Astronomical and Astrophysical Transactions (*AApTr*), 2018, Vol. 30, Issue 3, pp. 343–350,
ISSN 1055-6796, Photocopying permitted by license only, ©Cambridge Scientific Publishers

Fluctuating governing parameters in galaxy dynamo

E.A. Mikhailov*, V.V. Pushkarev

Department of Physics, Moscow M.V. Lomonosov University, Leninskie gori 2, 1, 119991 Moscow, Russia

Received 31 July, 2016

Evolution of galaxy magnetic fields is described by the so-called dynamo theory. The equations include governing parameters that describe alpha-effect, differential rotation and turbulent diffusion. Usually these coefficients are connected with average kinematic characteristics of the interstellar medium. This approach is possible for "calm" galaxies. However, if there is intensive star formation, supernova explosions and other active processes, the parameters of the interstellar gas change. The distribution of the regions where the velocities of the interstellar medium are different is quite random. So it is useful to use the model of the galaxy dynamo with random coefficients. Previous papers described fluctuating alpha-effect, but it is necessary to take into account the turbulent diffusion, too.

We use the so-called no-z approximation to study the magnetic field. It takes into account that the galaxy disc is thin, so the z-derivatives can be changed by algebraic expressions. We describe the fluctuations of turbulent diffusion coefficients.

We have shown that the magnetic field can be changed a lot even for quite small fluctuations of the dynamo governing parameters (associated with weak star formation). For example, the star formation that is several times higher than in the Milky Way can destroy the regular magnetic field.

Keywords: Magnetic fields, fluctuations, dynamo theory, star formation

1 Introduction

Now there is no doubt that some galaxies have regular magnetic fields. Faraday rotation measurement on modern radio telescopes shows that they have values of several μG (Beck et al. 1996). Their evolution is described by the so-called dynamo mechanism that is based on transition of kinematic energy of turbulent motions to the energy of the magnetic field. This process is connected with joint action of alpha-effect (characterizes turbulent motions) and differential rotation. They compete with turbulent diffusion that makes the field decay. If the alpha-effect and differential rotation are intensive, the field grows, otherwise it decreases (Arshakian et al. 2009).

*Email: ea.mikhajlov@physics.msu.ru

To model the magnetic field we use the so-called no-z model. It takes into account that the galaxy disc is very thin. So we can neglect the component of the magnetic field that is perpendicular to the galaxy disc plane. Also we can change some partial derivatives of the magnetic field and alpha-effect by algebraic expressions (Moss 1995; Phillips 2001).

The dynamo theory equations include some average characteristics of the interstellar medium. This approach is quite possible for "calm" galaxies where the parameters of the turbulence do not change very much. However, there are a lot of galaxies which have intensive star formation, supernova explosions, outflows from stars etc (Mikhailov et al. 2012). So we should use for such objects some special models.

Some previous works used the parameterizations of these characteristics, that connected their values with star formation rate or star formation surface density. It was obtained that for weak star formation its influence on the magnetic field can be neglected, but if it reaches some critical value (it is a few times higher than the star formation rate in the Milky Way), the field decays. The field regenerates only after the end of the starburst (Mikhailov et al. 2012, Mikhailov 2014).

Another possible way is connected with using the model with random coefficients. The regions of ionized medium exist for a short time and their evolution can be approximately described by stochastic laws. We can describe the alpha-effect fluctuations and obtain similar results (Mikhailov & Modyaev 2015). This is not the only way: the turbulent motions are connected not only with the alpha-effect. They also change the diffusion processes. So it is important to use a model of the magnetic field with fluctuations of the turbulent diffusion coefficient.

2 Dynamo model

The magnetic field growth is based on the alpha-effect and the differential rotation. The alpha-effect transforms the angular component of the field B_φ to the radial one B_r. The differential rotation transforms the radial component B_r to the angular one B_φ (Beck et al. 1996).

The evolution of the magnetic field can be described by the Steenbeck–Krause–Raedler equation (Moffat 1978):

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}(\alpha \mathbf{B}) + \mathrm{rot}\,[\mathbf{V},\mathbf{B}] + \eta \Delta \mathbf{B}, \tag{1}$$

where α characterizes the alpha-effect, $\mathbf{V}$ is the velocity of the large-scale motions, η is the turbulent diffusion coefficient that can be calculated as (Arshakian et al. 2009):

$$\eta = lv/3,$$

where l is the typical length of the turbulence, v is the typical velocity of the small-scale motions (describes the turbulence).

If we take into account that the galaxy disc is very thin, we can use for the magnetic field the following model (Phillips 2001):

$$\mathbf{B}(t, r, \varphi, z) = \mathbf{B}(t, r, \varphi, 0) \cos\left(\frac{\pi z}{2h}\right),$$

where h is the galaxy disc thickness. So the second partial derivative of the magnetic field in the disc plane will be the following (Phillips 2001):

$$\frac{\partial^2 \mathbf{B}}{\partial z^2} = -\frac{\pi^2}{4h^2}\mathbf{B}.$$

For the alpha-effect we can use the approximation:

$$\alpha(t, r, \varphi, z) = \alpha(t, r, \varphi, 0)z/h,$$

so its z-derivative in the disc plane is:

$$\frac{\partial \alpha}{\partial z} = \frac{\alpha}{h}.$$

As for the regular motions we can assume

$$\mathbf{V} = r\Omega \mathbf{e}_\varphi,$$

where Ω is the angular velocity of the galaxy rotation. Below we will also assume that the field has the angular symmetry (Phillips 2001).

If we neglect the z-component of the magnetic field and describe its derivative by the solenoidality condition, equation (1) can be written as follows (Moss 1995; Phillips 2001):

$$\frac{\partial B_r}{\partial t} = -\frac{\alpha B_\varphi}{h} - \eta\frac{\pi^2 B_r}{4h^2} + \eta\Delta_r B_r, \tag{2}$$

$$\frac{\partial B_\varphi}{\partial t} = -r\frac{\partial \Omega}{\partial r}B_r - \eta\frac{\pi^2 B_\varphi}{4h^2} + \eta\Delta_r B_\varphi. \tag{3}$$

To simplify the model we can use the local approach. We can neglect the Laplacian in the right part in equations (2)–(3), so the partial time derivatives will become total ones. If we use the dimensionless time measured in h^2/η, the equations can be written as (Phillips 2001):

$$\frac{dB_r}{dt} = -R_\alpha B_\varphi - \frac{\pi^2}{4h^2}B_r, \tag{4}$$

$$\frac{dB_\varphi}{dt} = -R_\omega B_r - \frac{\pi^2}{4h^2}B_\varphi, \tag{5}$$

where R_α described alpha-effect and R_ω characterizes differential rotation. The typical values of these parameters are $R_\alpha \sim 1$, $R_\omega \sim 10$ (Moss 1995).

The magnetic field cannot grow infinitely. Its generation is explained by the transition of the kinematic energy of turbulent motions to the field energy (Molchanov et al. 1985). So if these energies have the similar values (Arshakian et al. 2009):

$$\frac{B^{*2}}{8\pi} = \frac{\rho v^2}{2},$$

the field growth will stop. The equipartition value of the field will be:

$$B^* = 2v\sqrt{\pi\rho}.$$

For most galaxies the equipartition field is about several μG (Beck 1996; Arshakian 2009).

If we measure the field in equipartition values, this effect can be included into the equations:

$$\frac{dB_r}{dt} = -R_\alpha B_\varphi (1 - B_r^2 - B_\varphi^2) - \frac{\pi^2}{4h^2} B_r, \tag{6}$$

$$\frac{dB_\varphi}{dt} = -R_\omega B_r - \frac{\pi^2}{4h^2} B_\varphi. \tag{7}$$

We will describe both the linear and nonlinear models.

3 Fluctuations of the diffusion coefficient

If we describe the magnetic field in the galaxies with intensive star formation, we should take into account that there are a lot of regions of ionized hydrogen where the velocity of the turbulent motions will differ. We assume that $v = 10$ km/s with probability $(1 - p)$ and $v = 30$ km/s with probability p.

The turbulent diffusion coefficient that is proportional to this velocity will change too. If we define the value of this coefficient in "calm" galaxies as η_0, its value in regions of ionized hydrogen will be $\eta_1 = 3\eta_0$. If we use as the time unit h^2/η_0, equations (4)–(5) will be rewritten as:

$$\frac{dB_r}{dt} = -R_\alpha B_\varphi - kB_r; \tag{8}$$

$$\frac{dB_\varphi}{dt} = -R_\omega B_r - kB_\varphi; \tag{9}$$

where $k = k_0 = 2.5$ with probability $(1 - p)$, $k = k_1 = 7.5$ with probability p. The probability p can describe the volume fraction of the ionized hydrogen in the galaxy. If we measure the star formation surface density Σ in $M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$, the probability will be (Mikhailov & Modyaev 2015):

$$p \approx 12\Sigma.$$

We assume that the turbulent diffusion coefficient takes the constant value for some small time Δt and after that renews.

The field grows exponentially with velocity:

$$\gamma = -k \pm \sqrt{R_\alpha R_\omega}.$$

The main features of the field will be described by the highest value. If we take $R_\alpha = 1$ and $R_\omega = 10$, the values for k_0 and k_1 will be:

$$\gamma_0 = 0.66,\ \gamma_1 = -4.33.$$

The magnetic field for large time is:

$$B(n\Delta t) = B(0) \exp(\gamma(0)\Delta t) ... \exp(\gamma((n-1)\Delta t)).$$

With probability $C_n^m p^m (1-p)^{n-m}$ the field is:

$$B(n\Delta t) = B(0)\exp(m\gamma_1\Delta t)\exp((n-m)\gamma_0\Delta t).$$

Using these values, we can average the field and its square. The mean value of the field is:

$$< B(n\Delta t) >= B(0)\exp(\gamma_0 n\Delta t)(1-p+p\exp((\gamma_1-\gamma_0)\Delta t))^n,$$

the mean square field is:

$$< B^2(n\Delta t) >^{1/2}= B(0)\exp(\gamma_0 n\Delta t)(1-p+p\exp(2(\gamma_1-\gamma_0)\Delta t))^{n/2}.$$

So the typical field will grow with velocity:

$$\lambda_0 = p\gamma_1 + (1-p)\gamma_0.$$

The mean field velocity growth is:

$$\lambda_1 = p\gamma_1 + (1-p)\gamma_0 + p(1-p)(\gamma_0-\gamma_1)^2\Delta t/2,$$

the mean square field velocity growth is:

$$\lambda_2 = p\gamma_1 + (1-p)\gamma_0 + p(1-p)(\gamma_0-\gamma_1)^2\Delta t.$$

So the higher momentums grow faster than lower ones. It is called the intermittency effect (Zel'dovich et al. 1987).

The numerical value of the typical magnetic field velocity growth is:

$$\lambda_0 = 0.66 - 5p.$$

So the critical value of the probability is $p_{cr} = 0.13$. If $p < p_{cr}$, the magnetic field growth becomes lower, but the field is generated. If $p > p_{cr}$, the field decays. As for the star formation rate, $\Sigma = 0.011\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$. These results should be checked numerically.

4 Numerical modelling

We have modelled the magnetic field numerically. The typical solutions for different Σ are shown in Fig. 1. The different momentums of the field (typical solution, mean field and mean square field) are shown in Fig. 2. The velocities are described in Table 1. We can see that there is an intermittency effect.

We can say that the critical value for the probability is a bit higher than the asymptotic estimate ($p_{cr} = 0.16$, $\Sigma = 0.013\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$. This value is several times higher than the one in the Milky Way.

The nonlinear solutions are shown in Fig. 3.

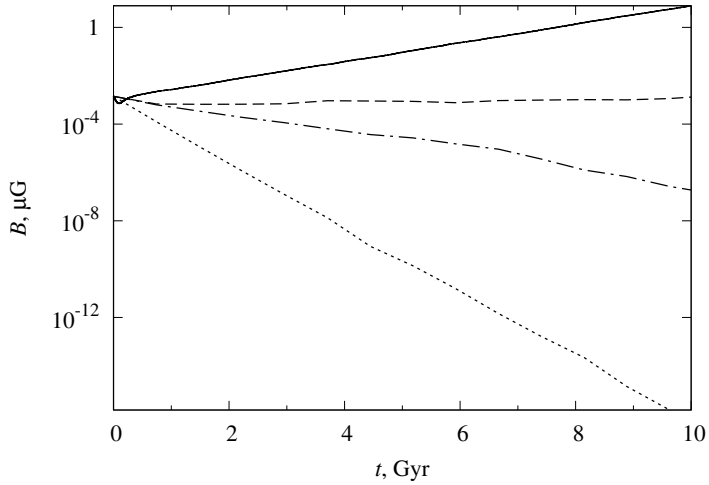


Figure 1 Typical solutions for different star formation densities in linear case. Solid line shows $\Sigma = 0$, long-dashed line – $\Sigma = 0.01\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$, dot-dashed line $\Sigma = 0.02\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$, short-dashed line – $\Sigma = 0.05\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$.

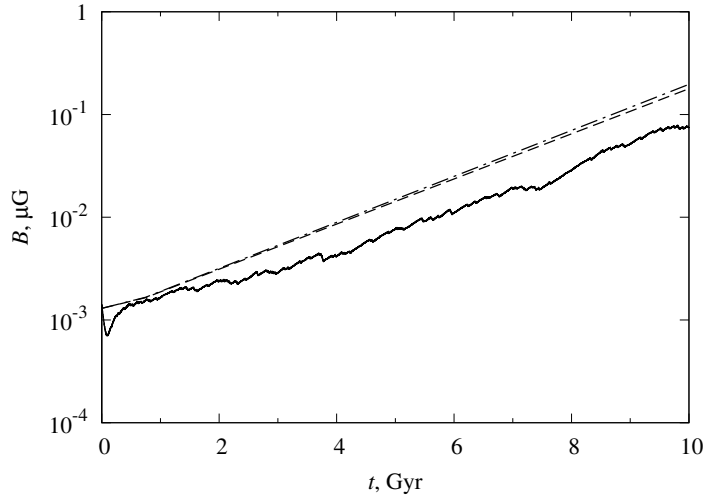


Figure 2 Different momentums of the field for $\Sigma = 0.005\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$. Solid line shows typical solution, dashed line – mean field, dot-dashed line – mean-square field.

Table 1 Velocities of the magnetic field growth (in Gyr^{-1}).

	Theoretical estimates			Numerical results		
$\Sigma, M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$	B	$< B >$	$< B^2 >^{1/2}$	B	$< B >$	$< B^2 >^{1/2}$
0	0.892	0.892	0.892	0.899	0.899	0.899
0.01	0.081	0.100	0.120	0.074	0.116	0.135
0.02	-0.730	-0.696	-0.663	-0.782	-0.680	-0.650
0.05	-3.16	-3.12	-3.07	-3.18	-3.10	-3.06

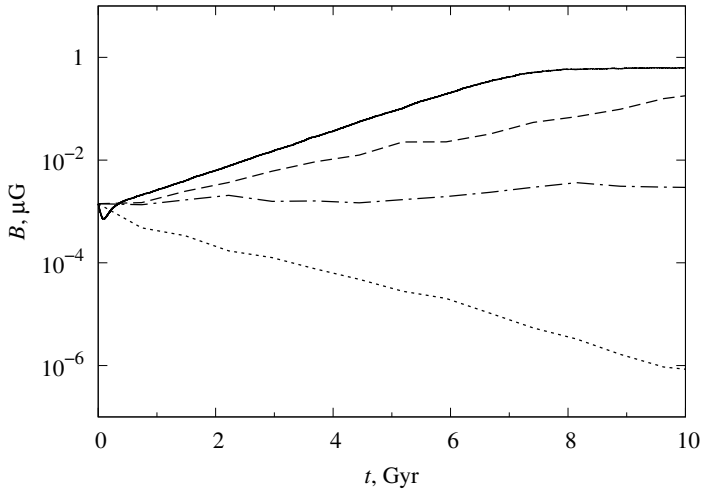


Figure 3 Typical solutions for different star formation densities in nonlinear case. Solid line shows $\Sigma = 0$, long-dashed line – $\Sigma = 0.005\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$, dot-dashed line $\Sigma = 0.01\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$, short-dashed line – $\Sigma = 0.02\ M_\odot \mathrm{kpc}^{-2}\mathrm{yr}^{-1}$.

Conclusion

We have described a model for the galaxy magnetic field with fluctuating turbulent diffusion coefficient. We have shown that for large fluctuations the field will stop its generation. We have also demonstrated the intermittency effect: mean square field grows faster than mean field and the typical solution.

We suggested a possible way to use this model for describing the mechanism of the influence of the star formation on the galaxy magnetic field. It was obtained that for small intensity of the star formation it does not change the **large-scale component of the** magnetic field very much. But if the intensity reaches some critical value, the field decays. It is quite similar to the results obtained in the deterministic model (Mikhailov 2014). **Of course, we do not speak about the small-scale magnetic field, that has typical scale-length of several tens of parsecs. It is quite possible that this component can even grow in the galaxies with active star formation (Condon 1992).** We **also** emphasize that it is not the only way to study this effect. Maybe we should take into account fluctuations of both the turbulent diffusion coefficient and the alpha-effect coefficient.

Acknowledgements We thank the anonymous referee for useful recommendations. The reported study was supported by RFBR, research project No. 16-32-00056 mol_a.

References

Arshakian T., Beck R., Krause M., Sokoloff D. (2009) A&A 494, 21
Beck R., Brandenburg A., Moss D. et al. (1996) ARA&A 34, 155

Condon J.J. (1992) ARAA 30, 575
Mikhailov E.A. (2014) AstL 40, 398 (PAZh 40, 445)
Mikhailov E.A., Modyaev I.I. (2015) BaltA 24, 194
Mikhailov E. A., Sokoloff D. D., Efremov Yu. N. (2012) AstL 38, 543 (PAZh 38, 611)
Moffat H.K. (1978) Magnetic field generation in electrically conducting fluids. Cambridge: Cambridge University Press.
Molchanov S.A., Ruzmaikin A.A., Sokolov D.D. (1985) Sov. Phys. Usp. 28, 307 (UFN 145, 4, 593)
Moss D. (1995) MNRAS 275, 191
Phillips A. (2001) GApFD 94, 135
Zel'dovich Ya.B., Molchanov S.A., Ruzmaikin A.A., Sokolov D.D. (1987) Sov. Phys. Usp. 30, 353 (UFN 152, 5, 3)