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Inverse bremsstrahlung in dwarf atmospheres: the absorption coefficients and Gaunt factors

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In this work we determine the electron–ion inverse bremsstrahlung characteristics for the case of the white dwarf atmospheres where such plasma characteristics as plasma density and temperature change in wide region. It is presented that determination of these characteristics, i.e. the absorption coefficients and Gaunt factors can be successfully performed within the whole range of electron densities and temperatures which is relevant for the corresponding atmospheres. The results are obtained for the DB white dwarf models in the wavelength region 100 nm $< \lambda <$ 3000 nm. These results can be of interest and use in investigation of different stellar and laboratory plasmas.

Keywords: Atomic processes, plasmas, stars: atmospheres

1 Introduction

Our previous study was devoted to the investigation of the opacity of the weakly ionized parts of the solar photospheres, lower chromospheres and different helium-rich white dwarf atmospheres. In this context processes with participation of ions and atoms of hydrogen were treated in the photosphere and lower chromospheres of the Sun (for example, see Mihajlov et al. (1986); Ignjatović et al. (2014a); Srećković et al. (2014)) and processes with participation of helium ion and atoms were treated in the helium-rich white dwarf atmospheres (for example, see Mihajlov & Dimitrijević (1992); Mihajlov et al. (2013); Ignjatović et al. (2014b)). The conclusion of this investigation was that these processes can be very important due to opacity of weakly ionized layers of the considered stellar atmospheres. In accordance with the just mentioned the regions of the stellar atmosphere, where the ion–atom radiative processes are significant, were only weakly ionized layers of these atmospheres. Practically this excludes the possibility of influence of the ion–atom radiative processes in the regions

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with the higher degree of ionization. As it is also known, the increase of the ionization degree is connected with the fast increase (proportional to square of electron density) of absorption caused by the relevant free–free (ff) radiative processes, i.e. the processes of electron–ion inverse bremsstrahlung.

After examining the existing literature (Berger, 1956; Karzas & Latter, 1961; D'yachkov, 1990; Hazak et al., 2002; van Hoof et al., 2014; Armstrong et al., 2014), it could be seen that most of presented papers are devoted to the determination of the corresponding Gaunt factor for the inverse bremsstrahlung process, which reflect the real situation in this field. Namely, as it is known the exact relation for the differential cross section for the direct bremsstrahlung process has existed for a long time (Sommerfeld, 1953). This automatically led to the possibility of the exact term for the inverse bremsstrahlung cross section $\sigma_{i.b.}^{(ex)}(E, \varepsilon_{ph})$, for the considered here inverse bremsstrahlung in the case of the single charged ions. However, the practical applicability of the mentioned strict relation was complex, which suggested the search for practical ways to describe inverse bremsstrahlung. Therefore, the existence of simple and widely used quasi classical (Kramer's) cross section for the direct and inverse bremsstrahlung process has been taken into account. The meaningful idea was in the presentation of $\sigma_{i.b.}^{(ex)}(E, \varepsilon_{ph})$ in a form

$$\sigma_{i.b.}^{(ex)}(E, \varepsilon_{ph}) = \sigma_{i.b.}^{q.c.}(E, \varepsilon_{ph}) \cdot g_{i.b.}(E, \varepsilon_{ph}), \tag{1}$$

where $g_{i.b.}(E, \varepsilon_{ph})$ is the adequately defined Gaunt factor. Because of that, a further step was to obtain a simple approximation for the Gaunt factor $g_{i.b.}(E, \varepsilon_{ph})$ in relation (1). Here, the mentioned exact and approximate absorption coefficients are denoted with $k_{i.b.}^{(ex)}(\lambda, T; N_e)$ and $k_{i.b.}^{q.c.}(\lambda, T; N_e)$, where N_e is free electron density. It is taken into account that in our case (single charged ion) the ion density is equal to electron density N_e. Consequently, those coefficients are connected with the relation

$$k_{i.b.}^{(ex)}(\lambda, T; N_e) = k_{i.b.}^{q.c.}(\lambda, T; N_e) \cdot G_{i.b.}(\lambda, T), \tag{2}$$

where $G_{i.b.}(\lambda, T)$ is the required Gaunt factor. The aim of this work is to find such relations for $\sigma_{i.b.}^{(ex)}(E, \varepsilon_{ph})$ which could be applicable in the case of different stellar atmospheres, where the values of electron density change from $N_e \sim 10^{13}$ cm^{-3} to $N_e \sim 10^{20}$ cm^{-3}. This range of electron densities includes the region which is usually characterized as higher density one. The derivation of the $\sigma_{i.b.}^{(ex)}$ which is applicable in this region represents a separate problem, and a relatively small number of papers was devoted to solving this problem.

2 Theory

2.1 Screening model of the considered plasma

Since the main aim of this work is the determination of the absorption coefficient $k_{i.b.}^{(ex)}(\lambda, T; N_e)$ and Gaunt factor $G_{i.b.}(\lambda, T)$ which could be applicable to the case of higher density plasmas, at this point it is needed to make an observation on the methods of determination in previous papers devoted to the dense plasma. Namely,

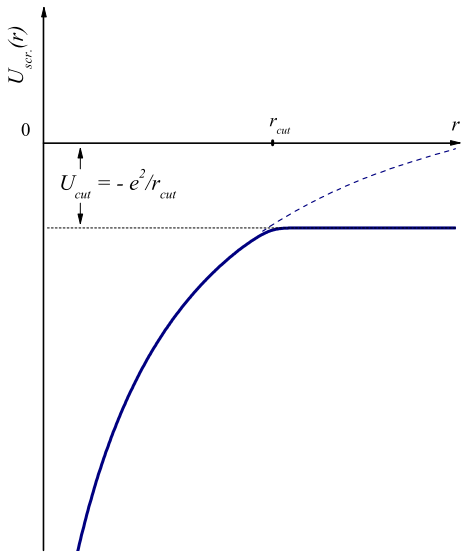


Figure 1 Behaviour of potential $U_{scr.}(r)$, where r_{cut} is the cut-off parameter.

the electron–ion scattering is treated there as a scattering of the electron onto the adequate Debye potential. However, in Mihajlov et al. (1986), as well as in our previous work (Mihajlov et al., 2011a,b), it was already taken into account that the model of the plasma with the scattering of the electron on Debye potential is not described adequately in the electron–ion scattering models. Because of that, Mihajlov et al. (2011a) applied the model of plasma which is illustrated here in Fig. 1. This figure implies that in this screening model the potential energy of free electron $U_{scr.}(r)$ is strictly Coulomb: $U_{scr.}(r) = -e^2/r$ in the region $r < r_{cut}$, where r_{cut} is the corresponding cut-off radius, and $U_{scr.}(r) = const = U_{cut}$ in the region $r > r_{cut}$, where U_{cut} is equal to the average energy of the free electron in the considered system. In order to complete the described model we have to determine the cut-off parameter r_{cut} and the energy parameter U_{cut}. Here we will use the fact that these parameters can be determined using Fig. 2 from Mihajlov et al. (2011a).

Figure 1 shows that in this model the cut-off radius is determined so that mentioned mean energy of free electrons in the area $r > r_{cut}$ is equal to $-e^2/r_{cut}$. In further considerations we take $-e^2/r_{cut}$ as zero of energy and describe such a plasma using model cut-off potential described by the relations

$$U_{cut}(r) = \begin{cases} -\dfrac{e^2}{r} + \dfrac{e^2}{r_{cut}} & , \quad 0 < r \le r_{cut} \\ 0 & , \quad r_{cut} < r \end{cases} , \tag{3}$$

which is specially adopted for describing the electron ion scattering inside plasma. Here, we use the procedure of the determination of inverse bremsstrahlung cross-section which is very improved in comparison to one from Mihajlov et al. (2015). By means of this procedure in this work we have performed calculations of inverse bremsstrahlung Gaunt factor and (ff) spectral absorption coefficients in the wide

range of electron densities and temperatures which can be used in connection with the different stellar plasmas.

2.2 Inverse bremsstrahlung cross section

Here, $W_{E;l}(r)$ is the full radial function of the considered system, where r is the distance from the beginning of the coordinate system. We will use this function in the usual form

$$W_{E;l}(r) = \frac{P_{E;l}(r)}{r}, \tag{4}$$

where $P_{E;l}(r)$ is the solution of the radial Schrödinger equation.

Keeping in mind the fact that the potential (3) is of finite radius, for determination of the cross section $\sigma_{i.b.}^{(ex)}$ for the inverse bremsstrahlung process we can use the standard expressions from Sobelman (1979),

$$\sigma_{i.b.}^{(ex)}(E;E') = \frac{8\pi^4}{3}\frac{\hbar e^2 k}{q^2}\sum_{l'=l\pm1} l_{max}|\hat{D}_{E,l;E'l'}|^2, \tag{5}$$

where matrix element $\hat{D}_{E,l;E'l'}$ is given by

$$\hat{D}_{E,l;E'l'} = \int_0^\infty P_{E';l'}(r)\cdot r\cdot P_{E;l}(r)dr. \tag{6}$$

The radial function $P_{E;l}(r)$ is the solution of the Schrödinger equation

$$\frac{d^2P_{E;l}(r)}{dr^2} + \left[\frac{2m}{\hbar^2}\left(E - U_{cut}(r)\right) - \frac{l(l+1)}{r^2}\right]P_{E;l}(r) = 0, \tag{7}$$

and $U_{cut}(r)$ is cut-off Coulomb potential given by Eq. (3). In the considered case we need only the function of the free states. The radial functions of such states are presented here. Let us note that cut-off Coulomb potential model gives the possibility of direct determination of the cross section for the inverse bremsstrahlung process without any additional approximations. For that purpose we use in Eq. (5) the matrix element of the gradient of the potential energy given by

$$|\hat{D}(r)_{E,l;E'l'}|^2 = \frac{\hbar^4}{m^2\left(E-E'\right)^4}|\nabla U_{E,l;E'l'}|^2, \tag{8}$$

$$\nabla_r U_{E,l;E'l'} = \int_0^{r_{cut}} P_{E,l}(r)\cdot\nabla_r U(r)\cdot P_{E'l'}(r)dr. \tag{9}$$

Namely, in the case of the cut-off potential (3) in the last expression integration over r is carried out only in the interval from 0 to r_{cut}. The given method enabled the fast and reliable calculation of the discussed cross section and, consequently, the corresponding spectral absorption coefficient.

From Eq. (3) one can see that here these functions $P_{E;l}(r)$ are described in different ways in the three following cases: $r < r_{cut}$, $0 \le E < e^2/r_{cut}$ case **a**); $r < r_{cut}$, $E > e^2/r_{cut}$ case **b**); $r > r_{cut}$, $E > 0$ case **c**). For the determination of these

solutions we use the possibility which the standard mathematical library provides (GSL) (Galassi et al., 2015). By means of above described equations for the radial function $P_{E;l}(r)$ is determined inverse bremsstrahlung cross section $\sigma_{i.b.}^{(ex)}(E;E')$ given by Eqs. (5), (8) and (9) for all relevant $l \geq 0$. We sum over l from 0 to l_{max}. We especially developed the procedure of l_{max} determination.

2.3 Inverse bremsstrahlung absorption coefficient

In accordance with the above text we will look for $\kappa_{i.b.}^{(ex)}$ in the form

$$\kappa_{i.b.}^{(ex)}(\varepsilon_\lambda; N_e, T) = N_e^2 \cdot \int_0^\infty \sigma_{i.b.}^{(ex)}(E;E')v \cdot f_T(v) \cdot 4\pi v^2 dv \cdot \left(1 - \exp\left[-\frac{\hbar\omega}{kT}\right]\right), \tag{10}$$

where $f_T(v)$ is the corresponding Maxwellian distribution function for a given temperature T, and the expression in parentheses was introduced in order to take into account the effect of stimulated emission. The quasi classical Kramer's $k_{i.b.}^{q.c.}(\lambda, T; Ne)$ is given by the known expression (Sobelman (1979))

$$k_{i.b.}^{q.c.}(\lambda, T; Ne) = N_e^2 \cdot \frac{16\pi^{5/2}\sqrt{2}e^6}{3\sqrt{3}cm^{3/2}\varepsilon_{ph}^3} \frac{\hbar^2}{(kT)^{1/2}} \left(1 - \exp\left[-\frac{\hbar\omega}{kT}\right]\right), \tag{11}$$

where $\varepsilon_{ph} = 2\pi\hbar c/\lambda$. According to this, averaged Gaunt factor $G_{i.b.}(\lambda, T)$ is determined here from Eq. (2), where $k_{i.b.}^{(ex)}(\lambda, T; Ne)$ and $k_{i.b.}^{q.c.}(\lambda, T; Ne)$ are given by Eqs. (10) and (11).

Within this work the ff absorption coefficients $\kappa_{i.b.}^{(ex)}(\varepsilon_\lambda; N_e, T)$ are determined by Eq. (10), where the spectral absorption cross section $\sigma_{i.b.}^{(ex)}(E;E')$ is given by Eqs. (5), (8) and (9) with the radial function $P_{E;l}(r)$ taken directly from Mihajlov et al. (1986).

3 Results and discussion

In this work, the calculations of the Gaunt factor $G_{i.b.}(\lambda, T)$ and absorption coefficients $k_{i.b.}^{(ex)}(\lambda, T; Ne)$ were carried out for the electron densities in the range from $1 \cdot 10^{13}\,\mathrm{cm}^{-3}$ to $3 \cdot 10^{19}\,\mathrm{cm}^{-3}$ and temperatures from $3 \cdot 10^3\,\mathrm{K}$ to $1 \cdot 10^5\,\mathrm{K}$, in the observed wavelength region 100 nm $< \lambda <$ 3000 nm covering the corresponding dwarf atmosphere models. This is illustrated by Figs. 2 and 3 where the plots of considered absorption processes including the inverse bremsstrahlung processes for the case of a DB white dwarf with $\log g = 8$, $T_{\mathrm{eff}} = 12000$ K and $T_{\mathrm{eff}} = 16000$ K from the atmosphere model of Koester (2015, private communication) are shown. From these figures one can see that within the considered DB white atmospheres investigated ff radiative processes strongly influence the atmosphere opacity, especially in the cases of white dwarf atmospheres with the higher effective temperature.

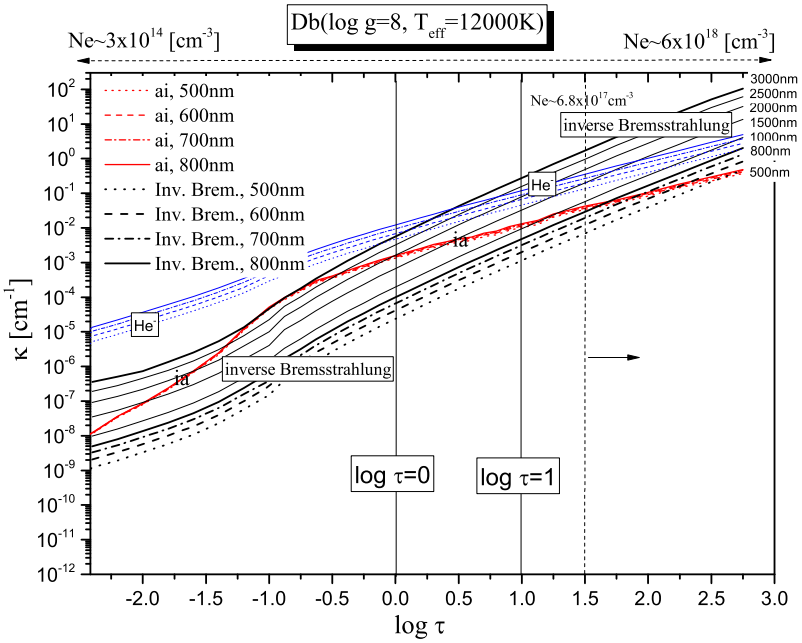


Figure 2 Absorption coefficients of considered absorption processes for the case of a DB white dwarf with $T_{\rm eff} = 12000$ K and $\log g = 8$.

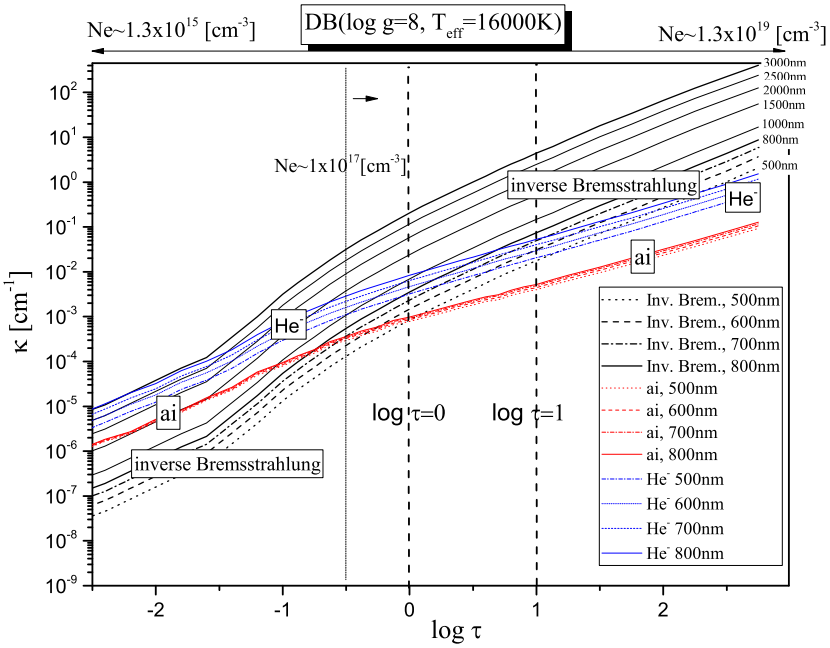


Figure 3 Same as in Fig. 2 but for the case of a DB white dwarf with $T_{\rm eff} = 16000$ K and $\log g = 8$.

To conclude, we highlight that the presented exact quantum mechanical method can be used to obtain the spectral coefficients for inverse bremsstrahlung process for the broad class of weakly non-ideal plasmas as well as for plasma of higher non-ideality. It is expected that the cut-off Coulomb potential model results are more accurate in comparison with other methods, which is so far used for cases of stellar plasma with higher non-ideality. Also, the presented method can be of interest in connection with the application in the cases of some laboratory plasmas.

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