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R^n gravity as a viable alternative to dark matter: application to stellar dynamics

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The role of $f(R)$ gravity, as well as the other modifications of standard Einstein's gravity, is to explain the accelerated expansion, structure formation of the Universe, and some other phenomena at extragalactic scales (such as e.g. flat rotation curves of spiral galaxies) without adding unknown forms of dark energy or dark matter. In $f(R)$ model, the Ricci scalar in the Einstein–Hilbert action is replaced by a general function of it. Its power-law form R^n is analyzed here using observed orbits of S-stars around the Galactic Center and also their computer simulations. We review the various consequences of the R^n gravity parameters (r_c, the characteristic radius, i.e. the scalelength depending on the gravitating system properties; and β, the universal constant) on stellar dynamics and investigate their constraints from the observed S-star orbits. The presented results show that these observations could put reliable constraints on the parameters of R^n gravity.

Keywords: Modified theories of gravity, experimental tests of gravitational theories

1 Introduction

Extended Theories of Gravity (ETGs) are physical theories that attempt to describe the phenomena of gravitation in competition with Einstein's theory of General Relativity (GR), by preserving the undoubtedly positive results of Einstein's theory. Instead of introducing Dark Matter (DM), a hypothetical type of matter which does not emit or interact with electromagnetic radiation, some theories which modify the

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laws of gravity, could explain in a very natural fashion several astrophysical and cosmological observations. For ETGs, see review in Capozziello & De Laurentis (2011).

The $f(R)$ modified gravity is the straightforward generalization of Einstein's General Relativity, where the function $f(R)$ is not linear in the Ricci scalar R ($f(R) \neq \mathrm{R}$) as in the Hilbert–Einstein action. Here we use R^n gravity, the power-law version of $f(R)$ modified gravity. About considerations of the power-law fourth-order theories of gravity see in Capozziello et al. (2007). In the weak field limit, R^n gravity potential (generated by a pointlike mass m at the distance r) is (Capozziello et al. 2007):

$$\Phi(r) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right], \tag{1}$$

where r_c is scalelength depending on the gravitating system properties and β is universal constant which depends on power n:

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}. \tag{2}$$

For consideration of the power-law fourth-order theories of gravity, as well as the determination of the parameter space of $f(R)$ gravity, see Capozziello et al. (2007), Borka et al. (2012), Borka et al. (2013), Zakharov et al. (2014), D. Borka et al. (2015), V. Borka Jovanović et al. (2016).

Sagittarius A (or Sgr A) is a complex radio source that consists of three components, which overlap: Sgr A East (the supernova remnant), Sgr A West (the spiral structure) and Sgr A* (a very bright compact radio source). Sgr A* is very compact and motionless source, and its location coincides with the dynamical center of the Galaxy. Although the compact source, Sgr A* has mass of order $\sim 10^6 M_\odot$ (Genzel et al. (2010)) and it is assumed to be a black hole. The supermassive black hole (SMBH) Sgr A* at the Galactic Center (GC) is surrounded by a cluster of stars orbiting around it: S-star cluster. Light from these stars is bent by the gravitational field of the black hole (see detailed explanation e.g. in Bozza & Mancini (2012)). S-stars are orbiting with large velocities ($v > 1000$ km/s), and have very eccentric orbits around central SMBH at GC. In the paper by Gillessen et al. (2009a), these stars (in the innermost arcsecond of our galaxy) are used as test particles to probe the potential in which they move. S2 star is one of the brightest members of the S-star cluster. It has about 15 Solar masses and seven times its diameter, with orbital period about 16 yr.

Here we use the astrometric observations of S2 star to probe and constrain R^n gravity potential. We organized this paper as an overview of our previous results regarding R^n gravity, but with an accent on the method we used: we want to give our method explained in detail, supplemented by the figures and description.

2 Results

2.1 Method

Our goal is also to give the method of calculation, well described and applicable to all S2-like star orbits. The data we used are publicly available as the supplementary

online data to the electronic version of Gillessen et al. (2009b): observations obtained using NTT/VLT and Keck optical telescopes. These data are collected from: New Technology Telescope (3.6 m, La Silla Observatory, Chile), Very Large Telescope (4×8.2 m, Paranal Observatory, Chile) (Eckart & Genzel 1996) and Keck telescope (2×10 m, Keck Observatory, Hawaii, USA) (Ghez et al. (1998)). First, we performed two-body simulations in the R^n gravity potential. Then, we compare the obtained theoretical results for S2-like star orbits in the R^n potential with the two independent sets of observations of the S2 star: the NTT/VLT and Keck observations.

For the same set of input parameters (t_0, x_0, y_0, v_{x0}, v_{y0}), we calculate Newtonian and modified (in this case R^n) trajectories and find a distance between them. The procedure is the following:

(i) values of initial positions and velocities we choose in that way to obtain orbits with excentricity and period which correspond to S2 star,
(ii) for both potentials, Newtonian and modified, as a starting point we take the pericenter (point at the trajectory which is closest to the focus, i.e. black hole in GC),
(iii) we use ϵ: the maximal allowed deviation between modified and Newtonian trajectory; it depends on the precision of observations,
(iv) after choosing some certain value of ϵ, we calculate theoretical deviation Δr (in AU), and compare it with ϵ (in $''$) multiplied with the distance between the observer and the S2 star,
(v) we search only for those orbits in R^n for which all deviations are less than some given value ϵ during the first orbital period, and in that way we determine the R^n gravity parameter space (r_c,β).

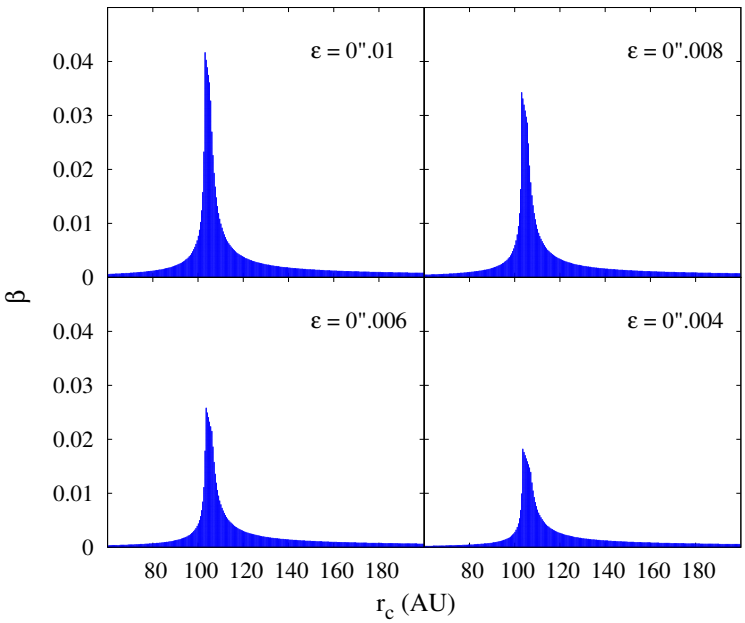


Figure 1 The parameter space (r_c,β) for R^n gravity in case of S2 star, for different values of ϵ: 0.01, 0.008, 0.006 and $0''.004$.

Parameters should be chosen in the way that during the first orbital period (because we have observations only for 1T), the distances of trajectories should not differ more than for some certain value ϵ.

We determined the parameter space (r_c,β) for S2-like stars in R^n gravity (see Fig. 1), assuming the mass of S2-like star to be $M_\star = 1\ M_\odot$, the BH mass, located at GC, to be $M_{BH} = 4.3 \times 10^6\ M_\odot$, and for the distance from the observer to GC we take 8.3 kpc. For stellar orbital elements we used: eccentricity $e = 0.8831$ and major semiaxis $a = 1034$ AU (the values taken from Gillessen et al. (2009b)). For some other values of ϵ, see the parameter space in Borka et al. (2012). From Fig. 1 it can be seen that in the case of S2 star, the parameters are about: $r_c \sim 100$ AU and β is somewhat greater than zero. But in further text it is explained that this inequality is important because it gives the orbits which are obviously different from the $\beta = 0$ (Newtonian) case. Here, it is useful to say that if we assume something different initial parameters (like mass, distance, eccentricity ...), the (r_c,β) space would differ slightly. But, in all cases, the trend is that, as the precision of observations increases (i.e. ϵ becomes smaller), this parameter space significantly reduces.

2.2 S2 star orbits in R^n and Newtonian potential

There is observed precession of orbits of S-stars. R^n gravity potential induces the precession of S2-like star orbits in the opposite direction with respect to GR. It has an effect similar to the extended mass distribution and produces a retrograde shift

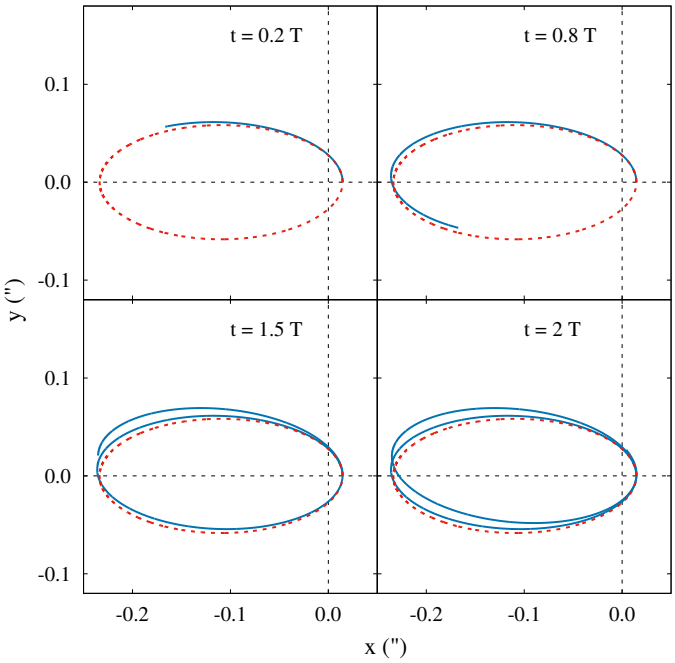


Figure 2 The simulated orbits of S2 star around a massive black hole in R^n gravity (blue solid line) and in Newtonian gravity (red dashed line) for $r_c = 104$ AU and $\beta = 0.03$, for different number of orbital periods: 0.2, 0.8, 1.5 and 2 T.

that results in rosette-shaped orbits (see our paper Borka et al. (2012)). In this paper we present some examples of orbits, in the case of $r_c = 104$ AU and $\beta = 0.03$, in Figs. 2 and 3. In the case when the simulated revolution of S2 star is in positive mathematical direction (counter clockwise), the simulated precession of S2 star orbit has negative mathematical direction (clockwise), and vice versa.

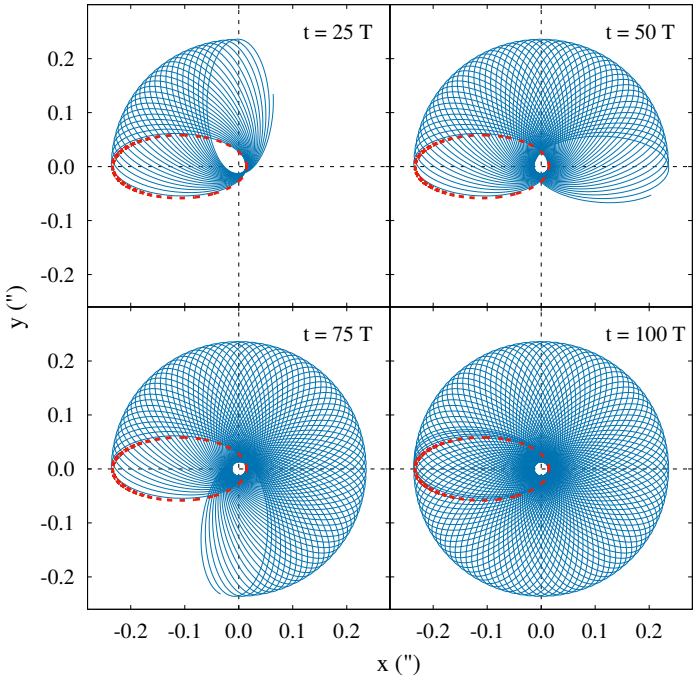


Figure 3 The same as Fig. 2, but for more orbital periods: 25, 50, 75 and 100 T.

We also compare the obtained theoretical results for S2-like star orbits in the R^n potential with two independent sets of observations of the S2 star, obtained by NTT/VLT and Keck telescope (see online data for Gillessen et al. (2009b)). We explained the fitting procedure in Borka et al. (2012). Our comparison here is for $r_c = 104$ AU and $\beta = 0.01$. It is useful to underline that if we want to show the star revolution in the same direction for more orbits, the chosen initial velocities have to be of the same sign: $\mathrm{sgn}(v_{x0,1}) \times \mathrm{sgn}(v_{x0,2}) = 1 = \mathrm{sgn}(v_{y0,1}) \times \mathrm{sgn}(v_{y0,2})$. In order to compare our calculated true orbits with observed positions, orbits have to be projected onto the observer's sky plane i.e. we calculated the apparent orbits (see Eq. (10) in Borka et al. (2012)). We present this in Figs. 4 and 5. When comparing with NTT/VLT observations, we only present the obtained apparent orbit, while in case of Keck data we had to move the apparent orbit for $\Delta\alpha = -2''.5$ and $\Delta\delta = -4''$; this offset is because they use the collection of more data sets and they moved coordinate system when combining Keck data (see for details Gillessen et al. (2009b)).

There is avery good agreement between the theoretical orbit and the observations. Our calculations showed, and it is also noticeable from Figs. 4 and 5, that the Newtonian orbit does not fit the observations very well, and there is a need to include a modified gravity potential.

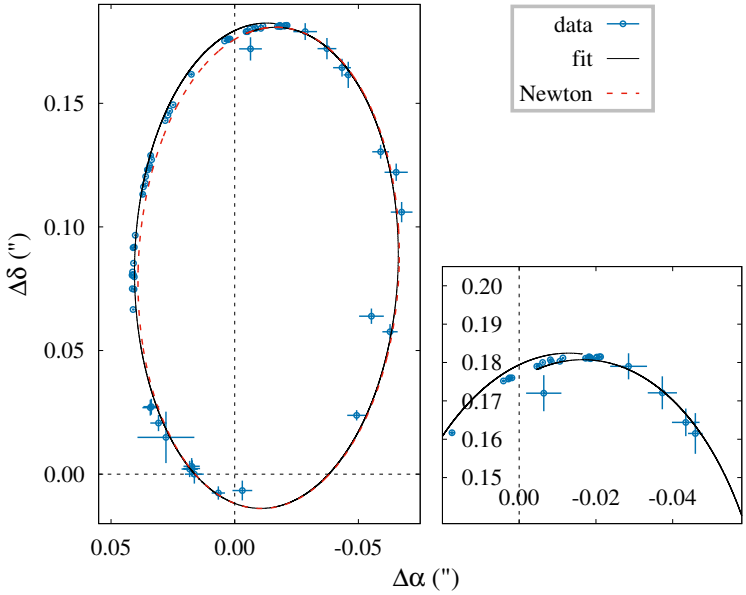


Figure 4 Comparison of the observations (NTT/VLT) and our fit. **(*Left*:)** The fitted orbit (black solid line) and Newtonian orbit (red dashed line) of the S2 star around a massive black hole in R^n gravity for $r_c = 104$ AU and $\beta = 0.01$, with the NTT/VLT astrometric observations added. **(*Right*:)** Zoomed part of the figure, showing precession.

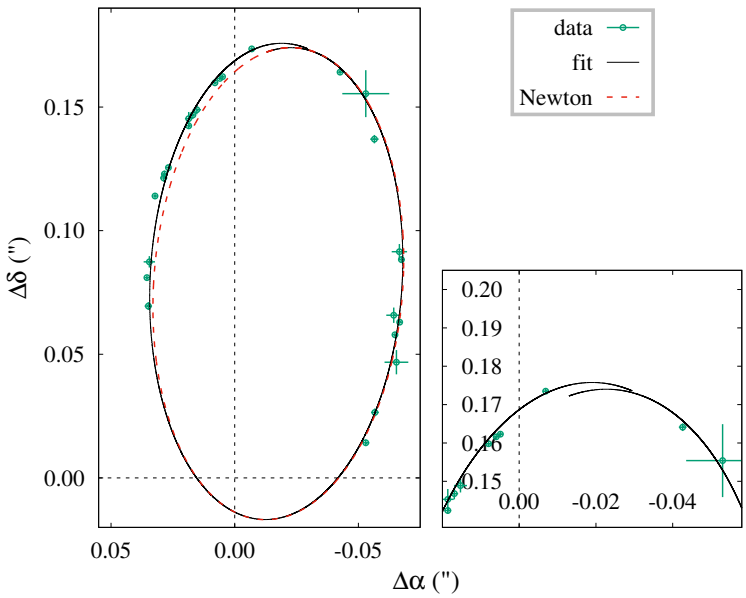


Figure 5 The same as Fig. 4, but for Keck observations.

Conclusions

Our main conclusions are the following:

- $f(R)$ theories of modified gravity represent good alternatives to DM, and they are a good basis on which to construct an effective theory of gravity
- $f(R)$ theories, in particular R^n, can explain (without DM hypothesis) moving of S2 star around SMBH at GC, although the different orbital precession in regard to GR is obtained
- we also described our method of calculation in detail, and it could be applicable to all S2-like star orbits.

For now, GR is the best theory of gravitation with the largest number of experimentally confirmed predictions, but there is a need to review and extend Einstein Relativity to scales where it has not been properly tested, with the aim to better explain observed phenomena at galactic scales, such as flat rotation curves of spiral galaxies, their BTFR, and FP of ellipticals. So, in this sense, our future work would be revising and extending the theory of gravitational interaction in order to overcome the shortcomings of GR at galactic and cosmological scales (thus, to explain observations and phenomenology without introducing "ad hoc" ingredients).

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