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On the possibility of deriving the absolute value of magnetic field vector from Stokes I and V

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In this work we describe a simple technique to evaluate, along with the line-of-sight component B_z of the magnetic field, the absolute value of the magnetic field strength $|B|$ from I and V Stokes parameters. The absolute value of the vector magnetic field can be derived from the extrema positions of Stokes $I \pm V$ profiles. One can simply evaluate the modulus of the transversal magnetic field from these two quantities, B_z and $|B|$. The spectropolarimetric data acquired by Hinode was used to test the proposed technique. Our results are in a good agreement with the magnetic field measurements provided by the Hinode team.

Keywords: Sun, magnetic field, active region

1 Introduction

A new solar spectropolarimeter is currently being tested at the Crimean Astrophysical Observatory. During the first years of its operation, the spectropolarimeter will provide data on Stokes I and V parameters in several photospheric and chromospheric spectral lines. A straightforward utilization of such data is the derivation of line-of-sight (LOS) magnetic field at several heights in the solar atmosphere. This approach is based on the measurements of the Zeeman splitting of spectral lines. If the magnetic field is directed along the line-of-sight, a spectral line splits into two circularly polarized lines. The fraction of the circularly polarized radiation in the total radiation intensity, i.e. Stokes V/I, is proportional, to a first approximation, to the LOS magnetic field.

However, we can derive some other characteristics of magnetic field from Stokes I and V profiles. Let us consider the Zeeman splitting in more detail. The energy levels of an atom split into several sublevels if the atom is placed in an external magnetic field. This results in the splitting of the spectral line into several components determined by the selection rules. It is worth noting that the splitting (in terms of energy)

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of energy levels in the atom (we do not consider the dependency on the quantum numbers of the upper and lower levels here) does not depend on the mutual orientation of the magnetic field and the line-of-sight. The splitting is proportional exclusively to the absolute value of the magnetic field (see the next section). The mutual orientation of the line-of-sight direction and magnetic field affects exclusively the redistribution of intensity between the spectral components of different polarization rather than its position on the wavelength scale. Therefore, we can derive the absolute value of the magnetic field vector if we succeed, by some means, in determining the splitting of the spectral components of the Zeeman multiplet.

The aim of this work is to show that the Stokes I and V can be used to derive the absolute value of magnetic field vector. Similar approaches were developed earlier when the only data available were circularly polarized spectral observations (e.g. Rayrole, 1967; Brants, 1985). The technique described in the following sections can still be useful to derive additional information from, for example, Mount Wilson Solar Magnetograph (Ulrich et al., 2002) measurements as well as from the stellar observations of Stokes $I \pm V$ profiles.

2 Data and methods

2.1 Data

To test our approach, in this work we used observations made by the Spectropolarimeter–Solar Optical Telescope (SOT-SP) onboard Hinode. Hinode/SOT-SP is a slit spectrograph with a slit height of 150 arcsec. Observations are carried out in the photospheric Fe I 6301.5 Å and 6302.5 Å spectral lines. The spatial resolution of the instrument is 0.16×0.15 arcsec^2 (Lites et al., 2013).

Ready for scientific analysis data on full-Stokes vector acquired by Hinode/SOT-SP are courteously provided by the High Altitude Observatory team (HAO, https://www2.hao.ucar.edu/csac/csac-data). The data are four Stokes profiles acquired for every slit position. To determine the LOS magnetic field, B_z, and the absolute value of magnetic field vector, $|\vec{B}|$, we used data on Stokes I and V profiles.

In order to check the validity of our methods, we compared the calculated magnetic field quantities with those provided by the HAO team. The magnetic field vector is obtained by the HAO team as a result of inversion of the full-Stokes profiles by the MERLIN code (Lites et al., 2007) that uses Milne–Eddington atmosphere approximation to derive the magnetic field, velocity and other quantities. Throughout the rest of the paper, we will denote the magnetic field quantities provided by the HAO team with the 'Hin' superscript, for example, $B_z^{\rm Hin}$ for the LOS magnetic field.

In the present study, we used Hinode/SOT-SP observations of active regions (ARs) NOAA 12172, 12645, and 12104 acquired on 2014.09.26 at 17:00 UT, 2017.04.01 at 11:00 UT, and 2014.07.07 at 01:55 UT, respectively. All the selected ARs were located near the central meridian at the chosen times, therefore we suppose that the line-of-sight coincides with the z-axis in the local Cartesian coordinate system. The LOS magnetogram of AR NOAA 12172, derived by HAO, is shown in Fig. 1 (left).

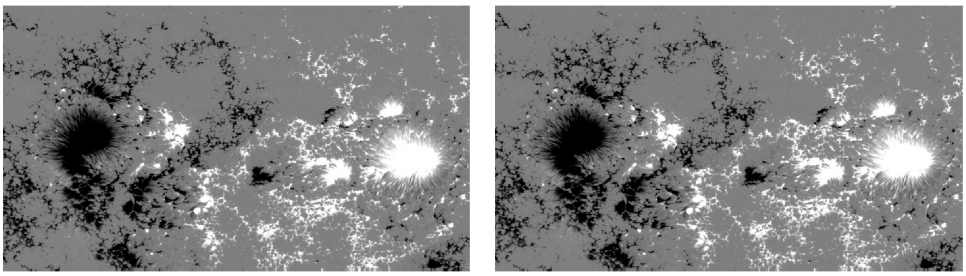


Figure 1 LOS magnetograms of AR NOAA 12172 (acquired on 2014.09.26 at 17:00 UT) provided by the HAO team (left) and obtained in this work in the framework of the weak-field approximation (right, see text). The field of view is 250 arcsec×150 arcsec. The magnetograms are scaled from –1000 G (black) to 1000 G (white).

2.2 Deriving LOS component of the magnetic field

Generally speaking, deriving the magnetic field from the measured Stokes profiles implies the solution of radiative transfer equations for the solar atmosphere in the presence of the magnetic field (e.g., Degl'Innocenti and Landolfi, 2004). However, if the Zeeman splitting is significantly smaller than the line Doppler shift, the weak field approximation could be used instead (Degl'Innocenti and Landolfi, 2004). In the framework of this approximation, Stokes V is linearly proportional to the longitudinal component of the magnetic field $B_z = |\vec{B}| \cos\theta$:

$$V(\lambda) = -\frac{dI}{d\lambda} g \Delta\lambda \cos\theta = -\frac{dI}{d\lambda} g \frac{\lambda_0^2 e}{4\pi m_e c^2} B_z, \tag{1}$$

where λ_0 is the unshifted wavelength of the spectral line center, $\Delta\lambda$ is the Zeeman splitting, g is the Landé factor, e and m_e are electron charge and mass, respectively, c is the speed of light, θ is the angle between magnetic field vector and line-of-sight (inclination). The linear proportionality in equation (1) holds as long as the magnetic field value remains low. For the spectral lines used in this work a significant deviation from the linear relationship between Stokes V and B_z is observed at values higher than approximately 2500 G. By integration, we can rewrite equation (1) in the following form:

$$\int_a^b V(\lambda) d\lambda = -\Big(I(b) - I(a)\Big) g \frac{\lambda_0^2 e}{4\pi m_e c^2} B_z. \tag{2}$$

Landé g-factor should be defined experimentally, since it may deviate significantly from the theoretical value if the LS-coupling in the atom breaks (Degl'Innocenti, 1982; Solanki and Stenflo, 1985). Alternatively, one can introduce a calibration constant $C_z = g\frac{\lambda_0^2 e}{4\pi m_e c^2}$ that can be determined, for example, by a cross-calibration of two instruments. In this work, C_z was chosen in such a way as to achieve the best

agreement between $B_z^{\rm Hin}$ and the calculated longitudinal magnetic field

$$B_z = C_z \frac{\int\limits_a^b V(\lambda) d\lambda}{\Big(I(a) - I(b)\Big)}. \tag{3}$$

The value of longitudinal magnetic field B_z was calculated numerically as the area under the Stokes V curve divided by the Stokes I amplitude $I(b) - I(a)$ (shown schematically in Fig. 2a). To determine the area under the Stokes V curve, the integral $\int\limits_a^b V(\lambda)d\lambda - \int\limits_b^c V(\lambda)d\lambda$ was calculated. The limits of integration a and c were chosen at the boundaries of the spectrogram while the point b coincided with the minimum of Stokes I.

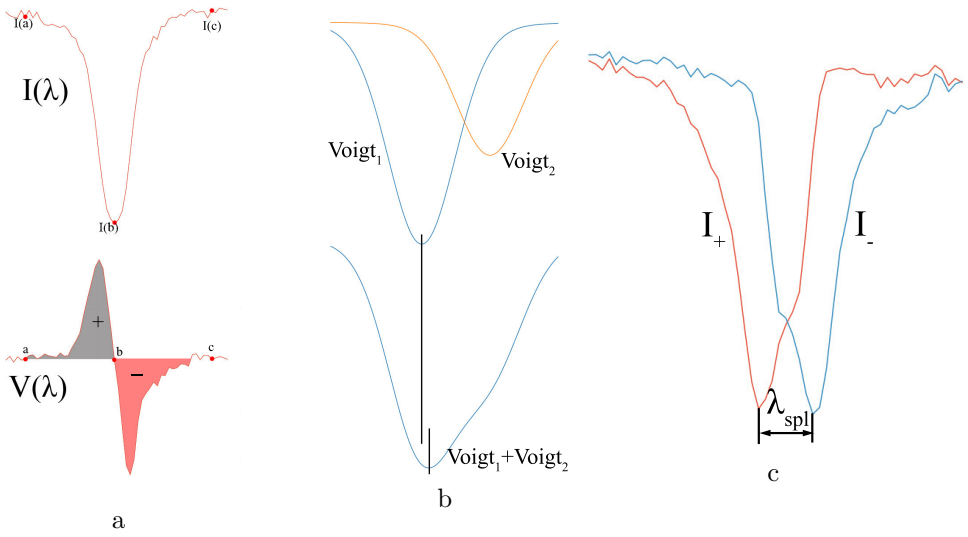


Figure 2 Illustration to show the acquisition of the longitudinal magnetic field by Eq. (3). a – the shaded area enclosed by the V-profile is proportional to the longitudinal magnetic field; integral limits a and c are selected on the edges of the spectrogram and the location of the point b is in the minimum of the Stokes I. b – a sketch of a shift of the minimum in the circular polarization spectrum due to blending of two close spectral lines; c – derivation of $\lambda_{\rm spl}$, a proxy for the spectral line splitting.

2.3 Deriving the absolute value of magnetic field vector

Splitting of spectral lines centers in the Zeeman multiplet (throughout this paper we consider only the simple Zeeman triplet) is proportional to the magnetic field strength (e.g. Degl'Innocenti and Landolfi, 2004):

$$\Delta\lambda = g \frac{\lambda^2 e}{4\pi m_e c^2} |\vec{B}|. \tag{4}$$

As it was mentioned above, spectral line splitting is a result of energy levels splitting in an atom. Thus, it depends only on the properties of the atom and on magnetic field strength. The angle between the magnetic field vector and line-of-sight can only impact spectral component polarization and amplitudes rather than its positions.

In this work, we propose to evaluate the spectral component splitting $\lambda_{\rm spl}$ as a difference between Stokes $I+V$ and $I-V$ profiles extrema (see Fig. 2c). The main source of uncertainties in this case is spectral line blending. Indeed, if the magnetic field is not completely longitudinal, the profile of the shifted circularly polarized component ('Voight1' in Fig. 2b) would blend with the profile of the unshifted spectral component ('Voight2' in Fig. 2b). This blending will result in shifting of the extrema of the composite profile ('Voight1 + Voight2' in Fig. 2b). The shifting becomes more significant as the inclination of the magnetic field vector to the line-of-sight increases. Therefore, this approach diminishes the absolute value of highly inclined magnetic field. Note also that (i) the approach is suitable for simple Zeeman triplets only; (ii) $\lambda_{\rm spl}$ equals $\Delta\lambda$ from the left part of equation (4) for completely longitudinal magnetic field, hence we consider $\lambda_{\rm spl}$ as some proxy of the real splitting.

Since Hinode/SOT-SP data are provided as sets of Stokes I and V profiles, to determine $\lambda_{\rm spl}$ we calculated sets of I_+ and I_- profiles corresponding to both circular polarizations:

$$I_\pm = \frac{1}{2}(I \pm V). \tag{5}$$

Then, for both profiles, I_+ and I_-, the locations of their extrema were derived (see Fig. 2c). Discretization of measurements along the wavelength (SOT/SP provides only about 30 data points along the Fe I 6302.5 Å spectral line width) causes a problem for an accurate acquisition of the extremum location. To mitigate the problem, approximately 10 data points around the extremum were fitted by a Gaussian profile and the peak of the profile was accepted as an extremum location.

If the σ-component profile is not significantly contaminated by the π-component, the extremum position of the corresponding $I_\pm$ profile will approximately coincide with the position of the σ-component center. In such a case the difference between the extrema positions of I_+ and I_- profiles (see Fig. 2c), $\lambda_{\rm spl}$, can be used as a proxy for double Zeeman splitting $\lambda_{\rm spl} \approx 2\Delta\lambda$. Using equation (4), we can write for the absolute value of the magnetic field vector, $|\vec{B}|$:

$$|\vec{B}| = \frac{4\pi m_e c^2}{g\lambda^2 e}(\lambda_{\rm spl}/2). \tag{6}$$

Further, having the longitudinal magnetic field component B_z and absolute value of the magnetic field vector, we can easily obtain an estimate for the absolute value of transverse magnetic field component:

$$|\vec{B}_t| = \sqrt{|\vec{B}|^2 - B_z^2}. \tag{7}$$

To check the validity of our method numerically, we performed the following numerical experiment. For simplicity, let us suppose that each spectral line in the Zeeman multiplet can be described by a Gaussian profile. If one observes the Zeeman

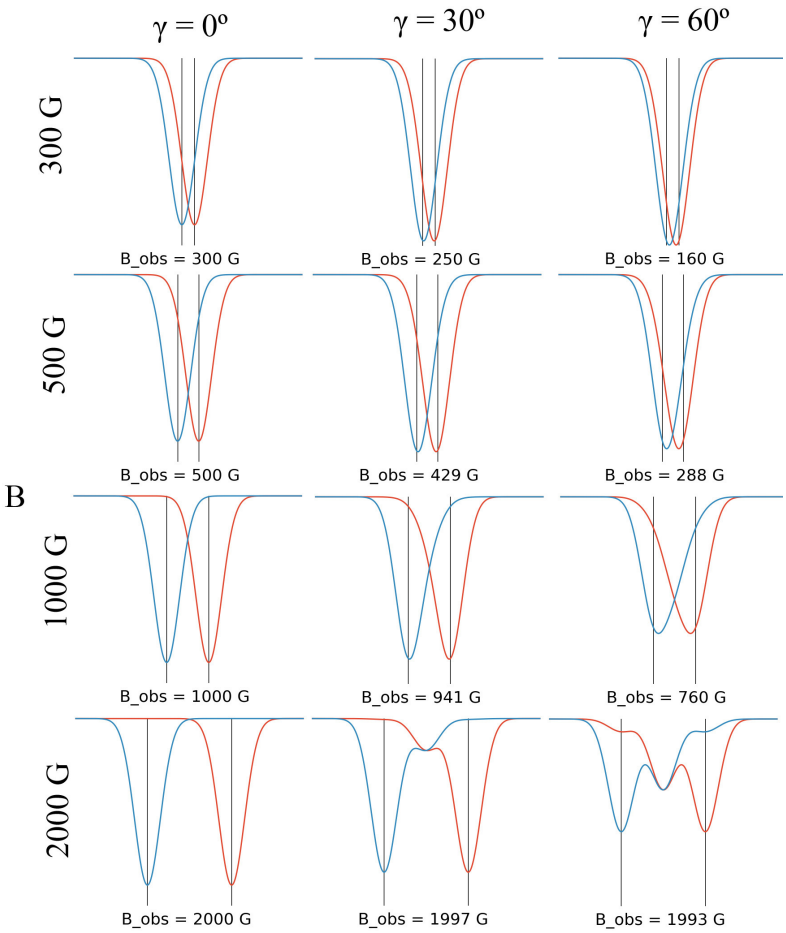


Figure 3 Simulated $I \pm V$ Stokes profiles as observed in left- (blue curves) and right-hand (red curves) circular polarizations. The simulations are carried out for magnetic field strength (from top to bottom) $|\vec{B}| = 300$, 500, 1000, and 2000 G and for inclination (from left to right) $\gamma = 0°$, 30°, and 60°. The deduced evaluation of the magnetic strength by the proposed algorithm is shown in each frame as $B_{\rm obs}$. Vertical lines display positions of the centers of σ-components of the Zeeman multiplet. One can see that the distance between the σ-components extrema ($\lambda_{\rm spl}$) decreases as the inclination increases (at a constant magnetic field strength).

multiplet in a circular polarization, the multiplet will be composed of three spectral lines with relative amplitudes (Brey and Loughhead, 1964)

$$A_{\sigma+} = \frac{0.25\eta_0(1 \mp \cos\gamma)^2}{1 + 0.5\eta_0(1 + \cos^2\gamma)}, A_{\lambda_0} = \frac{0.5\eta_0 \sin^2\gamma}{1 + \eta_0 \sin^2\gamma}, A_{\sigma-} = \frac{0.25\eta_0(1 \pm \cos\gamma)^2}{1 + 0.5\eta_0(1 + \cos^2\gamma)}, \quad (8)$$

where $A_{\sigma+}$ and $A_{\sigma-}$ are amplitudes of the shifted σ-components, A_{λ_0} is the amplitude of the unshifted π-component, η_0 is a line depth in the center of unperturbed line and γ is the inclination angle. The shift of the σ-components in the magnetic field $|\vec{B}|$ can

be evaluated from equation (4). In the simulation we set the effective Landé factor equal to 2.5 and the linewidth of each spectral component to 80 mÅ.

Using equations (4) and (8), we simulated several Zeeman multiplet profiles for different magnetic field strengths, $|\vec{B}|$, and for three values of inclination γ (Fig. 3). The simulated Stokes $I \pm V$ profiles were used as an input data in our algorithm. The derived values of magnetic field strength, $B_{\rm obs}$, are shown in each frame in Fig. 3. One can see that the initial value of magnetic field strength and the deduced one coincide at zero inclination. As expected, the algorithm diminishes the value of magnetic field strength as the inclination increases. This happens due to blending of σ-components with the wings of the unperturbed component: blending shifts the extrema of σ-components toward the center of the multiplet. The relative diminishing of the magnetic field strength is essentially significant for low real magnetic field values.

Nevertheless, the algorithm allows us to deduce the evaluation of the magnetic field strength. Consequently, some information on the transverse magnetic field from the circular polarization measurements only can be obtained as well.

3 Results

In Fig. 1 (right) for NOAA AR 12172 we show a B_z-magnetogram obtained in this study using the weak field approximation (3) in comparison with that delivered by the HAO-team (left frame). A good visual agreement is confirmed by the scatter plot (Fig. 4, left), where our data are plotted versus the corresponding $B_z^{\rm Hin}$. The Pearson correlation coefficient consists of 0.992. The calibration coefficient in Eq. (3)

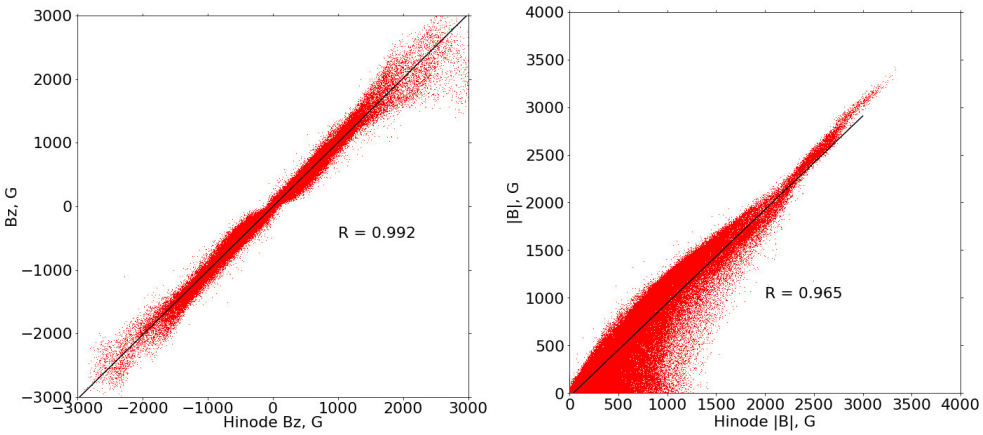


Figure 4 Magnetic field data for NOAA AR 12172. *Left* – Longitudinal magnetic field calculated in the present study with the weak-field approximation (B_z) versus the longitudinal magnetic field derived by the HAO-team ($B_z^{\rm Hin}$). *Right* – A proxy of the absolute values of the magnetic field vector (vertical axis) versus the absolute values of the magnetic field vector as derived by the HAO-team (horizontal axis). The black lines show the best linear fits to the data points. The Pearson correlation coefficients R are shown.

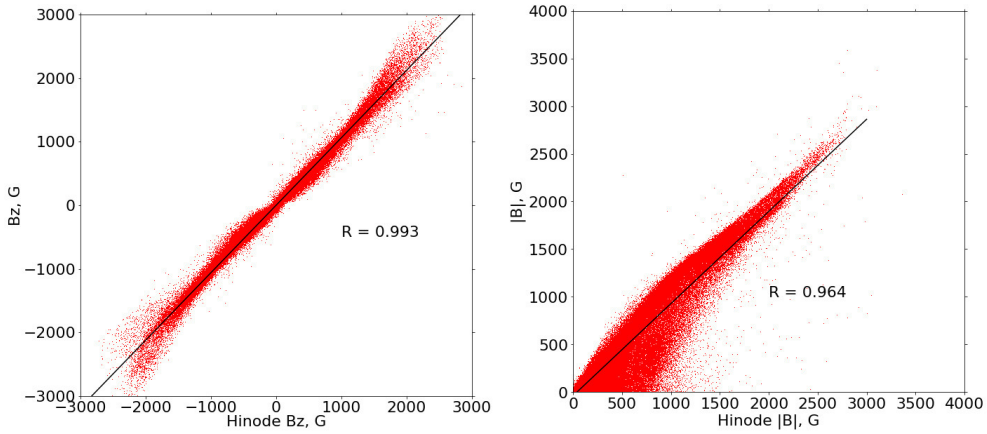


Figure 5 The same as in Fig. 3 for NOAA AR 12645.

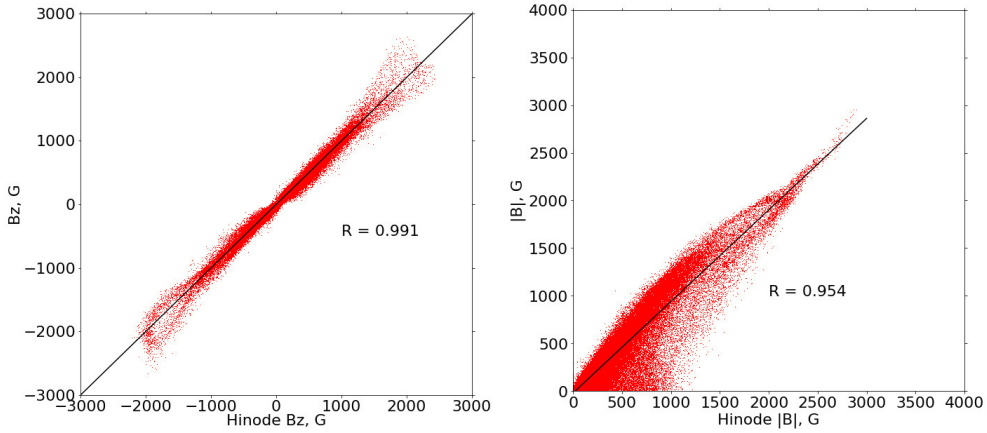


Figure 6 The same as in Fig. 3 for NOAA AR 12104$\beta\gamma$.

was selected so that the linear regression coefficient between B_z and $B_z^{\rm Hin}$ equals 1.0, C_z in Eq. (3) equals 8063.79 G/Å.

The right frame in Fig. 4 shows the scatter plot of the absolute values of the magnetic field vector obtained from Eq. (6) versus that obtained for the same pixels by the HAO-team for the same magnetogram of NOAA AR 12172. Correlation is also very high, $R = 0.965$. The slope of the linear regression line is very close to unity, which indicates that no additional changes for the calibration coefficient are needed. An excess of data points at low magnitudes of the magnetic field can be explained by the aforementioned blending of a non-shifted component when an angle between the magnetic field vector and the line-of-sight is large enough, see Fig. 2b.

To test the proposed approach, we selected two ARs, NOAA 12645 and 12104$\beta\gamma$, and applied for the spectropolarimetric data the aforementioned technique to derive the longitudinal magnetic field component and the absolute values of the magnetic field vector. The calibration constants derived from the AR NOAA 12172 data were

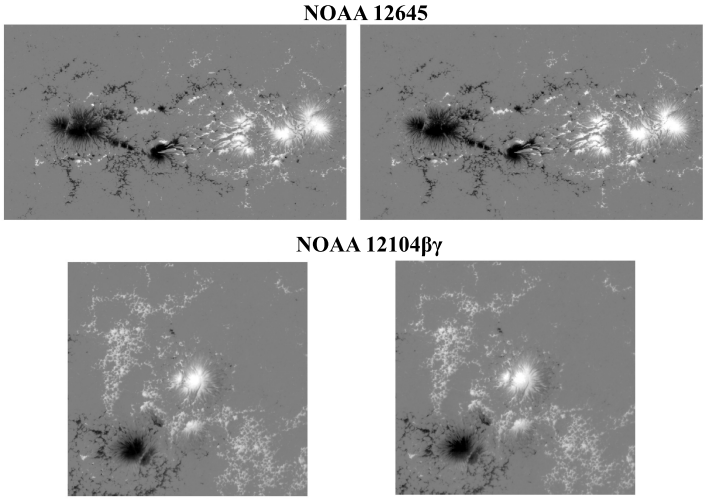


Figure 7 Comparison of the same-AR-magnetograms of the longitudinal magnetic field derived by different calibration methods. Left frames – results delivered by the HAO-team; right frames – results obtained in the present study. The magnetograms are scaled from 1000 G (white) to –1000 G (black).

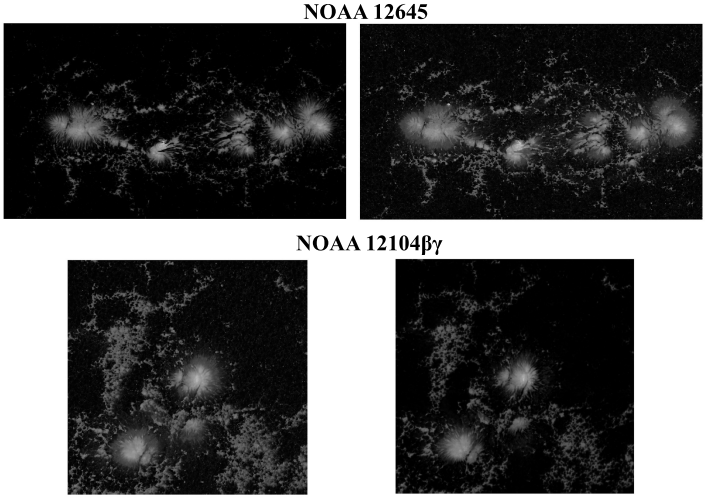


Figure 8 The same as in Fig. 6 for the absolute values of the magnetic field vector $|B|$. The magnetograms are scaled from 0 G (black) to 2000 G (white).

utilized (we thus assumed that the calibration constants depend from the spectral line and instrumental features only). The scatter plots for ARs 12645 and 12104 similar to that presented in Fig. 4 for AR 12172, are shown in Figs 5 and 6, respectively. As in the case of AR 12172, we see very good correlations between the data derived in the present study and their counterparts delivered by the HAO-team.

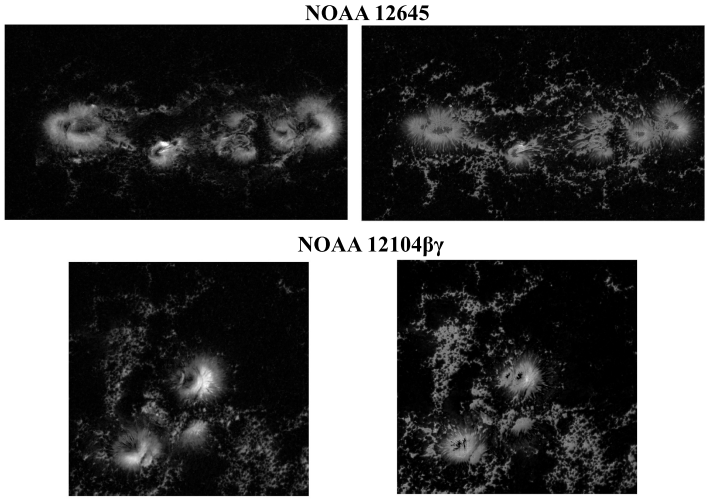


Figure 9 The same as in Fig. 7 for the absolute values of the transverse component of the magnetic field vector $|B_t|$. Notations are the same as in Fig. 7.

For visual comparison of 2D magnetic field structures, in Figs 7–9 we plot the magnetograms obtained by our method side-by-side with those delivered by HAO-team. A good agreement is obvious. It can be noted, however, that in penumbra, where the inclination of the magnetic field vector is large, our method gives a slightly lower estimate of the absolute value of the magnetic field vector, see Fig. 8. This can be due to the aforementioned blending of spectral lines. It is worth noting that inside penumbra both methods show enhanced magnitudes of the absolute value of the transverse magnetic field (see Fig. 9), which is physically reasonable.

Conclusions

We suggest here a simple method to derive from I and V Stokes parameters not only the longitudinal component of the magnetic field, but also to estimate the absolute value of the magnetic field vector and, therefore, the absolute value of the transverse magnetic field component. Our method is based on measurements of the splitting of spectral components in Zeeman multiplet. The splitting is derived as a wavelength difference between two extrema of the Stokes V profiles. The method was elaborated and then tested using Hinode/SOT-SP data and results of HAO-team calibration. Data for three ARs were utilized: NOAA 12172, 12645, and 12104$\beta\gamma$. Spectro-polarimetric data for the first one allowed us to perform our calibration, to compare results with those delivered by the HAO-team and, thus, to derive the calibration coefficients. The second and third ARs were used to test the method: using obtained calibration coefficients, magnetograms of longitudinal magnetic field, of absolute values of the magnetic field vector and of absolute values of the transverse magnetic field, were calculated and then compared with the HAO-team results. The comparison shows a

very good agreement between the data sets. One shortcoming of our method is the lowering of accuracy in areas of highly inclined fields.

This simple and reliable calibration routine will be used to process observational data from the new solar spectro-polarimeter recently installed and tested on Tower Solar telescope (TST-1) of Crimean Astrophysical Observatory.

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References

Brants J.J. (1985) Solar Phys., V. 95, P. 15

Brey R., Loughhead R. "Sunspots", Chapman and Hall, ltd, London, 1964.

Degl'Innocenti, E.L. (1982) Solar Phys., V. 77, P. 285

Degl'Innocenti E.L., Landolfi M. (2004) "Polarisation in spectral lines", Kluwer Acad. Publ., P. 397

Lites B., Casini, R., Garcia, J. et al (2007) Memorie della Societa Astronomica Italiana, V. 78, P. 148

Lites B.W., Akin D.L., Card G. et al (2013) Solar Physics, V. 283, I. 2, P. 579

Rayrole J. (1967) Annales d'Astrophysique, V. 30, P. 257

Solanki S.K., Stenflo J.O. (1985) A&A, V. 148, P. 123

Ulrich R.K., Evans S., Boyden J.E. et al (2002) The AJ Supplement Series, V. 139, P. 259