig}\left(f-\Delta f_m,j\right)} = g\left(T^j_{sys}+\Delta T^j_{sys}+T_s\right)\\
\overline{P^{c_on}_{ref}\left(f-\Delta f_m,j\right)} = g\left(T^j_{sys}+T_{cal}\right)\\
\overline{P^{c_off}_{ref}\left(f-\Delta f_m,j\right)} = g\left(T^j_{sys}+\Delta T^j_{sys}\right)
\end{array} \tag{19}$$

$$\begin{array}{l}
\overline{P^{c_on}_{sig}\left(f+\Delta f_m,j\right)} = g\left(T^j_{sys}+T_{cal}\right)\\
\overline{P^{c_off}_{sig}\left(f+\Delta f_m,j\right)} = g\left(T^j_{sys}+\Delta T^j_{sys}\right)\\
\overline{P^{c_on}_{ref}\left(f+\Delta f_m,j\right)} = g\left(T^j_{sys}+T_{cal}+T_s\right)\\
\overline{P^{c_off}_{ref}\left(f+\Delta f_m,j\right)} = g\left(T^j_{sys}+\Delta T^j_{sys}+T_s\right)
\end{array}$$

For calculation of the antenna temperature of the source according to equation (1) we take into account that it contains an error reflecting presence of ΔT_{sys}. Substituting values of P signals from equation (19) into (1) we obtain

$$T^j_s = T_s + \Delta T^j_s = \frac{T_s}{T_{cal}-\Delta T^j_{sys}} T_{cal}$$

or

$$\Delta T^j_s = \frac{\Delta T^j_{sys}}{T_{cal}-\Delta T^j_{sys}} T_s$$

Hence we find relative error of the antenna temperature of the source in this case

$$\delta_{T^j_s} = \frac{\Delta T^j_s}{T_s} = \frac{1}{\frac{T_{cal}}{T^j_{sys}}\frac{1}{\delta_{T^j_{sys}}}-1} \approx \frac{T^j_{sys}}{T_{cal}}\delta_{T^j_{sys}} \tag{20}$$

where $\delta_{T^j_{sys}} = \frac{\Delta T^j_{sys}}{T^j_{sys}}$ is relative error of system temperature for one HS period with index j. Hereinafter we assume that system temperature T^j_{sys} is the same and equals T_{sys}.

Antenna temperature of the source averaged according to m of HS regions is

$$\tilde{T}^m_s = T_s + \Delta\tilde{T}^m_s = \frac{1}{m}\sum_{j=1}^{m} T^j_s = T_s + \frac{1}{m}\sum_{j=1}^{m}\Delta T^j_s$$

Relative error of $\tilde{T}^m_s$ is

$$\tilde{\delta}_{T^m_s} = \frac{\Delta\tilde{T}^m_s}{T_s} \approx \frac{T_{sys}}{T_{cal}}\frac{1}{m}\sum_{j=1}^{m}\delta_{T^j_{sys}} = \frac{T_{sys}}{T_{cal}}\tilde{\delta}_{T^m_{sys}} \tag{21}$$

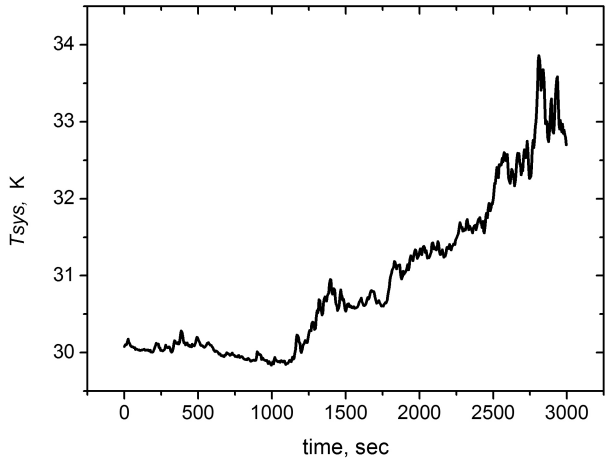


Figure 5 Time dependence of observed system temperature. Data were obtained in the worst weather conditions (overcast) and at the small height angle.

where

$$\tilde{\delta}_{T^m_{sys}} \equiv \frac{1}{m}\sum_{j=1}^{m} \delta_{T^j_{sys}} \cdot 100\% \tag{22}$$

For calculation of $\tilde{\delta}_{T^m_{sys}}$ and $\delta_{T^j_{sys}}$ a corresponding test was carried out at RT-32 radio telescope. The telescope was pointed at the source-free region and tracked it for an hour. During this periods system temperature accumulated for 5 seconds and was saved on the hard drive every 5 seconds. Then, using this data system temperature was averaged during $2t_q$ and thus T^j_s formed according to different t_q. This test was carried out a few times in different weather conditions and at different height angles. Next we consider the case of the worst weather conditions (overcast) and a small height angle. Figure 5 shows time dependence of the observed system temperature. Relative errors $\delta_{T^j_{sys}}$ are calculated using $\delta_{T^j_{sys}} = \frac{T^{j+1}_{sys} - T^j_{sys}}{T^j_{sys}}$. Relative errors $\tilde{\delta}_{T^m_{sys}}$ are calculated using (22). Figure 6 shows time dependence of $\tilde{\delta}_{T^m_{sys}}$ for different t_q. Time scale is determined using $time = m \cdot 2t_q$. 0 corresponds to the starting point of measurement.

Figures 5 and 6 show that $\tilde{\delta}_{T^m_{sys}}$ increases with instability of system temperature as well as t_q. To calculate relative error of the antenna temperature of the source averaged according to $m$of HS regions it is convenient to find the value of $\tilde{\delta}_{T^m_{sys}}$ typical for these weather conditions and t_q. Considering time dependence of $\tilde{\delta}_{T^m_{sys}}$ as a set of random values, triple standard deviation of $\tilde{\delta}_{T^m_{sys}}$ is used as such parameter. This value is $\tilde{\delta}_{T_{sys}} = 0.09\%$ for $t_q = 10$ sec, $\tilde{\delta}_{T_{sys}} = 0.18\%$ for $t_q = 20$ sec and $\tilde{\delta}_{T_{sys}} = 0.25\%$ for $t_q = 30$ sec. According to (21) relative error of antenna temperature of the source is

$$\delta_{T_s} = \frac{T_{sys}}{T_{cal}} \tilde{\delta}_{T_{sys}} \tag{23}$$

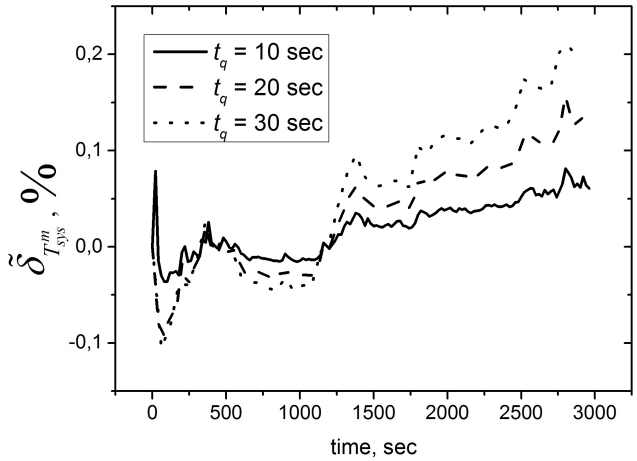


Figure 6 Time dependences of relative error of system temperature $\tilde{\delta}_{T^m_{sys}}$ for different t_q.

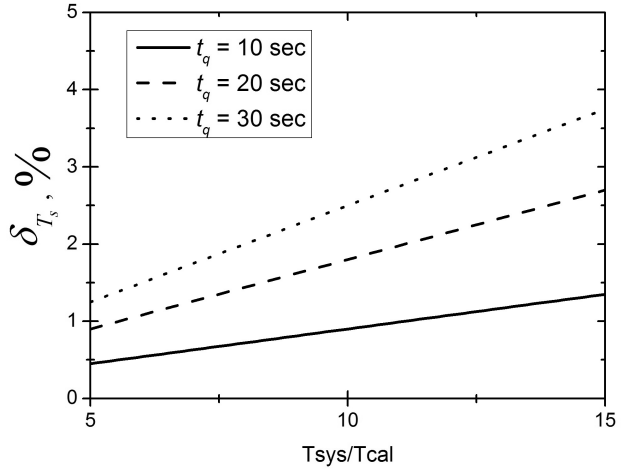


Figure 7 Dependence of relative error of the source antenna temperature on $\frac{T_{sys}}{T_{cal}}$ for different t_q

Figure 7 shows dependence of relative error of the antenna temperature of the source δ_{T_s} on $\frac{T_{sys}}{T_{cal}}$ for different t_q. This estimate of relative errors is the maximum limit. It is obtained in the in the worst weather conditions. Similar experiments carried out in good weather show that δ_{T_s} is a few times smaller than the values in Fig. 7.

6 Estimating of t_q

The minimum time t_q can be determined as follows. In equation (1) all denominators containing noise signals have to be greater than 0. That is $\overline{P^{c_on}_{sig}} >> \sigma_{P^{c_on}_{sig}}$, $\overline{P^{c_on}_{ref}} >>$

$\sigma_{P_{ref}^{c_on}}$, $\overline{P_{sig}^{c_off}} >> \sigma_{P_{sig}^{c_off}}$ and $\overline{P_{ref}^{c_off}} >> \sigma_{P_{ref}^{c_off}}$ must be met. Thus, $t_q >> \frac{1}{\Delta f}$. This time is very short.

Maximum value of time t_q is defined from Allan test. The Allan time and variance of the receiver system were measured at good and bad weather during May 2019. Allan time value was estimated as $\sim$ 20 sec in cloudy and 70–80 sec in good weather.

In practice, the value of t_q is determined from the following reasons. Decrease in time t_q allows us to reduce error δ_{T_s}. In the same time, substantial decrease in t_q leads to time losses caused by switching noise diode between on and off modes and changing oscillator frequency. Based on this, we set $t_q = 20$ sec.

7 Discussion

Increase in the ratio of the system temperature to the temperature of noise diode $\frac{T_{sys}}{T_{cal}}$ leads to opposite processes. On the one hand, additional time percentage δ_{time} necessary for compensating T_{cal} influence decreases. It means that standard deviation of the antenna temperature of the source σ_{T_s} (equation (6), Fig. 2) decreases. On the other hand, errors $\delta_{k_{ref}}^{const}$ or $\delta_{k_{ref}}^{linear}$ arising from fitting during calculation of k_{ref} and k_{sig} (equation (14) or (18), Fig. 3 or Fig. 4) increase. Also there is increase in error of δ_{T_s} at instability of T_{sys} (equation (23), Fig. 7). Thus choosing the ratio $\frac{T_{sys}}{T_{cal}}$ requires the compromise. Usually T_{sys} exceeds T_{cal} several times. We set $\frac{T_{sys}}{T_{cal}} \approx 8$. The choice of t_q was discussed in the previous section.

Thus, the tuning of the receiving system of the RT-32 radio telescope for observations in the C-band ($\frac{T_{sys}}{T_{cal}} \approx 8$, $t_q = 20$ sec, $\Delta f = 1$ kHz, $N = 7000$, observation time $\sim$1 hour) leads to the following errors in the source antenna temperature that arise during the calibration process: additional standard deviation percentage is $\delta_{\sigma_{T_s}} \sim 6\%$, errors at the fitting of k_{ref} and k_{sig} is less than 1%, error of instability of T_s is $\delta_{T_s} < 1.5\%$ in the worst weather conditions (overcast) and less than 1% in the good weather. These are small errors and, therefore, are difficult to verify in the process of real observations. Note that these errors do not include pointing errors, error of $DPFU$, error of T_{cal} and zenith opacity error.

8 Conclusions

We have considered the calibration process of single-dish data of RT-32 radio telescope. Signal from the noise diode was used as calibration signal. Frequency modulation mode has been considered.

Dependence of standard deviation of source antenna temperature σ_{T_s} on the calibrated signal value has been calculated.

Errors $\delta_{k_{ref}}^{const}$ and $\delta_{k_{ref}}^{linear}$ arising at the fitting of k_{sig} and k_{ref} at different dependences of k_{sig} and k_{ref} on frequency are determined. These errors are negligible at the spectral resolution of $\Delta f = 1$ kHz and number of spectral channels of $N = 7000$. Errors of the antenna temperature of the source arising at the instability of the noise diode, receiver system gain and system temperature have been determined.

Relative error of the antenna temperature of the source δ_{T_s} mostly depends on the instability of the system temperature when the noise diode is switched. This dependence is determined in different weather conditions. We have determined that increase in temperature of the noise diode at the same system temperature leads to increase of σ_{T_s}, decrease of $\delta_{k_{ref}}^{const}$, $\delta_{k_{ref}}^{linear}$ and δ_{T_s}. Decrease in t_q allows one to reduce δ_{T_s} but minimal t_q has its limits too.

According to dependencies $\sigma_{T_s}$and δ_{T_s}on $\frac{T_{sys}}{T_{cal}}$, we set $\frac{T_{sys}}{T_{cal}} \approx 8$ and $t_q = 20$ sec. In this case, the errors $\delta_{k_{ref}}^{const}$ and $\delta_{k_{ref}}^{linear}$ are less than 1%, and δ_{T_s} does not exceed 1.5% in bad weather. In good weather this error is much smaller.

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