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Turbulent and fractal nature of solar and stellar magnetism

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Since the Babcock–Leighton phenomenological solar dynamo concept, a variety of solar dynamo models (in particular, mean-field dynamo models) were proposed to explain the 11-year cyclicity of solar magnetic activity. Similar magnetic cycles are observed on stars of the solar type. As the observational and computational facilities improved, ubiquitous irregularities in a broad range of scales appeared implying that the mean-field dynamo alone is not enough to explain the observed picture. A concept of small-scale near-surface dynamo was suggested to explain a magnetic carpet – population of mixed-polarity small magnetic elements in an undisturbed photosphere. Besides, an origin of irregularities on larger scales (scales of active regions, ARs) is still an open question. A question appears: is it necessary to introduce two independent mechanisms to explain the magnetism on the Sun and stars? Using the magneto-graphic observations of the Sun, we have shown that solar magnetic field possesses properties of multifractality, its near-surface plasma evolves in a state of developed turbulence, the flux dispersion occurs in a regime of anomalous diffusion (super-diffusivity). Therefore, the process of magnetic field generation can be explained by unique dynamo mechanism operating as a non-linear dynamical dissipative system in a broad range of scales: from a typical AR size down to the size of smallest observable magnetic features. Pragmatic consequences can be as follows: i) strong fluctuations (e.g., flares) are unpredictable and not rare; ii) transport processes occur on a percolation cluster, which differs significantly from transport in continuous medium; iii) Gaussian statistics and central limit theorem are not applicable, individual probability density functions should be utilized for statistical estimates.

Keywords: Sun: magnetic fields – Sun: photosphere – Sun: sunspots – Sun: activity

1 Introduction

The whole variety of observed solar magnetic fields is produced as a result of continuous interaction between plasma flows and magnetic fields inside the convective zone (CZ), the process referred to as the solar dynamo. The basic idea of the solar dynamo

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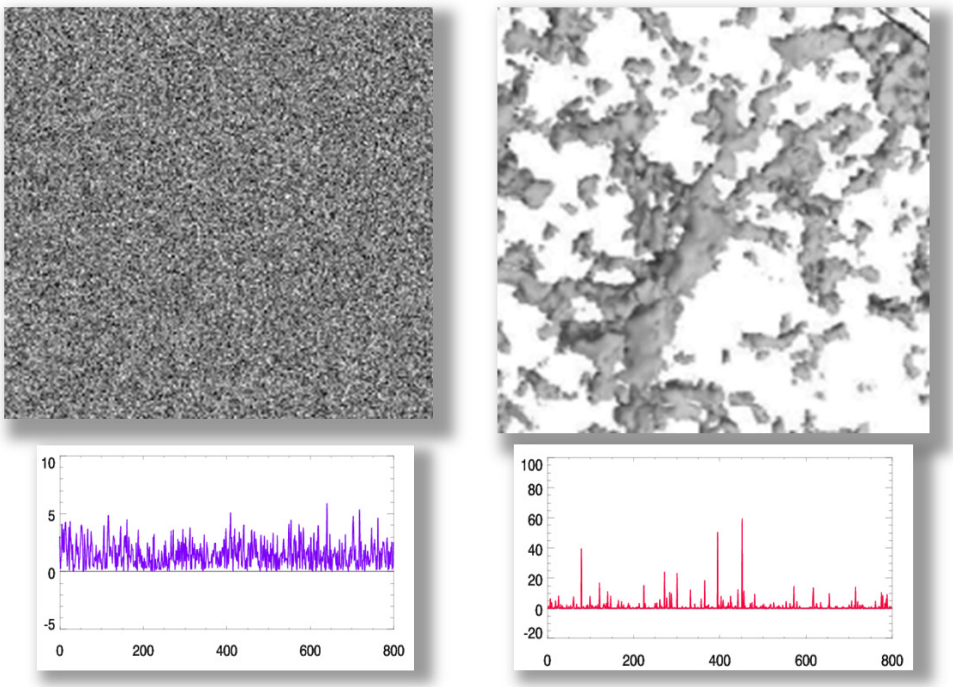


Figure 1 Illustration of difference between Gaussian and multifractal processes. Left – typical patterns of spatial and time behaviour for a Gaussian process: an array of random fluctuations with extremely rare large peaks; distribution functions are normal. Right – the same for a multifractal process in a NDDS: extended zones of low activity are intermittent with narrow patches of high activity, strong fluctuations are not rare and determine high statistical moments causing heavy-tailed distribution functions.

is that the poloidal component of the field is transformed to the toroidal, which, in turn can reproduce the poloidal component again thus closing the dynamo loop. The conceptual model of the solar dynamo was proposed by Babcock (1961) and Leighton (1964), referred to hereafter as BL-model. A basic idea of this phenomenological scenario was later transformed into a so-called mean-field dynamo theory, and details are discussed in numerous reviews (e.g., Charbonneau (2010, 2014), Miesch (2010, 2012), Choudhuri (2013)). Mean-field dynamo describes well the solar cycle, the evolution of the large-scale magnetic field of the Sun (as well as of a sun-like star with a convective zone). Although we know that there are no two absolutely similar solar cycles, yet, self-organization of the large-scale dynamo remains impressive.

We observe a drastically different picture in the photosphere on small spatial and temporal scales. The large-scale dynamo is hardly of much help in the explanation of the magnetic chaos in the quiet sun photosphere. To do that, a concept of small-scale, local, or turbulent dynamo was elaborated (e.g., Petrovay and Szakaly (1993), Cattaneo (1999), Vögler and Schüssler (2007), Pietarila Graham et al. (2010)). A purpose of this review is to analyze, is it necessary to involve two independent dynamo mechanisms to understand the solar magnetism? In other words, are there observations

Table 1 Spectrum of physical processes.

Process:	Deterministic	Multifractal	Gaussian
Energy dissipation:	No dissipation	Inhomogeneous dissipation	Homogeneous dissipation
Structurization:	One or several expicitly defined structures	Hierarchy of random structures; extended zones of low activity are intermittent with narrow patches of extremely high deviations; large fluctuations are not rare	Array of random, predominantly small peaks with extremely rare large peaks
Tools:	A set of differential equations; the solution is uniquely determined by boundary/initial conditions	Multifractal analysis; percolation theory; theory of catastrophs; strange attractors; self-organized criticality	The central limit theorem; Gaussian statistics; the least mean square technique

allowing us to consider the solar magnetic field generation as a united process in a non-linear dynamical dissipative system (NDDS)? Such a system displays order and self-organization on large scales, self-similarity on intermediate scales and chaos on smallest scales. Generally, any turbulent medium is a NDDS. A set of useful tools is elaborated to explore such systems in different fields of research, including physics, biology, sociology. We can benefit by using them in the explanation of the magnetic field generation on the Sun and Sun-like stars.

A brief introduction into the NDDS concept can be as follows. When starting to analyze any physical system, one has to answer a fundamental question: whether dissipation of energy can be neglected? In the case of affirmative answer, we deal with deterministic process without dissipation (see Table 1, 2nd column). A deterministic system can be usually represented by one or several explicitly defined structures, and can be described by a set of differential equations. The uniqueness of the solution is fully determined by initial conditions. In the case of negative answer, the easiest (and for a long time the most common) assumption was to consider dissipation as a homogeneous (in time and space) process with Gaussian properties (see the last column in Table 1). In the case of a Gaussian process, a structure is an array of random, predominantly small peaks with extremely rare large peaks. Distribution functions are Gaussian. There are tools for dealing with such systems: the least mean square technique based on the central limit theorem.

In between these limits, a vast majority of natural processes is hosted. Processes where dissipation is allowed, but it is not homogeneous but rather highly inhomogeneous in spatial domain (multifractal), and highly intermittent, burst-like, in time. Large fluctuations are not rare, distribution functions are not Gaussian but rather heavy-tailed. A set of equations includes the dissipation term, and a solution (if any) is not determined by boundary/initial conditions. The tools for Gaussian fields analysis are not applicable to treat such systems. One has to simplify the problem, or to

find other ways to analyse such systems, which are referred to as non-linear dynamical dissipative systems (NDDSs).

Essential properties of NDDS can be summarized as follows: multifractality/intermittency in time and space (e.g., Abramenko 2008); self-similarity and self-organized criticality (Aschwanden et al. 2018); anomalous diffusivity (Klafter and Sokolov 2005); interconnection of processes on different scales (e.g. Frisch 1995). In application to the upper boundary of the solar convection zone (the observable layer), the properties can be reformulated as requirements for NDDS: a state of developed turbulence for magnetized plasma in the photosphere; multifractal structure of the magnetic field; anomalous turbulent magnetic diffusivity; signatures of magnetic field generation on a broad range of scales (from smallest observable magnetic elements to a typical scale of an active region). Below, by using solar magnetographic data, I show how these requirements are met for the Sun.

2 Turbulent state of photospheric plasma

As it was mentioned above, a turbulent medium is a particular case of NDDS. The turbulent state of photospheric magnetized plasma can be best diagnosed via the power spectra of 2D-structures of the magnetic and velocity fields. Presence of a broad linear range (so-called inertial range) in a spectrum is a signature of turbulent state (Frisch 1995). Magnetic power spectra were retrieved from magnetograms of ARs by many authors (e.g., Abramenko et al. 2001, Abramenko 2005b, Abramenko and Yurchyshyn 2010a, Katsukawa and Orozco Suarez 2012, Mandage and McAteer 2016). ARs demonstrate a power-law magnetic spectrum $E(k) \sim k^{-\alpha}$ with well-pronounced inertial interval and the power index α varying around the classical Kolmogorov value of 5/3. Examples of ARs magnetic power spectra are shown in Fig. 2 (by Abramenko and Yurchyshyn 2010a). An interesting feature was revealed in Abramenko (2005) and Abramenko and Yurchyshyn (2010a): ARs with a spectrum which is steeper than Kolmogorov's spectrum are prone to strong flaring. A flare index, A, is shown in Fig. 2. The index was introduced by Abramenko (2005) and it indicates a specific flare productivity, namely, an AR with $A = 1.0$ (100) produced, on average, one C1.0 (X1.0) flare per day during the passage across the solar disc. This finding indicates that the state of turbulence in the photosphere is relevant to eruptive energy release in the upper atmosphere.

In the undisturbed photosphere (outside ARs) the magnetic power spectra are more shallow with α close to 1, see Fig. 3, where examples of spectra are presented. The inertial range is still observable, however the character of turbulence seems to be different with respect to that in ARs.

Magnetic and kinetic power spectra for undisturbed photosphere were compared in Abramenko et al. (2012), where it was shown that as the resolution of an instrument improves, the shallow magnetic energy spectrum tends to extend toward higher wavenumbers (smaller scales), the slope remaining the same. The behaviour of the kinetic spectrum is different: the spectrum shows a steady maximum at 1–3 Mm followed by the Kolmogorov-type regime with $\alpha = 5/3$. This relation between kinetic and magnetic spectra seems to be favourable for small-scale turbulent dynamo action (see, e.g., Biskamp 1993).

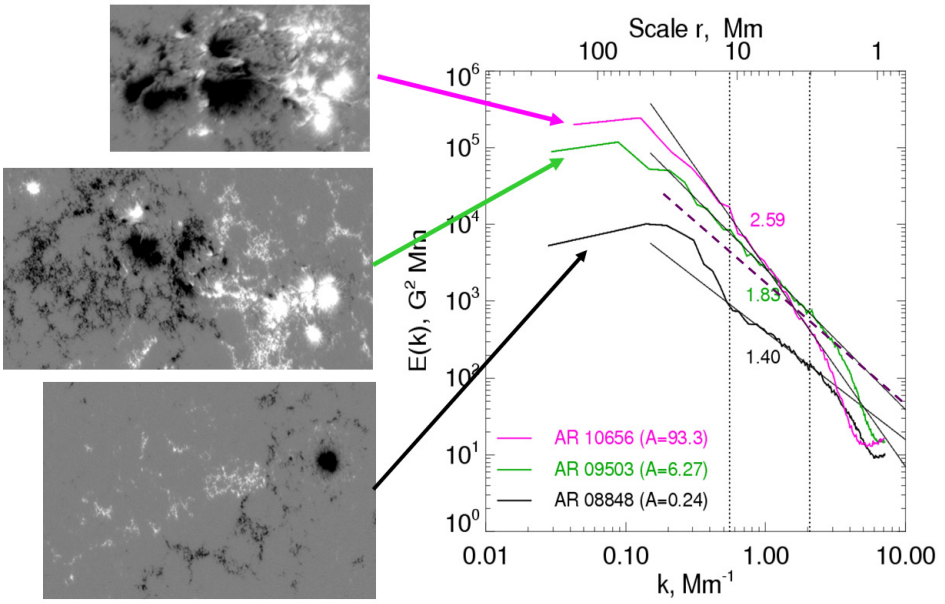


Figure 2 Magnetograms of three ARs (left) and calculated power spectra (right). NOAA ARs numbers and flare index, A, of ARs are shown. Numbers 1.40, 1.83 and 2.59 are the power indices α in a relationship $E(k) \sim k^{-\alpha}$. A dashed line indicates the classical Kolmogorov turbulence with $\alpha = 5/3$. Vertical dotted segments indicate a linear range of the spectrum, where the best linear fit was calculated.

3 Multifractal nature of magnetic fields

Multifractality of solar magnetic structures was first reported by Lawrence et al. (1993). Using a box counting method, authors acquired the spectrum of fractal dimensions $f(\alpha)$ versus the singularity strength (Holder exponent) α for a quiet-sun area. Based on the fact of magnetic multifractality, the authors made a suggestion that the quiet-sun magnetic field is generated by local small-scale dynamo, whereas ARs magnetic field is due to global mean-field dynamo. Since this pioneering study, many authors explored multifractal and intermittent nature of solar magnetic fields (e.g., Abramenko et al. 2002, Abramenko 2005a, Conlon et al. 2008, McAteer et al. 2010, Abramenko and Yurchyshyn 2010b, McAteer 2015). Multifractality of solar magnetograms was also explored in terms of Hausdorff dimension by Makarenko et al. (2012), who provided a strong mathematical basis for calculations of the spectrum of multifractality.

Usually, a technique to derive the multifractality spectrum involves the calculation of a fractal dimension on the basis of the box counting method. The latter is based on a thresholding technique, that is rather problematic when applied to noisy data. Solar magnetograms always contain a lot of noise. So, an alternative technique, based on the structure functions, which do not require any thresholding, was suggested in Abramenko et al. (2002). Structure functions for the line-of-sight magnetic field

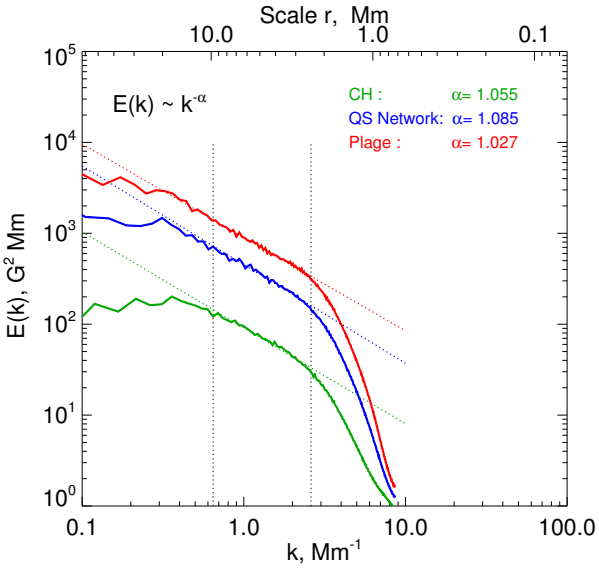


Figure 3 Magnetic power spectra in undisturbed solar photosphere: areas inside a coronal hole, in quiet sun network and inside a plage zone observed on February 10, 2015 on the disc center with SDO/HMI. The vertical dotted segments show the scale interval, where the best linear fits were calculated. The values of the power index α are shown.

component B_z can be presented as

$$S_q(r) = \langle |B_z(\mathbf{x}+\mathbf{r}) - B_z(\mathbf{x})|^q \rangle, \tag{1}$$

where $\mathbf{r}$ is a separation vector, $\mathbf{x}$ is a current point on a magnetogram, and q is a statistical moment. In Abramenko et al. (2002) multifractality in AR magnetic field was revealed and it was shown that multifractality is higher for high flare activity. (Note that there exists a direct relationship between the multifractality spectrum from structure functions and the classical spectrum of multifractality $f(\alpha)$, see Abramenko and Yurchyshyn (2010b)). Scaling of structure functions allows us to introduce a new technique to reveal the range of scales where the multifractatity is held, and the degree of multifractality. Namely, a ratio of the sixth-order structure function to the cube of the second-order structure function determines a so-called flatness function (Abramenko 2005a):

$$F(r) = S_6(r)/(S_2(r))^3 \sim r^{\kappa}. \tag{2}$$

For structures without multifractality, $F(r)$ does not depend on the scale (it is flat for all r); however, for multifractal structures, it grows by a power law in the scale interval, Δr (see Fig. 4), where the multifractality takes place. The higher the degree of multifractality, the steeper the function $F(r)$ is. The absolute value of the power index κ is accepted as a measure of multifractality (Abramenko 2005a, Abramenko and Yurchyshyn 2010b). An example of the flatness function for an AR magnetogram is shown in Fig. 4.

Further studies have shown that magnetic fields of all ARs possess the property of multifractality, however the degree of multifractality varies and seems to be related

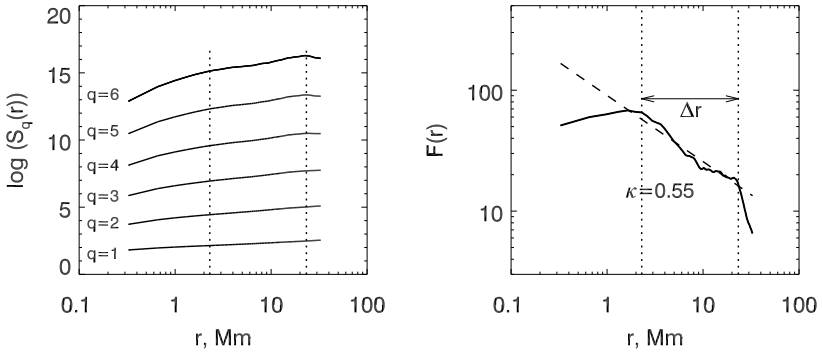


Figure 4 Structure functions $S_q(r)$ (*left*) calculated from a magnetogram of NOAA AR 10930 recorded on 2006 Dec 12 at 00:48UT by SOT-FG/Hinode instrument. *Right:* flatness function $F(r)$ calculated from the structure functions. Vertical dotted lines mark the interval of multifractality, Δr, where flatness grows according to the power law when r decreases. The interval Δr is also marked in the left frame. The multifractality index κ is the slope of $F(r)$ calculated within Δr: the steeper the flatness function, the higher the degree of multifractality.

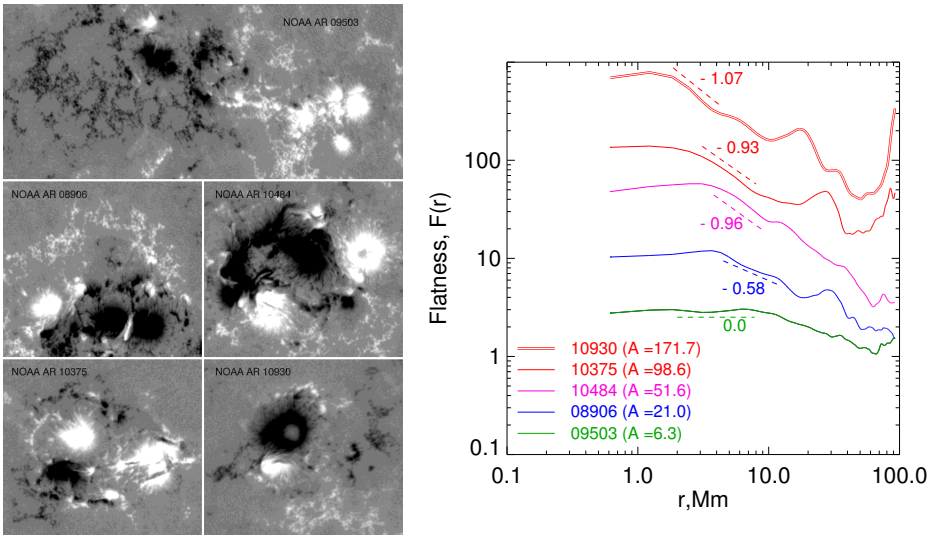


Figure 5 SOHO/MDI/HR magnetograms (*left*) and flatness functions (*right*) for five ARs. The $F(r)$ functions are shifted up into a "well-readable" position. The dashed segments show the multifractality regions for small scales, as well as the corresponding indexes, κ. Flare index, A, is marked for each AR. All magnetograms are shown at the same scale, and the width of the AR NOAA 10375 corresponds to 186 arcsec, or 135 Mm on the Sun.

to flaring activity of an AR (Abramenko and Yurchyshyn 2010b). As soon as the degree of multifractality represents the complexity (and therefore, non-potentiality) of the AR magnetic structure, the revealed correlation between the multifractality index and flaring productivity is natural and agrees with the NDDS standpoint: high fluctuations are the manifestation of high degree of multifractality (see Fig. 1).

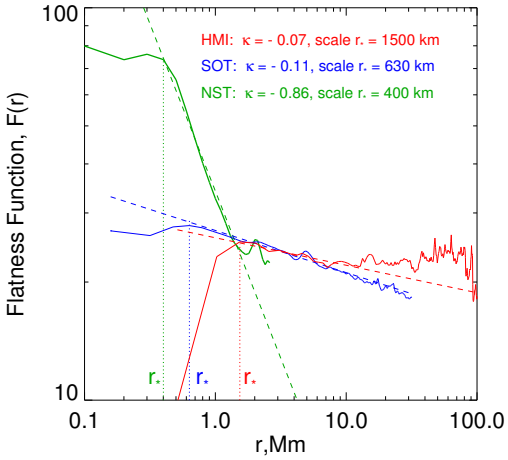


Figure 6 Flatness functions calculated from magnetograms obtained for three coronal holes with different telescopes: SDO/HMI magnetogram (August 12, 2011); SOT/SP/Hinode magnetogram (March 10, 2007); BBSO/NST magnetogram (June 2, 2012). The dashed lines are the best linear fits to the data on scales exceeding $r_\star$, the cutoff of the flatness function on small scales. The slopes of the fits, the multifractality indexes κ, are shown. By Abramenko (2013).

Examples of magnetograms and flatness functions for ARs of various flare activity are shown in Fig. 5. Data from SOHO/MDI instrument obtained in the high resolution mode were used.

Outside ARs, in undisturbed photosphere, multifractality of the magnetic field is also present, however it is much weaker when considered on the same scales as the aforementioned ARs were. At the same time, as the resolution improves, multifractality of small-scale quiet sun magnetic field becomes well pronounced. Figure 6 presents a comparison of flatness functions for three coronal hole areas (weakest magnetic environments on the Sun) as derived from three instruments. Multifractality is almost not seen under weak resolution: for the SDO/HMI (resolution of 1 arcsec) and SOT/SP/Hinode (resolutuon of 0.32 arcsec) data, the multifractality index κ is small: –0.07 and –0.11, respectively. The cutoff, $r_\star$ of $F(r)$ on small scales, caused by the insufficient resolution, occurs on scales of 1500 km and 630 km, respectively. Thus, only weak multifractality is detected on scales above approximately 1000–1500 km. At the same time, the BBSO/NST data (resolution of 0.1 arcsec) show high degree of multiftactality on scales from 1500 down to 400 km with $\kappa = -0.86$. So, the degree of multifractality seems to increase as the scale decreases, which is favourable for the small-scale local dynamo action.

Interestingly, recently a signature of multifractality was found in time series of magnetic activity of 34 M dwarf stars observed by *Kepler* mission (de Freitas et al. 2017).

4 Anomalous diffusion

When a medium under study is a multifractal, the property of multifractality is held in both spatial and temporal domains. One of the ways to explore the temporal multifractality is to examine the turbulent diffusion regime from observational data. The diffusion process cannot be normal diffusion in a multifractal medium, it is anomalous diffusion (Lawrence and Schrijver 1993, Klafter and Sokolov 2005).

For a long time, a mechanism for transporting the magnetic flux over the solar surface on small scales was supposed to be *normal diffusion*, when the mean-squared displacement of flow tracers (displacement spectrum) varies with time, τ, as $\langle(\Delta l)^2\rangle = 4K\tau \sim \tau^\gamma$, where K is the diffusion coefficient (a scalar), and $\gamma = 1$ (Fig. 7). When index γ deviates from unity, diffusion is called *anomalous diffusion*. A regime with $\gamma > 1$ is referred to as *super-diffusion*, while $\gamma < 1$ indicates *sub-diffusion*. Parameters $\langle(\Delta l)^2\rangle$ and γ can be derived from observations and allow us to determine the diffusion coefficient as a function of time and spatial scales (exact formulae can be found in Abramenko et al. (2011)). With reference to a turbulent process, the diffusion coefficient K is usually noted as η_T – turbulent diffusivity.

Under low resolution, detection of small displacements is problematic, and this might be a reason of sub-diffusivity in quiet sun reported by Cadavid et al. (1999), see Fig. 7, left panel. Recent studies of solar diffusivity, based on new-generation high resolution data, undoubtedly demonstrated that diffusion in quiet sun occurs in a regime of super-diffusivity (e.g., Abramenko et al. 2011, Lepreti et al. 2012, Giannatasio et al. 2013, 2014a,b, Jafarzadeh et al. 2017, Abramenko 2017). The regime of super-diffusivity implies that the coefficient of turbulent diffusion varies in direct proportion with time and spatial scales, see Fig. 7, right panel. This means

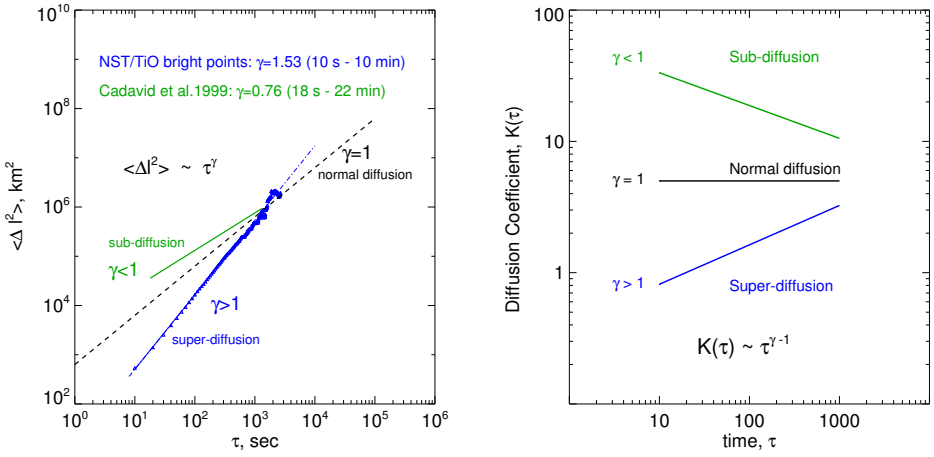


Figure 7 Left – displacement spectra for different diffusion regimes; sub-diffusion is schematically presented per Cadavid et al. (1999) data, super-diffusion is shown using BBSO/NST high resolution data described in Abramenko et al. (2011); normal diffusion is presented schematically by a dashed line. Right – schematic illustration of the diffusion coefficient behaviour under different diffusivity regimes; arbitrary units.

that on large scales (e.g., scales of supergranulation, above 30 000 km) high diffusivity can be achieved (facilitating meridional flows, see, e.g., Wang et al. 2002), whereas on smallest scales, the diffusion coefficient decreases as the scale decreases, which is favourable for the local turbulent dynamo action. Observational results on the direct proportion between the diffusivity and scales for various magnetic environments on the Sun can be found, in particular in Abramenko et al. (2011, Fig. 10), Giannattasio et al. (2013, Fig. 5), Abramenko (2017, Fig. 8).

To this end, the property of multifractality, one of the essential properties of NDDS, is attributable for the solar surface magnetic fields.

5 Concluding remarks

A list of NDDS properties, which are fulfilled for the Sun is not restricted by the aforementioned ones and can be easily continued. For example, a presence of percolation properties for solar structures is obvious (Balke et al. 1993, Seiden and Wentzel 1996, Pustil'nik 1999, Schatten 2007). A new field of research is now developing: to model solar processes with slow stochastic driving and burst-like energy release: Self-Organized Criticality (SOC) concept (Longcope and Noonan 2000, Lopez Fuentes and Klimchuk 2006, Aschwanden 2011a,b). A theory of catastrophes as transitions in the phase space is also applicable to the magnetic sun (Isenberg and Forbes 2007, Kitchatinov and Olemskoy 2011, Hubbard and Brandenburg 2012). Meanwhile, the most impressive argument and support for the NDDS concept seems to be a huge progress in exploring fluctuation dynamo (also referred to as small-scale, local, or turbulent dynamo). This mechanism of magnetic field generation was recently revealed on the Sun both from observations (Hagenaar et al. 2003, Lites et al. 2008, Ishikawa and Tsuneta 2009, Abramenko 2013, Sokoloff et al. 2015) and from numerical simulations (Vogler and Schussler 2007, Pietarila Graham et al. 2010, Rempel 2014). Observations show signatures of fluctuation dynamo for a broad range of scales: from smallest observable scales (Abramenko 2013) to scales of a typical AR (Sokoloff et al. 2015). So, there are all means to consider the magnetic field generation not as two independent processes (global mean-field dynamo and small-scale dynamo), but rather as a united process in a non-linear dynamical dissipative system – convective envelope of our star.

Thus, our understanding of the dynamo process made a new circle along the dialectic spiral: from the classical Babcock–Leighton model that explains the global dynamo and solar cycle, to the acceptance of the small-scale local turbulent dynamo, which might operate inside the near-surface layer and be responsible for generation of small-scale magnetic fields forming the magnetic carpet. And yet, this scenario of the simultaneous action of two dynamos seems to be oversimplified. Finally, closing the circle, we suggest that the solar dynamo is both chaotic and cyclic, and performs as a non-linear dynamical dissipative system.

Pragmatic consequences can be as follows: i) strong fluctuations of energy release (flares) are unpredictable; ii) Gaussian statistics and central limit theorem are not applicable, individual probability density functions should be utilized for statistical estimates; iii) transport processes occur on a percolation cluster, which differs significantly from transport in continuous medium. In particular, when estimating

magnetospheric accretion of matter from the inner disk regions onto a T Tauri star, the fractal nature of the photospheric magnetic field on the star surface should be counted (Rei et al. 2018): the shock front bumps into a complex magnetic structure of intermittent closed and open magnetic field lines, that unavoidably will affect the physics of interaction between the shock and the photosphere.

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