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Statistical investigations into low-power solar flares

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From the International Flare Patrol data over 1972–2010, we conducted statistical investigations into Hα solar flares. 91% of all the flares occurring on the Sun were established to refer to low-power flares, of which the bulk is SF flares (64%). In terms of evolution features, low-power flares do not differ from major solar flares. They, as well as powerful flares, are accompanied by filament activations and disappearances, have the explosive phase, and multiple intensity bursts. Among them, there are flares covering sunspot umbrae, two-ribbon, and white flares.

From the data on temporal parameters of more than 85,000 flares, we determined the mean times of rise and decay, and durations for solar flares of various types, classes of area and of importance. The lifetime of 94.6% flares was established not to exceed 60 minutes; for powerful flares, this time is no more than 8.3 hours. On rarest occasions, flares may last about 12 hours. The most short-lived are the flares with one increased-brightness center in the flare region; the longest are two-ribbon flares and the flares accompanied by multiple intensity bursts. Explosive type flares possess the shortest flash-phase. Such flares, in most cases, have one increased-brightness center, and they are most often observed in sites of filament disappearance.

Keywords: Solar flares, solar activity

1 Introduction

Spatial and temporal parameters for solar flares in the optical wavelength range were studied explicitly during the formation of the international flare patrol in 1930s–1970s. Lately, the instrumental basis of the Solar Survey has advanced significantly and there are stations with standard automated telescopes. Instruments' resolution, efficiency, and accuracy of measurements have increased. By using CCD matrices in astronomy, observations in the Hα line have improved essentially. But despite the above changes and a great data level stored during this time, there were practically no detailed investigations into this field. We targeted to define mean statistical parameters for solar flares with a focus on low-power flares, the most numerous, yet poorly studied class of solar flares.

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2 Importance distribution of solar flares

For our statistical analysis, we compiled an electronic database from the Solar Geophysical Data (SGD) and Quarterly Bulletin on Solar Activity (QBSA) catalogs over 1972–2010. Special software corrected errors, corrigenda, inaccuracies. We excluded the data obtained visually and with a poor image quality. The cases, when stations did not report the area class, the brightness class, and importance flare were taken into account. Because near the solar disk limb estimates for flare parameters are not so reliable (especially, for small flares), we did not address S and 1 area class flares with the >65° longitude. For major flares, there was no such a limitation. In group reports from stations, we refused generalized data. Upon applying the above limitations, we used the flare parameters of the station that had most reports in its database. As a result of such a selection, the total number of flares was 98225, of which 88518 had the S area class, 8012 referred to class 1, 1509 flares – to area class 2, and 186 flares – to classes 3 and 4.

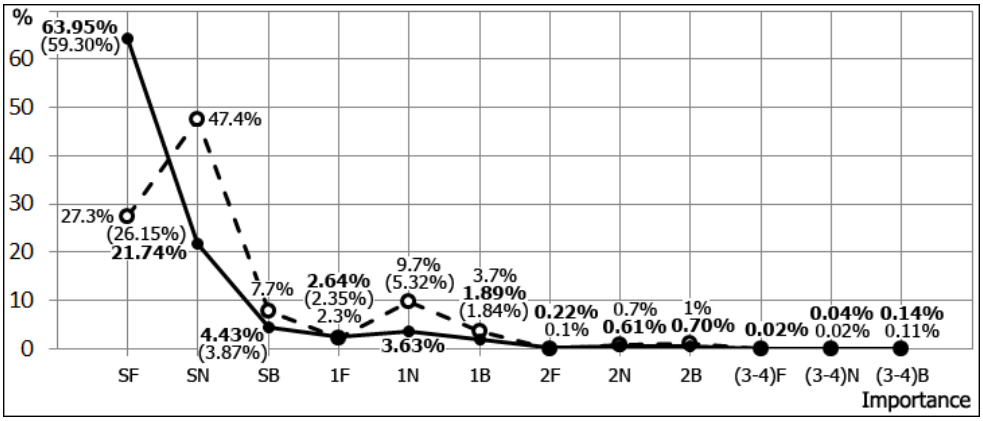


Figure 1 Importance distribution of solar flares.

Mitra et al. (1972) were the first to investigate the importance distribution of solar flares. In 2001, Temmer et al. (2001) performed similar studies. Figure 1 shows the importance distribution obtained by Borovik and Zhdanov (2017a). The data from Temmer et al. (2001) are given in brackets, the dash line presents the distribution from Mitra et al. (1972).

According to Mitra et al. (1972), the bulk of flares occurring on the Sun has the SN importance (47.4%). Flares with importance 1N (9.7%), 2B (1%) and (3–4)B (0.11%) show small maxima. From the distribution by Borovik and Zhdanov (2017a), the overwhelming majority of flares refer to the SF importance (63.95%); and there is no maximum for SN-flares. The excess of 1N-importance flares (3.63%) is on factor of 2.7 less, than that from Mitra et al. (1972), and on factor of 1.5 less, than that in Temmer et al. (2001).

The histogram shows a well-defined trend of reducing the number of flares with

the growth in the optical importance. The trend is accompanied by a gradual increase in the number of flares with a higher brightness class.

3 Rise, decay, and duration of solar flares

We analyzed statistically the temporal parameters of flares. Table 1 presents the mean time for flare brightness rise to maximum with the α confidence interval, time for decay and duration of flares, depending on the flare area, brightness, and importance. To add more statistical significance to large flares, we united area classes 2, 3, and 4 into group 2–4. Figure 2 shows variations in the mean, modal, and median parameters for the distributions.

Table 1 Mean temporal parameters for solar flares.

	Rise, min	Decay, min	Duration, min
SF	4.6±0.1	12.8±0.1	17.4±0.1
SN	5.4±0.1	16.6±0.2	22.1±0.3
SB	5.9±0.3	20.0±0.6	25.9±0.7
1F	10.5±0.6	29.7±1.0	40.3±1.4
1N	10.6±0.5	33.2±1.0	44.0±1.3
1B	10.8±0.7	40.1±1.4	51.5±1.8
(2–4)F	14.5±2.6	41.3±5.0	57.1±6.7
(2–4)N	16.0±1.6	52.0±3.3	69.3±4.5
(2–4)B	15.5±1.3	63.0±3.1	85.5±4.5
S	4.9±0.1	14.1±0.1	18.9±0.1
1	10.6±0.3	33.8±0.6	44.7±0.8
2–4	15.6±0.9	56.2±2.1	76.0±3.0
$\sum$	5.5±0.1	16.2±0.1	21.7±0.2
F	4.8±0.1	13.0±0.1	17.8±0.1
N	6.3±0.1	18.9±0.2	25.3±0.3
B	8.0±0.3	28.4±0.6	37.3±0.9

The results evidence that, with an increase in the area class, the brightness rise mean time increases from 4.9 to 15.6 minutes, decay time – from 12.8 to 63 minutes, duration – from 17.4 to 85.5 minutes. Modal times displace from 1 to 6 minutes for the rise, from 5 to 35 minutes for the decay, from 8 to 47 minutes for the duration. The median values vary from 3 to 10 minutes, from 9 to 55 minutes, and from 13 to 69 minutes, respectively. A similar trend is traced for the brightness characteristics of flares. An exclusion is the rise time, whose growth $\bar{t}$ with the increase in the flare importance is not explicit. The conclusion in earlier papers asserting that the bulk of flares (irrespective of their area class) reach the brightness maximum in 5–10 minutes after their onset (Smith and Smith 1966) is not corroborated.

Tables 2–5 present the solar flare temporal parameters obtained at different times by different authors.

The solar flare temporal parameters obtained in Borovik and Zhdanov (2017b, 2018a,b) differ from the results by other authors in most cases. The reason is, most likely, scarce data used by other authors for their statistical analyses.

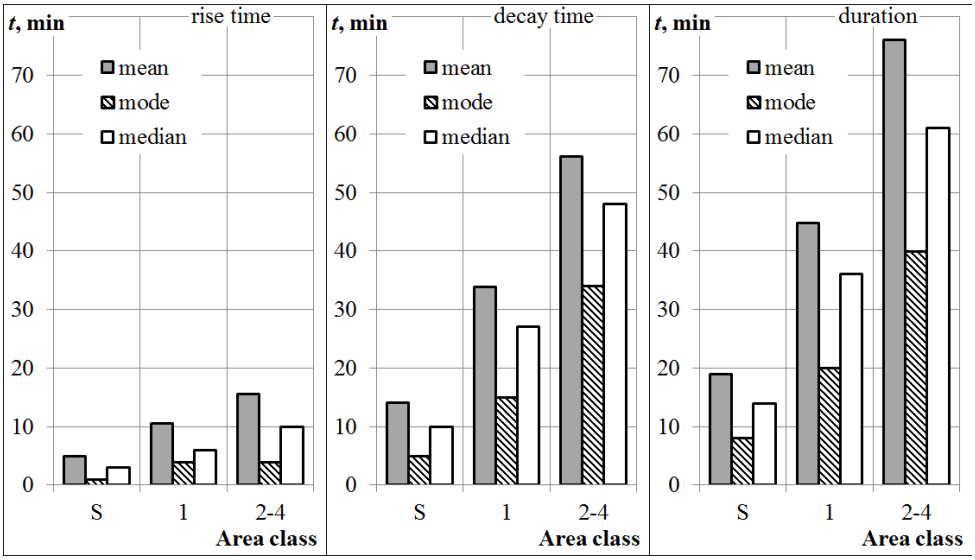


Figure 2 Variation in statistical parameters for solar flares with the area class growth.

Table 2 Mean time of solar flare main phase*.

	Quantity			Mean, min			Mode, min			Median, min		
	I	II	III	I	II	III	I	II	III	I	II	III
S	2660	85649	76985	5.7	–	4.9	<2	–	1	–	3	3
1	211	9176	6335	10.1	–	10.6	5	–	4	–	5	6
2	88	1014	1156	15.5	–	15.1	4	–	4	–	–	10
3	–	101	115	–	–	21.2	–	–	6	–	–	17
2–4	–	1120	1281	–	–	15.6	–	–	4	–	8	10
$\sum$	2959	95945	84601	–	5.1	5.5	–	1	1	–	3	3

*Columns I, the results from Smith and Smith (1966); columns II, from Temmer et al. (2001); columns III, Borovik and Zhdanov (2018a).

Table 3 Mean time of solar flare brightness decay time*.

	Quantity		Mean, min		Mode, min		Median, min	
	I	II	I	II	I	II	I	II
S	85649	76984	–	14.1	–	4	10	10
1	9176	6325	–	33.8	–	12	22	27
2–4	1120	1285	–	61.0	–	35	45	49
$\sum$	95945	84559	15.5	16.2	8	4	11	11

*Columns I, the results from Temmer et al. (2001); columns II, from Borovik and Zhdanov (2017b).

Table 4 Mean duration of solar flares*.

Authors	Period	Quantity of flares	Mean duration, min						
			1−	1	2	3	3+	2, 3, 3+	∑
			S	1	2	3	3+	2–4	
Newton & Barton (1937)[1]	1935–1936	–	–	–	–	–	–	–	20.0–40.0
Waldmeier (1938)	1935–1937	357	–	21.0	38.0	61.0	–	–	27.0
Giovanelli (1948)	1937–1938	24	–	–	–	–	–	–	30.0
Waldmeier (1948)	1935–1944	927	–	20.3	33.4	62.4	–	–	24.8
Ellison (1949)	1935–1947	109	–	17.0	29.0	62.0	≈ 180	–	–
Warwick (1954)	1951–1953	357	31.0	60.0				–	40.0
Dodson et al. (1956)[2]	1949–1952	194	28.0	43.0	66.0	84.0		–	–
Waldmeier & Bachmann (1959)	1945–1954	1604	–	21.1	44.8	84.9	–	–	28.1
Smith H. & Smith E. (1966)[3]	1957–1958	–	16.8	32.2	68.8	146.0	–	–	39.4
Reid (1968)	1958–1965	2907	16.5	28.2	–	–	–	60.5	–
Ružičková–Topolová (1974)[4]	1957–1965	661	–	–	71.3	129.9	305.5	–	–
Wilson (1983)	1980	1348	–	–	–	–	–	–	29.8
Antalová (1985)	1970–1974	460	27.0	55.0	78.0	–	–	–	–
	1975–1979	561	30.0	51.0	78.0	201.0	–	–	–
Wilson (1987)	1975	850	16.6	38.7	62.7	–	–	–	18.1
Barlas & Altaş (1992)[5]	1947–1990	3569	24.0	38.0	–	–	–	78.0	–
Borovik & Zhdanov (2018)[5]	1972–2010	577	21.7	37.3	–	–	–	71.8	24.2
Temmer et al. (2001)	1975–1999	75739	18.9	35.7	66.3	116.0	–	–	20.6
Borovik & Zhdanov (2018)	1972–2010	85968	18.9	44.7	72.8	106.9	85.6	76.0	21.7

*The table was compiled from Smith & Smith (1966) and Temmer et al. (2001), corrected and supplemented by Borovik and Zhdanov (2018b). The table top shows the flare area classes as per the 1956 First International Classification for Hα flares.

[1]Smaller value refers to less intensive flares; bigger value, to more intensive.

[2]Duration is calculated by flare photometric curves.

[3]Data were obtained from the International Geophysical Year (IGY).

[4]Addressed were only large solar flares.

[5]Addressed were only spotless solar flares.

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Table 5 Mode and median for duration distributions of solar flares.

Authors	Period	Mode, min						Median, min					
		1–	1	2	3	2, 3, 3+	$\sum$	1–	1	2	3	2, 3, 3+	$\sum$
		S	1	2	3	2–4		S	1	2	3	2–4	
Smith H. & Smith E. (1966)[3]	1935–1944	–	7.5	22.5	57.5	–	7.5	–	13.0	22.5	70.0	–	15.0
	1945–1954	–	7.5	22.5	50.0	–	7.5	–	16.0	33.0	40.0	–	23.5
	1957–1958	10.0	12.5	62.5	62.5	–	25.0	13.0	23.5	55.0	130.0	–	25.0
Temmer et al. (2001)	1975–1999	–	–	–	–	–	8	13	28	–	–	57	15
Borovik & Zhdanov (2018)	1973–2010	8	20	49	70	49	8	13	36	59	86	61	15

Table 6 Mean rise, decay, and duration time for separate types of solar flares.

type	class S			class S			class S		
	rise, min	decay, min	duration, min	rise, min	decay, min	duration, min	rise, min	decay, min	duration, min
V	3.9±0.2	15.4±0.4	19.1±0.5	8.0±1.2	36.6±3.3	44.7±4.2	12.0±3.1	66.3±12.0	83.2±15.5
D	4.0±0.2	11.2±0.4	15.2±0.4	7.5±1.3	21.0±2.7	27.7±3.4	13.6±6.1	46.4±12.3	65.1±17.5
H	5.1±0.3	14.3±0.5	19.3±0.7	9.4±1.1	29.5±2.0	38.8±2.6	15.6±3.4	49.8±05.5	65.0±07.2
G	5.8±0.5	15.9±1.2	21.7±1.5	9.2±2.2	28.6±6.9	37.3±8.1	15.3±5.8	56.5±27.1	71.8±29.1
E	6.0±0.2	16.7±0.4	22.5±0.5	9.7±0.6	31.3±1.2	40.9±1.5	14.4±2.0	52.5±05.0	68.6±06.3
U	7.6±0.8	25.4±1.7	32.7±2.0	13.2±1.5	50.0±3.0	63.7±3.9	19.7±2.5	80.2±05.4	109.1±08.0
K	10.9±0.8	33.5±1.6	46.0±2.1	19.5±2.5	50.6±3.9	72.8±5.6	25.0±4.7	67.6±07.5	97.2±10.9

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Table 7 Mean temporal parameters of superlong solar flares ($t > 60$ min).

	Rise, min	Decay, min	Duration, min
S	87.5±4.1	84.3±1.6	86.3±1.1
1	91.4±6.5	91.4±2.4	95.8±1.9
2–4	92.4±7.9	108.9±4.4	118.8±4.2
$\sum$	89.3±3.2	91.4±1.4	93.6±1.1

4 Flares with superlong phases

Early studies showed that the flash-phase and the main phase of flares continue for no more than one hour (Smith H., Smith E. 1966; Švestka 1976; Altyntsev et al. 1982). However, from the Flare Patrol data, it follows that there are many cases, when the flare phases last much longer (Table 7). With the growth in the area class, the relative percent of such flares increases.

The mean rise time of superlong flares in all the area classes are about equal, and make around 90 minutes. For 90% of such flares, the mean rise time is within 60–140 min. For low-power flares and area class 1 flares the maximal rise time is 5.8 hrs, for large flares – 3.6 hrs. About 80% of such flares differ in a shorter main phase. Their mean relative brightness rise time to the maximum is 0.67. 2.9% (3393) solar flares have superlong main phases. For 90% of such flares, they continue for 1 through 2–3 hrs. Maximal decay time for S- and class 1 flares is 8.9 and 11.7 hrs, respectively, for large flares – 8.3 hrs.

According to different sources, the overall duration of solar flares may vary from several minutes to 12 hrs, and even more. From our data, 94.6% of flares have duration of less than one hour, 5.4% (6675 flares) exist much longer. Lifetimes of 90% of superlong flares vary from 1 to 2.1–3.3 hrs. Maximal durations for S- and class 1 flares are 9.2 and 11.7 hrs, for large flares – 8.32 hrs. There are practically no flares with >12 hr duration.

The trend of temporal parameter growth with the area class increase manifests itself weakly or is completely absent.

5 Temporal parameters for separate solar flare types

Solar flares in their evolution exhibit a great variety of forms, features, and properties, for example: D – brilliant point; E – two or more brilliant points; G – no visible spots nearby; H – flare accompanied by high-speed dark filament; K – several intensity maxima; L – existing filaments suddenly active; M – white-light flare; R – asymmetry in H-alpha line suggests high speed mass ejection; S – brightness followed filament disappearance in same position; U – two bright branches, parallel or converging; V – an explosive phase; W – great area increase after time of maximum brightness; Z – major sunspot umbra covered by flare.

As established in Borovik and Zhdanov (2018a), such features are practically characteristic of all flares irrespective of their area class. This provides evidence that low-power flares do not differ from large flares in terms of their evolution features. Such flares, like powerful ones, are accompanied by filament activation and disappearance

(L, S, H); in active regions, they originate over sunspot umbrae (Z); they may have the explosive phase (V), and reiterated intensity bursts (K). Among small flares, there are two-ribbon (U) and spotless flares (G). Even extremely rarely observed white flares (M), accompanied by a powerful hard X-ray and microwave emission may have a very low optical importance in Hα line.

We analyzed temporal parameters for seven types of flares that have sufficiently high statistical significance (Table 6). For comparison, there are the parameters for all solar flares in the last line of the table. Because the confidence intervals for area classes 2–4 appeared sufficiently wide, we could draw confident conclusions, mainly, for S- and class 1 flares.

5.1 Rise time

Table 6 and Fig. 3 data show that, irrespective of the area class, the explosive-type flares (V) and the flares, inside the flare region of which there is one center of increased brightness (D) have the shortest rise time. For the S- and class 1 flares, that time is 4 and 8 min, respectively. The two-ribbon flares and the flares showing reiterated intensity bursts have the longest rises: (8 and 13 min) and (11 and 20 min), respectively. For large U- and K-flares, the rise time is about thrice the time for S-flares, and about twice the time for class 1 flares.

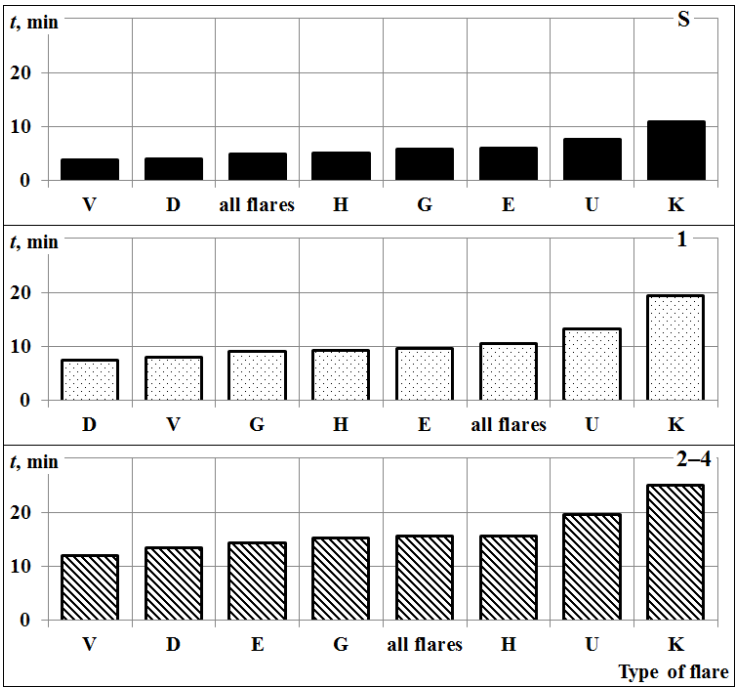


Figure 3 Mean rise time for separate types of flares depending on the area class. The histograms are arranged in the $\bar{t}$ ascending order.

H-flares are intermediate in Fig. 3 time series. The spotless flares (G) have a relatively low rise time. Spotless flares, as well as flares in active regions, may be accompanied by filament activation (L, H), with two-ribbon flares among them. Detailed studies of spotless flares were performed in Borovik & Myachin (2002, 2010) and Borovik et al. (2016).

5.2 Decay time

The flares with one center of increased brightness in the flare region (D) have the shortest decay time (as well as the rise time). The flares showing featuring several intensity bursts (K) and two-ribbon flares (Figure 4) have the longest decay time. For the S- and class 1 D-flares, it is equal to 11 and 21 min, respectively; for two-ribbon flares, it is 25 and 50 min; for K-flares, it is 34 and 51 min. Note that, with the area class growth, some types of flares change their relative position on Fig. 4 temporal scale. In particular, the explosive-type flares (V) move toward longer decay time, closer to two-ribbon and K-flares.

On average, the solar flare main phase, relative to the rise time, lasts about three-fold longer. Herewith, two types of flares are notable: two-ribbon with the 3.3, 3.8,

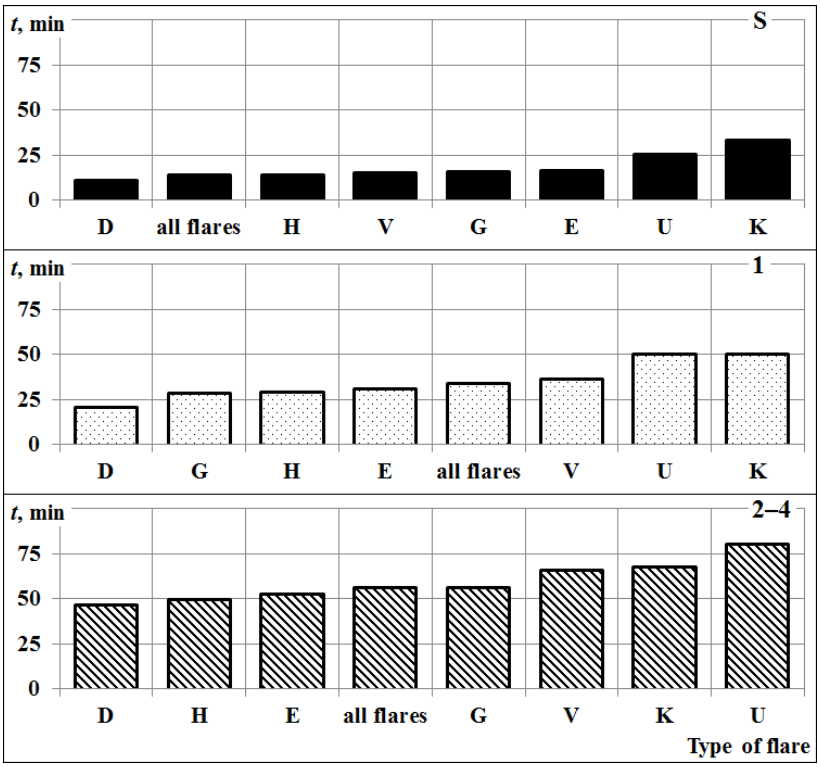


Figure 4 Mean decay time for separate types of flares depending on the area class. The histograms are arranged in the $\bar{t}$ ascending order.

4.1 phase ratio (in the area class ascending order), and the explosive-type flares with 4, 4.5, and 5.5, respectively.

5.3 Duration

Of all the flare types, D-flares have the shortest lifetime (Fig. 5). For S- and class 1 flares, their $\bar{t}$ is 15 and 28 min, which is about 1.5 times less than that for the flares with two and more centers (E). The longest are the flares showing reiterated intensity bursts (K). Their lifetime ($\bar{t}$ = 46 and 73 min) relative to, for example, explosive-type flares (V) ($\bar{t}$ = 19 and 45 min), is longer by a factor of 2.4–1.6, and, as compared with D-flares, the duration is longer by a factor of 3.0–2.6.

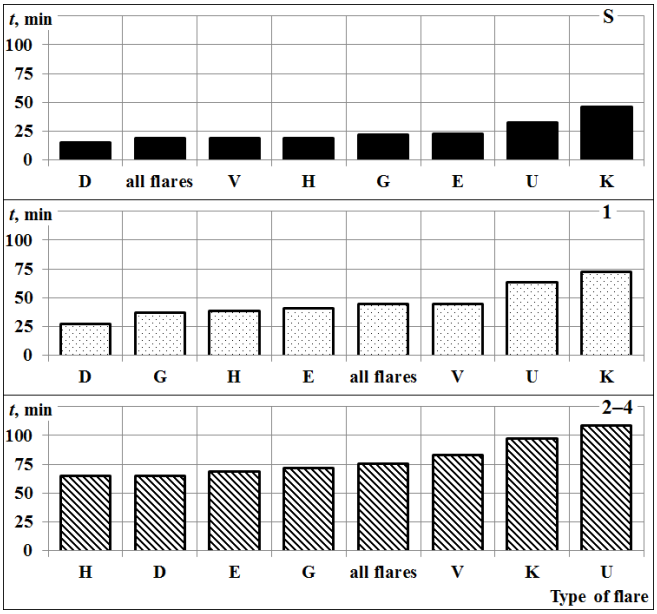


Figure 5 Mean duration for separate types of flares depending on the area class. The histograms are arranged in the $\bar{t}$ ascending order.

By duration, two-ribbon flares are close to K-flares. Their mean duration for S- and class 1 flares is 33 and 64 min, which is significantly higher than the mean values for all solar flares. With the area class growth, explosive-type flares (V) move toward longer duration, closer to two-ribbon and K-flares, as well as its occurrence in the case of decay.

Individual flare features appear to be closely related among themselves. Thus, explosive-type flares (V) involve, in most cases, only one center of increased brightness (D) and do not exhibit reiterated intensity bursts (K). More often, such flares are observed in filament disappearance sites (S). On the contrary, two-ribbon flares (U), as a rule, involve several centers of increased brightness (E), and, in most cases, demonstrate reiterated intensity bursts (K). Among them, there is low percentage of explosive flares (V) and high percentage of the flares covering sunspot umbrae (Z).

The model for two-ribbon solar flares (CSHKP), most developed for today, combining different observational and theoretical schemes (Carmichael 1964; Sturrock 1966; Hirayama 1974; Kopp, Pneuman 1976), assumes that the flare onset and rapid release of the energy accumulated in the magnetic field occur in the corona, as a result of magnetic reconnection. From a coronal source, the energy is transmitted along magnetic tubes into the chromosphere, where it manifests itself in the form of flare ribbons and magnetic loop footpoints in the hard X-ray radiation. As established in Švestka et al. (1982) and corroborated by other authors (Fletcher et al. 2011), X-ray loop footpoints are usually associated with the brightest sites of solar flares. Apparently, reiterated intensity bursts reflect the dynamics of the energy source(s), when the energy release in a separate magnetic tube of flare arcades corresponds to each brightness burst. Presumably, the number of increased brightness centers inside the flare region (footpoints of magnetic loops) plays an important role for flare duration.

The obtained evidence gives grounds to assume that the physical nature of explosive type flares differs from the nature of two-ribbon flares. Most likely, V-type flares relate to so-called Hyder flares, the flares that, as per the gravity model (Hyder 1967), originate as a result of the eruptive prominence substance fall (reenter) from the corona into the chromosphere.

Conclusion

As a result of our statistical analysis, we may draw the following conclusions:

- More than 90% of flares occurring on the Sun are low-power flares, of which the most numerous are SF flares (64%). The importance distribution of solar flares has a smooth decrease without essential deviations.
- For the first time, we obtained fuller and statistically more reliable data on the temporal parameters for solar flares of different area classes and importance. Those parameters may be used to estimate the energetics of solar flares and to build physical models for the latter.
- We corroborated the trend of flare temporal parameter growth with the area class increase. That trend is accompanied by rearranging the number of flares toward their brightness class increase.
- We established that 94.6% of solar flares have duration of no more than 1 hr. 90% of the flares with t>60 min exist within 2.1–3.3 hrs. In very rare cases, flares may last for about 12 hrs. The lifetime of major solar flares is no more than 8.3 hrs.
- We showed that the temporal parameters of solar flares depend not only on the area class, but also on their evolution features. The most short-lived are flares with one increased brightness center in the flare region, the longest are two-ribbon flares and the flares accompanied by reiterated intensity bursts. Explosive-type flares possess the shortest flash-phase. One may assume that, because the percentage ratio of various-type and of various area class flares varies with the cycle phase, and the total mean statistical parameters of solar flares will also vary: they will decrease toward the solar activity minimum and increase toward its maximum.

- We obtained the evidence that low-power flares do not differ from major solar flares in terms of their evolution features. They, like powerful flares, are accompanied by filament activation and disappearance, have the explosive phase, and reiterated intensity bursts. Among them, there are flares covering sunspot umbrae, two-ribbon, and white flares.

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